Solucionario

Trigonometría

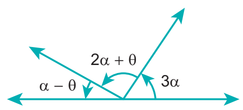
4.°

Unidad 1

ÁNGULO TRIGONOMÉTRICO
SISTEMAS DE MEDIDAS ANGULARES

APLICAMOS LO APRENDIDO (página 6) Unidad 1

1. Colocando los ángulos en sentido antihorario:



Del gráfico se tiene:

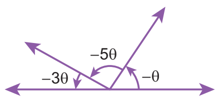
$(\alpha - \theta) + (2\alpha + \theta) + (3\alpha) = 180°$

$6\alpha = 180°$

$\therefore \alpha = 30°$

Clave C

2. Colocando los ángulos en sentido antihorario:



Del gráfico se tiene:

$(-3\theta) + (-5\theta) + (-\theta) = 180°$

$-9\theta = 180°$

$\therefore \theta = -20°$

Clave C

3. $E = \dfrac{1°\,2'}{2'} + \dfrac{2^g\,1^m}{1^m}$

$E = \dfrac{1° + 2'}{2'} + \dfrac{2^g + 1^m}{1^m}$

$E = \dfrac{(60') + 2'}{2'} + \dfrac{(200^m) + 1^m}{1^m}$

$E = \dfrac{62'}{2'} + \dfrac{201^m}{1^m} = 31 + 201$

$\therefore E = 232$

Clave D

4. $\dfrac{\pi}{125}\text{ rad} = \dfrac{\pi}{125}\text{ rad}\left(\dfrac{180°}{\pi\text{ rad}}\right) = \dfrac{180°}{125} = 1{,}44°$

Luego:

$1{,}44° = 1° + 0{,}44°\left(\dfrac{60'}{1°}\right) = 1° + 26{,}4'$

$1{,}44° = 1° + 26' + 0{,}4'\left(\dfrac{60''}{1'}\right)$

$1{,}44° = 1° + 26' + 24''$

$1{,}44° = 1°\,26'\,24''$

$\therefore \dfrac{\pi}{125}\text{ rad} = 1°\,26'\,24''$

Clave A

5. $(3x)° + \left(\dfrac{20x}{3}\right)^g = \dfrac{\pi}{2}\text{ rad}$

$(3x)° + \left(\dfrac{20x}{3}\right)^g \cdot \left(\dfrac{9°}{10^g}\right) = \dfrac{\pi}{2}\text{ rad}\left(\dfrac{180°}{\pi\text{ rad}}\right)$

$(3x)° + (6x)° = 90°$

$3x + 6x = 90$

$9x = 90$

$\therefore x = 10$

Clave B

6. $P = \sqrt{\dfrac{C+S}{C-S} - \sqrt[3]{8 + \dfrac{C+S}{C-S}}}$

Sabemos:

$\dfrac{S}{9} = \dfrac{C}{10} \Rightarrow S = 9k \;\wedge\; C = 10k$

Reemplazando en la expresión:

$P = \sqrt{\dfrac{10k+9k}{10k-9k} - \sqrt[3]{8 + \dfrac{10k+9k}{10k-9k}}}$

$P = \sqrt{19 - \sqrt[3]{8+19}}$

$P = \sqrt{19 - \sqrt[3]{27}} = \sqrt{19-3} = \sqrt{16} = 4$

$\therefore P = 4$

Clave C

7. $S = x + 4 \quad \ldots(I)$

$C = x + 5 \quad \ldots(II)$

Dividiendo (I) y (II):

$\dfrac{S}{C} = \dfrac{x+4}{x+5} \Rightarrow \dfrac{9}{10} = \dfrac{x+4}{x+5}$

$\Rightarrow 9x + 45 = 10x + 40$

$45 - 40 = 10x - 9x$

$\therefore x = 5$

Clave D

8. S: n.° de grados sexagesimales.

C: n.° de grados sexagesimales.
Ambos para un mismo ángulo, del enunciado se plantea:

$S = n$

$C = n + 1$

$\Rightarrow C = S + 1$

Se sabe: $\dfrac{S}{C} = \dfrac{9}{10} \Rightarrow \dfrac{S}{S+1} = \dfrac{9}{10}$

$10S = 9S + 9$

$\Rightarrow S = 9$

Entonces el ángulo mide 9°

Ahora: $9° \times \left(\dfrac{\pi\text{ rad}}{180°}\right) = \dfrac{\pi}{20}\text{ rad}$

Por lo tanto, el ángulo mide $\dfrac{\pi}{20}$ rad.

Clave B

9. $M = \dfrac{\pi^2(C-S)(C+S)}{380R^2}$

Sabemos:

$\dfrac{S}{180} = \dfrac{C}{200} = \dfrac{R}{\pi} = k$

$\Rightarrow S = 180k;\ C = 200k;\ R = \pi k$

Reemplazando en la expresión:

$M = \dfrac{\pi^2(200k - 180k)(200k + 180k)}{380(\pi k)^2}$

$M = \dfrac{\pi^2(20k)(380k)}{380\pi^2k^2} = \dfrac{20k^2}{k^2} = 20$

$\therefore M = 20$

Clave C

10. $\dfrac{R+3}{C+S} = \dfrac{C+S}{C^2 - S^2}$

$R + 3 = \dfrac{(C+S)(C+S)}{(C+S)(C-S)}$

$R + 3 = \dfrac{C+S}{C-S}$

Sabemos:

$S = 180k;\ C = 200k;\ R = \pi k$

Reemplazando tenemos:

$(\pi k) + 3 = \dfrac{(200k) + (180k)}{(200k) - (180k)}$

$\pi k + 3 = \dfrac{380k}{20k}$

$\pi k + 3 = 19$

$\pi k = 16 \Rightarrow k = \dfrac{16}{\pi}$

El número de radianes será:

$R = \pi k = \pi\left(\dfrac{16}{\pi}\right) = 16$

Por lo tanto, el ángulo mide 16 rad.

Clave E

11. Del gráfico, OB es bisectriz. Luego:

$x' = \alpha^g\,\theta^m$

$x' = \alpha^g + \theta^m$

$x' = \alpha^g + \left(\theta \cdot \dfrac{1}{100}\right)^g$

$x' = \left(\alpha + \dfrac{\theta}{100}\right)^g$

$x' = \left(\alpha + \dfrac{\theta}{100}\right)^{\cancel{g}} \times \dfrac{9°}{10^{\cancel{g}}}$

$x' = \left(9\alpha + \dfrac{9\theta}{1000}\right)°$

$x' = \left(9\alpha + \dfrac{9\theta}{1000}\right)60'$

$x' = \left(540\alpha + \dfrac{27\theta}{50}\right)'$

$\therefore x = 540\alpha + \dfrac{27\theta}{50}$

Clave B

12. Se tiene:

$x°z' = \left(\dfrac{6^g3^m}{9^m}\right)°\left(\dfrac{5'6''}{17''}\right) = \left(\dfrac{6^g + 3^m}{9^m}\right)°\left(\dfrac{5' + 6''}{17''}\right)$

$x°\,z' = \left(\dfrac{603^M}{9^m}\right)°\left(\dfrac{306''}{17''}\right)'$

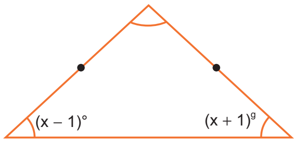
$x°\,z' = 67°\,18'$

$\Rightarrow x = 67 \wedge z = 18$

$\therefore x + z = 85$

Clave C

13. De los datos:



Luego:
$(x-1)° = (x+1)^g$
$(x-1)° = (x+1)^g \cdot \dfrac{9°}{10^g}$

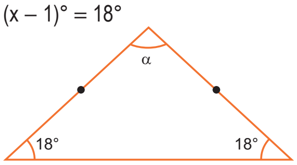
$(x-1)\cancel{°} = \dfrac{9}{10}(x+1)\cancel{°}$

$(x-1) = \dfrac{9x}{10} + \dfrac{9}{10}$

$10x - 10 = 9x + 9$

$\therefore x = 19$

Entonces:
$(x-1)° = 18°$



Sea α el tercer ángulo
$C\alpha = 180° - \alpha$

Del triángulo
$\alpha + 18° + 18° = 180°$
$36° = 180° - \alpha$

$\Rightarrow\ C\alpha = 36\cancel{°} = \dfrac{\pi rad}{180\cancel{°}}$

$\therefore C\alpha = \dfrac{\pi}{5} rad$

Clave A

14. Sea S, C y R los números que representan al ángulo de los sistemas sexagesimal, centesimal y radial de la fórmula general de conversión.

$\dfrac{S}{180} = \dfrac{C}{200} = \dfrac{R}{\pi} = k \qquad \text{... (1)}$

Por dato:

$\dfrac{S + C + R}{3} = 95 + \dfrac{\pi}{4} \qquad \text{... (2)}$

De (1):
$S = 180k,\ C = 200k,\ R = k\pi$

En (2):

$\dfrac{180k + 200k + k\pi}{3} = 95 + \dfrac{\pi}{4}$

$k\left(\dfrac{380 + \pi}{3}\right) = \dfrac{380 + \pi}{4}$

$\Rightarrow k = \dfrac{3}{4}$

Luego:

$\dfrac{S}{180} = k = \dfrac{3}{4} \ \Rightarrow\ \dfrac{S}{180} = \dfrac{3}{4}$

$S = 135$

El número de minutos sexagesimales será 60S
$60S = 60\ (135)$
$60S = 8100$

∴ El número de minutos sexagesimales del ángulo es 8100.

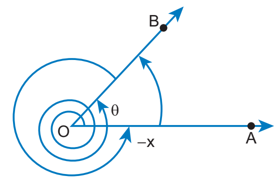
Clave D

PRACTIQUEMOS

Nivel 1 (página 8) Unidad 1

Comunicación matemática

1. Invertimos el sentido de x, además:



Del gráfico:
$-x + m\angle AOB = 360°$
$m\angle AOB = 360° + x$

Finalmente del gráfico:
$\theta - m\angle AOB = 2(m\angle 1 \text{ vuelta})$
$\theta - m\angle AOB = 2(360°)$
$\theta = 720° + m\angle AOB$
$\theta = 720° + 360° + x$
$\therefore x = \theta - 1080°$

Clave E

2. A) Si el triángulo es equilátero:
$m\angle B = m\angle A = m\angle C = 60°$

Luego:

$m\angle B = 60° = 60° \dfrac{\pi}{180°} rad = \dfrac{\pi}{3} rad$

Del gráfico: $m\angle B = x\ rad = \dfrac{\pi}{3}\ rad$

$\therefore x = \dfrac{\pi}{3}$

A es correcta.

B) Si CB = AB entonces:
$m\angle A = m\angle C$
$b^g = a°$

$b^g \cdot \dfrac{9°}{10^g} = a° \Rightarrow \dfrac{9b}{10} = a$

$\therefore \dfrac{a}{b} = \dfrac{9}{10} \ \vee\ \dfrac{b}{a} = \dfrac{10}{9}$

B es correcta.

C) En el triángulo equilátero:

$m\angle C = m\angle B = 60°$

$m\angle C = 60° = a° \ \Rightarrow\ a = 60$

$m\angle B = 60° = x\ rad$

$\dfrac{\pi}{180°}\ rad \cdot 60° = x\ rad$

$x\ rad = \dfrac{\pi}{3} rad \ \Rightarrow\ x = \dfrac{\pi}{3}$

$a + x = 60 + \dfrac{\pi}{3} = \dfrac{180 + \pi}{3}$

C es correcta.

D) Si el lado AC es mayor al lado AB se cumple:
$m\angle B > m\angle C$
$x\ rad > a°$

$x\ rad \cdot \dfrac{180°}{\pi\ rad} > a°$

$\dfrac{x180}{\pi} > a$

$\therefore \dfrac{x}{a} > \dfrac{\pi}{180}$

D es correcta.

E) Asumiendo que el ΔCBA sea equilátero:
$a° = 60° \Rightarrow a = 60$

$x\ rad = \dfrac{\pi}{3}\ rad \Rightarrow x = \dfrac{\pi}{3}$

$b^g = \left(\dfrac{200}{3}\right)^g \Rightarrow b = \dfrac{200}{3}$

Luego:

$a + x + b = 60 + \dfrac{\pi}{3} + \dfrac{200}{3} \approx 127{,}71 < 180 (\Rightarrow\Leftarrow)$

∴ La proposición E es falsa.

Clave E

Razonamiento y demostración

3. $18°\ 54' = 18° + 54'\left(\dfrac{1°}{60'}\right)$

$18°\ 54' = 18° + 0{,}9° = 18{,}9°$

$18°\ 54' = 18{,}9°\left(\dfrac{10^g}{9°}\right) = 21^g$

$\therefore 18°\ 54' = 21^g$

Clave C

4. $E = \dfrac{99° + 0{,}2\pi\ rad}{180^g - 27°}$

Pasamos los términos a un solo sistema angular:

$0{,}2\pi\ rad = 0{,}2\pi\ rad\left(\dfrac{180°}{\pi\ rad}\right) = 36°$

$\Rightarrow 0{,}2\pi\ rad = 36°$

$180^g = 180^g\left(\dfrac{9°}{10^g}\right) = 162°$

$\Rightarrow 180^g = 162°$

Reemplazando en la expresión E:

$E = \dfrac{99° + (36°)}{(162°) - 27°} = \dfrac{135°}{135°} = 1$

$\therefore E = 1$

Clave A

5. $E = \dfrac{1^g}{10^m} + \dfrac{1°}{3'} + \dfrac{1^m}{1^s}$

Sabemos:
$1° = 60';\ 1^g = 100^m;\ 1^m = 100^s$

Reemplazando tenemos:

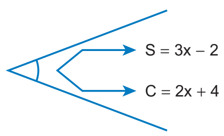
$$E = \frac{(100^m)}{10^m} + \frac{(60')}{3'} + \frac{100^s}{1^s}$$

$E = 10 + 20 + 100 = 130$

$\therefore E = 130$

Clave E

6.



Sabemos: $\frac{S}{9} = \frac{C}{10}$

$\Rightarrow \frac{3x-2}{9} = \frac{2x+4}{10}$

$15x - 10 = 9x + 18$

$6x = 28 \Rightarrow x = \frac{14}{3}$

Entonces:

$S = 3\left(\frac{14}{3}\right) - 2 = 12$

$\Rightarrow S = 12$

La medida del ángulo es entonces 12°.

Piden: la medida del ángulo en radianes.

$\Rightarrow 12°\left(\frac{\pi \text{ rad}}{180°}\right) = \frac{\pi}{15}$ rad

Clave B

7. $C = \frac{2°\,3'}{3'} + \frac{1°\,2'}{2'}$

Sabemos: $1° = 60'$

Entonces:

$C = \frac{2(60') + 3'}{3'} + \frac{(60') + 2'}{2'}$

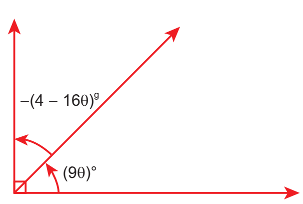
$C = \frac{123'}{3'} + \frac{62'}{2'}$

$C = 41 + 31 = 72$

$\therefore C = 72$

Clave C

8. Colocando los ángulos en sentido antihorario:



Del gráfico se tiene:

$-(4 - 16\theta)^g + (9\theta)° = 90°$

$\Rightarrow (9\theta)° - 90° = (4 - 16\theta)^g\left(\frac{9°}{10^g}\right)$

$9\theta - 90 = \frac{9}{10}(4 - 16\theta)$

$5\theta - 50 = 2 - 8\theta$

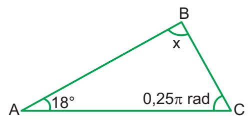
$13\theta = 52$

$\therefore \theta = 4$

Clave C

Resolución de problemas

9.



En el ΔABC se cumple:

$18° + x + 0{,}25\pi \text{ rad} = 180°$

$18° + x + 0{,}25\pi \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right) = 180°$

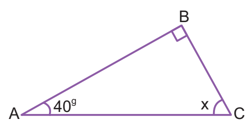
$18° + x + 45° = 180°$

$63° + x = 180°$

$\therefore x = 117°$

Clave C

10.



Piden: x en radianes

En el ⊿ABC se cumple:

$40^g + x = 90° = \frac{\pi}{2}$ rad

$\Rightarrow 40^g \cdot \left(\frac{\pi \text{ rad}}{200^g}\right) + x = \frac{\pi}{2}$ rad

$\frac{\pi}{5}$ rad $+ x = \frac{\pi}{2}$ rad

$\therefore x = \frac{3\pi}{10}$ rad

Clave D

11. De la fórmula: $\frac{S}{9} = \frac{C}{10}$

Reemplazando:

$\frac{2n}{9} = \frac{2n+2}{10}$

$20n = 18n + 18$

$2n = 18$

$n = 9$

Luego:

$(3n)° = 3(9)° = 27° \underbrace{\frac{\pi \text{ rad}}{180°}}_{\text{factor de conversión}}$

$\therefore (3n)° = \frac{3\pi}{20}$ rad

Clave D

12. Por dato:

$\left(\frac{5x}{60}\right)\pi \text{ rad} + \left(\frac{100x}{3}\right)^g = 180°$...(I)

Luego:

$\left(\frac{5x}{60}\right)\pi \text{ rad} = \left(\frac{5x}{60}\right)\pi \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right)$

$\Rightarrow \left(\frac{5x}{60}\right)\pi \text{ rad} = (15x)°$

$\left(\frac{100x}{3}\right)^g = \left(\frac{100x}{3}\right)^g\left(\frac{9°}{10^g}\right)$

$\Rightarrow \left(\frac{100x}{3}\right)^g = (30x)°$

Reemplazando en (I):

$\Rightarrow (15x)° + (30x)° = 180°$

$15x + 30x = 180$

$45x = 180$

$\Rightarrow x = 4$

Piden la medida del mayor de los ángulos.

$\Rightarrow (30x)° = (30 \,.\, 4)° = 120°$

Por lo tanto, el mayor ángulo mide 120°.

Clave E

Nivel 2 (página 9) Unidad 1

Comunicación matemática

13. I. El ángulo b^g posee sentido horario por lo que:

$b^g < 0^g \Rightarrow b < 0$ ...(1)

Además a° gira con sentido antihorario, luego:

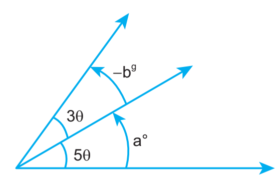
$a° > 0° \Rightarrow a > 0$ ...(2)

Por transitividad:

$b < 0 \wedge 0 < a \qquad \therefore b < a$

I es verdadero.

II. Cambiando el sentido de b^g, se tiene el gráfico:



Luego:

$\frac{a°}{(-b^g)} = \frac{5\theta}{3\theta}$

$3a° = -5b^g$

$3a° = -5b^g \cdot \underbrace{\frac{9°}{10^g}}_{\text{factor de conversión}}$

$a° = \left(-\frac{3b}{2}\right)°$

$a = -\frac{3b}{2}$

$2a = -3b \qquad \therefore 2a + 3b = 0$

II es verdadero.

III. Si: $\theta = 15°$

$a° = 5\theta \Rightarrow a = 75$

$-b^g = 3\theta \Rightarrow -b^g = 45°$

Luego

$b^g = -45°$

$b^g \cdot \frac{9°}{10^g} = -45° \Rightarrow b = -50$

Finalmente:

$a - b = 75 - (-50) = 125$

III es falso.

Clave B

14.

A) En la notación de grados, minutos y segundos de un ángulo, con los valores de minutos y segundos sexagesimales se cumple:

Si: $\alpha = a° \, b' \, c''$; $b, c \in [0; 60\rangle$

Además: $a; b; c \in \mathbb{Z}$

A es incorrecta.

B) De lo anterior, g se encuentra en el intervalo $[0; 60\rangle$

B es incorrecta.

C) Se sabe que f y g son enteros tal que:

$f, g \in [0; 60\rangle$

Entonces: $f_{máx.} = 59$; $g_{máx.} = 59$

La suma de los valores máximos de f y g será:

$f_{máx.} + g_{máx.} = 59 + 59 = 118$

C es incorrecta.

D) Se sabe.

$1° = 60'$

$1' = 60''$

$1° = 3600''$

Luego:

$\alpha = e° \, f' \, g'' = e° + f' + g''$

$\alpha = e° + f\,(60)'' + g''$

$\alpha = e\,(3600)'' + f\,(60)'' + g''$

$\alpha = (3600e + 60f + g)''$

El número de segundos de α es

$3600e + 60f + g$

D es incorrecta.

E) f es el n° de minutos del ángulo, por lo tanto:

$f \in [0; 60\rangle$ o $[0; 59]$

E es correcta.

Clave E

Razonamiento y demostración

15. Por dato:

$$2S + C - \frac{20R}{\pi} = 27$$

Sabemos: $\dfrac{S}{9} = \dfrac{C}{10} = \dfrac{20R}{\pi} = k$

$\Rightarrow S = 9k;\ C = 10k;\ R = \dfrac{\pi k}{20}$

Reemplazando tenemos:

$$2(9k) + (10k) - \frac{20}{\pi}\left(\frac{\pi k}{20}\right) = 27$$

$$18k + 10k - k = 27$$

$$27k = 27$$

$\Rightarrow k = 1$

Piden la medida del ángulo en el sistema inglés (sexagesimal).

$\Rightarrow S = 9k = 9(1) \Rightarrow S = 9$

Por lo tanto, el ángulo mide 9°.

Clave C

16. Por dato:

$$\frac{3S - C}{C - S} = \frac{17\pi}{2R}$$

Sabemos: $\dfrac{S}{180} = \dfrac{C}{200} = \dfrac{R}{\pi} = k$

$\Rightarrow S = 180k;\ C = 200k;\ R = \pi k$

Reemplazando tenemos:

$$\frac{3(180k) - (200k)}{(200k) - (180k)} = \frac{17\pi}{2(\pi k)}$$

$$\frac{340k}{20k} = \frac{17}{2k}$$

$$17(2k) = 17 \Rightarrow k = \frac{1}{2}$$

Piden la medida del ángulo en el sistema sexagesimal.

$\Rightarrow S = 180k = 180\left(\dfrac{1}{2}\right) \quad \Rightarrow S = 90$

Por lo tanto, el ángulo mide 90°.

Clave A

17. Por dato:

$$\frac{S}{3} + \frac{C}{2} = \frac{16R^2}{\pi} \qquad ...(I)$$

Sabemos: $\dfrac{S}{180} = \dfrac{C}{200} = \dfrac{R}{\pi}$

$\Rightarrow S = \dfrac{180R}{\pi} \ \wedge \ C = \dfrac{200R}{\pi}$

Reemplazando en (I), tenemos:

$$\frac{180R}{3\pi} + \frac{200R}{2\pi} = \frac{16R^2}{\pi}$$

$$60R + 100R = 16R^2$$

$$160R = 16R^2 \quad \Rightarrow R = 10$$

Por lo tanto, la medida del ángulo es 10 rad.

Clave D

18. Por dato:

$$\frac{10}{9C} - \frac{9}{10S} = \frac{R}{2\pi} \qquad ...(I)$$

Sabemos:

$$\frac{S}{180} = \frac{C}{200} = \frac{R}{\pi}$$

$\Rightarrow C = \dfrac{10S}{9} \ \wedge \ R = \dfrac{\pi S}{180}$

Reemplazando en (I), tenemos:

$$\frac{10}{9\left(\frac{10S}{9}\right)} - \frac{9}{10S} = \left(\frac{\pi S}{180}\right)\left(\frac{1}{2\pi}\right)$$

$$\frac{1}{S} - \frac{9}{10S} = \frac{S}{360}$$

$$\frac{S}{10S^2} = \frac{S}{360}$$

$$36 = S^2$$

$\Rightarrow S = 6$

Por lo tanto, el ángulo mide 6°.

Clave A

Resolución de problemas

19. Sean: α y β los ángulos

Por dato: α y β son complementarios

$\Rightarrow \alpha + \beta = 90° \qquad ...(1)$

$\alpha - \beta = 10^g$

$\Rightarrow \alpha - \beta = 10^g\left(\dfrac{9°}{10^g}\right) = 9°$

$\Rightarrow \alpha - \beta = 9° \qquad ...(2)$

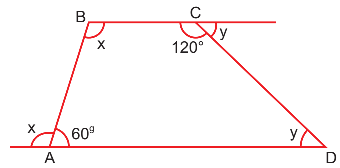
De (1) y (2): $\alpha = 49{,}5° \ \wedge \ \beta = 40{,}5°$

$\Rightarrow \alpha = 49{,}5°\left(\dfrac{\pi \text{ rad}}{180°}\right) = \dfrac{11\pi}{40}$ rad

$\therefore \alpha = \dfrac{11\pi}{40}$ rad

Clave B

20.



Por dato: ABCD es un trapecio.

Entonces se cumple:

$y + 120° = 180° \Rightarrow y = 60°$

$x + 60^g = 180°$

$x = 180° - 60^g \, . \left(\dfrac{9°}{10^g}\right) = 126°$

$\Rightarrow x = 126°$

Piden:

$(x - y)$ rad $= 126° - 60° = 66°$

$(x - y)$ rad $= 66°\left(\dfrac{\pi \text{ rad}}{180°}\right) = \dfrac{11\pi}{30}$ rad

$\therefore (x - y)$ rad $= \dfrac{11\pi}{30}$ rad

Clave E

21. Por dato: $5x^g$ y $(4x + 7)°$ son equivalentes:

$5x^g = (4x + 7)°$

$5x^g \, . \, \dfrac{9°}{10^g} = (4x + 7)°$

$45x° = (4x + 7)° \, 10$

$5x = 70$

$x = 14 \qquad ...(1)$

Luego:

Sea α el tercer ángulo del triángulo isósceles:

$\alpha + 5x^g + (4x + 7)° = 180°$

$\alpha + 2(4x + 7)° = 180°$

De (1):

$\alpha + 2(4 \times 14 + 7)° = 180°$

$\alpha + 2 \, . \, 63° = 180°$

$\alpha = 180° - 126°$

$\alpha = 54°$

Finalmente:

$\alpha = 54° \, . \, \dfrac{\pi \text{ rad}}{180°}$

$\alpha = \dfrac{3\pi}{10}$ rad

Clave D

22. Los ángulos internos de un cuadrilátero suman 360°, luego:

$$(3x)° + x^g + \frac{\pi x}{300}\text{rad} + (2x + 35)° = 360°$$

Llevamos todos los ángulos al sistema sexagesimal usando factores de conversión:

$$(3x)° + x^g \cdot \frac{9°}{10^g} + \frac{\pi x}{300}\text{rad} \cdot \frac{180°}{\pi\ \text{rad}} + (2x + 35)° = 360°$$

$$(3x)° + \left(\frac{9x}{10}\right)° + \left(\frac{3x}{5}\right)° + (2x + 35)° = 360°$$

$$3x + \frac{9x}{10} + \frac{3x}{5} + 2x + 35 = 360$$

$$x\left(3 + 2 + \frac{9}{10} + \frac{3}{5}\right) = 325$$

$$\frac{13}{2}x = 325$$

$$x = 50$$

Reemplazando x en las expresiones sexagesimales de los ángulos:

$$(3x)° = (3 \cdot 50)° = 150°;\ \left(\frac{9x}{10}\right)° = \left(\frac{9 \cdot 50}{10}\right)° = 45°$$

$$\left(\frac{3x}{5}\right)° = \left(\frac{3 \cdot 50}{5}\right)° = 30°;\ (2x + 35)° = ((2 \cdot 50) + 35)° = 135°$$

$\therefore$ El mayor de los ángulos es igual a 150°.

Clave E

Nivel 3 (página 10) Unidad 1

Comunicación matemática

23. De la expresión:

$$P = S + \frac{S}{P} = C - \frac{C}{P} \qquad ...(1)$$

$$\frac{S}{P} + \frac{C}{P} = C - S$$

$$P = \frac{S + C}{C - S} \qquad ...(2)$$

(2) en (1):

$$\frac{S + C}{C - S} = S + \frac{S}{\frac{S + C}{C - S}}$$

$$\frac{S + C}{C - S} = S + \frac{S(C - S)}{S + C} = \frac{S(S + C) + S(C - S)}{S + C}$$

$$\frac{S + C}{C - S} = \frac{S^2 + SC + SC - S^2}{S + C}$$

$$\frac{S + C}{C - S} = \frac{2SC}{S + C} \qquad ...(3)$$

De la relación:

$$\frac{20R}{\pi} = \frac{S}{9} = \frac{C}{10} = P \Rightarrow \begin{array}{l} S = 9P \\ C = 10P \\ R = \dfrac{\pi P}{20} \end{array}$$

En (3):

$$\frac{9P + 10P}{10P - 9P} = \frac{2(10P)(9P)}{9P + 10P}$$

$$\frac{19P}{P} = \frac{180P^2}{19P}$$

$$P = \frac{19^2}{180} = \frac{361}{180}$$

Luego:

I. $S = 9P = 9 \cdot \dfrac{361}{180} = \dfrac{361}{20} = 18{,}05$

$S < 90;\ 18{,}05 < 90$

I es verdadera.

II. $C = 10 \cdot \dfrac{361}{180} = \dfrac{361}{18} = 20{,}05$

$\therefore C = 20{,}05$

II es falsa.

III. $R = \dfrac{\pi P}{20} = \dfrac{361\pi}{3600}$

R representa el valor numérico de la medida del ángulo en el sistema radial, por lo tanto, no posee unidades.

III es falsa.

Clave C

24. Del gráfico cambiamos el sentido de β. El ángulo ω tal que:

$\omega = \alpha + (-\beta)$
$\omega = \alpha - \beta$

Posee un número entero de vueltas tal que: $\omega = \alpha + \beta = 360°\ p$

Por dato: p = n

Luego:

$\alpha + \beta = 360°n$
$\alpha = 360°\ n - \beta \qquad ...(1)$

Pero:

$\alpha = 360°\ n + b = 360°\ n - \beta$
$\therefore b = -\beta \qquad ...(2)$

I. De (I)

$\alpha = 360°\ n - \beta$

$\alpha + \beta = 360°\ n$

$\dfrac{\alpha + \beta}{n} = 360°$

I es verdadera.

II. De (2)

$b = -\beta$

II es verdadera.

III. El ángulo β posee giro horario, entonces:

$\beta < 0°$

De (2): $\beta = -b$

$-b < 0°$

$\therefore b > 0°$

III es falsa.

Clave A

Razonamiento y demostración

25. Por dato:

$$\sqrt[3]{\frac{R}{\pi}} + \sqrt[3]{\frac{S}{180}} + \sqrt[3]{\frac{C}{200}} = 3$$

Sabemos: $\dfrac{S}{180} = \dfrac{C}{200} = \dfrac{R}{\pi} = k$

$\Rightarrow S = 180k;\ C = 200k;\ R = \pi k$

Reemplazando tenemos:

$$\sqrt[3]{\frac{(\pi k)}{\pi}} + \sqrt[3]{\frac{(180k)}{180}} + \sqrt[3]{\frac{(200k)}{200}} = 3$$

$$\sqrt[3]{k}+\sqrt[3]{k}+\sqrt[3]{k}=3$$
$$3\sqrt[3]{k}=3$$
$$\sqrt[3]{k}=1$$
$$\Rightarrow k=1$$

Piden:

$$\sqrt[3]{\frac{\pi}{6SCR}}=\sqrt[3]{\frac{\pi}{6(180k)(200k)(\pi k)}}$$

$$\Rightarrow \sqrt[3]{\frac{\pi}{6SCR}}=\sqrt[3]{\frac{1}{216\,000\,k^3}}=\frac{1}{60k}$$

$$\therefore \sqrt[3]{\frac{\pi}{6SCR}}=\frac{1}{60(1)}=\frac{1}{60}$$

Clave D

26. Del gráfico, cambiamos el sentido de los ángulos. Todos con sentido antihorario.

Luego:

$$\left(\frac{10x^2}{3}-\frac{50x}{9}+\frac{20}{9}\right)^g+\frac{\pi}{3}\text{ rad}.\frac{180°}{\pi\text{ rad}}+(x^2+x-2)°=180°$$

Usando factor conversión a sexagesimales:

$$\left(\frac{10x^2}{3}-\frac{50x}{9}+\frac{20}{9}\right)^g.\frac{9°}{10^g}+\frac{\pi}{3}\text{rad}.\frac{180°}{\pi\text{rad}}+(x^2+x-2)°=180°$$

$(3x^2-5x+2)°+60°+(x^2+x-2)°=180°$

$4x^2-4x=120$

$x^2-x=30$

$x^2-x-30=0$

$x \quad -6$

$x \quad 5$

$(x-6)(x+5)=0$

$x=6 \qquad x=-5$

$\therefore$ Los valores de x son 6 y –5.

Clave D

27. $E=\frac{1°}{1'}-\frac{1^g}{1^m}+\frac{1'}{1''}\cdot\frac{1^m}{1^s}$

Sabemos:

$1°=60'$; $1'=60''$

$1^g=100^m$; $1^m=100^s$

Reemplazando en E tenemos:

$$E=\frac{60'}{1'}-\frac{100^m}{1^m}+\frac{60''}{1''}.\frac{100^s}{1^s}$$

$\Rightarrow E=60-100+60\,.\,100$

$\therefore E=5960$

Clave E

Resolución de problemas

28. Sean: α, β y θ los ángulos.

Por dato:

$$\alpha+\beta=\frac{\pi}{30}\text{ rad}\,.\left(\frac{180°}{\pi\text{ rad}}\right)$$

$\Rightarrow \alpha+\beta=6°$ ...(I)

$$\beta+\theta=\frac{\cancel{\pi}}{20}\text{ rad}\,.\left(\frac{180°}{\cancel{\pi}\text{rad}}\right)$$

$\Rightarrow \beta+\theta=9°$ ...(II)

$$2\alpha-\theta=\frac{\cancel{\pi}}{60}\text{ rad}\,.\left(\frac{180°}{\cancel{\pi}\text{rad}}\right)$$

$\Rightarrow 2\alpha-\theta=3°$ ...(III)

Restando (II) y (I):

$\Rightarrow \theta-\alpha=3°$ ...(IV)

De (III) y (IV): $\alpha=6° \wedge \theta=9°$

Reemplazando α en (I):

$\Rightarrow (6°)+\beta=6° \Rightarrow \beta=0°$

Piden:

$\alpha+\beta+\theta=6°+0°+9°=15°$

$\therefore \alpha+\beta+\theta=15°$

Clave C

29. Por dato:

$C=2a+b$; $S=a+b$ y $R=7\pi-\pi a$

Sabemos: $\frac{S}{180}=\frac{C}{200}=\frac{R}{\pi}$

Entonces:

$$\frac{a+b}{180}=\frac{2a+b}{200}=\frac{7\pi-\pi a}{\pi}$$

$$\Rightarrow \frac{a+b}{9}=\frac{2a+b}{10}$$

$\Rightarrow 10a+10b=18a+9b \quad \Rightarrow b=8a$

Luego:

$$\frac{a+b}{180}=\frac{7\pi-\pi a}{\pi}$$

$$\Rightarrow \frac{a+(8a)}{180}=7-a$$

$$\Rightarrow \frac{9a}{180}=7-a \Rightarrow \frac{a}{20}+a=7$$

$$\Rightarrow \frac{21a}{20}=7 \Rightarrow a=\frac{20}{3}$$

Reemplazando en R: $R=7\pi-\pi\left(\frac{20}{3}\right)=\frac{\pi}{3}$

Por lo tanto, el ángulo mide $\frac{\pi}{3}$ rad.

Clave B

30. Sean: α, β y θ los ángulos.

Por dato: $\alpha \xrightarrow{+20°} \beta \xrightarrow{+20°} \theta$

$\Rightarrow \beta=\alpha+20° \wedge \theta=\alpha+40°$

Además: $\beta+\theta=200°$

$\Rightarrow (\alpha+20°)+(\alpha+40°)=200°$

$2\alpha=140° \Rightarrow \alpha=70°$

$\Rightarrow \beta=90° \wedge \theta=110°$

Piden la suma de los tres ángulos.

$\alpha+\beta+\theta=70°+90°+110°$

$\alpha+\beta+\theta=270°\,.\left(\frac{10^g}{9°}\right)=300^g$

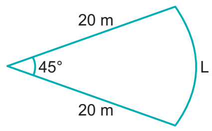
$\therefore \alpha+\beta+\theta=300^g$

Clave C

SECTOR CIRCULAR

APLICAMOS LO APRENDIDO (página 11) Unidad 1

1.



$$\theta = 45°\left(\frac{\pi\,\text{rad}}{180°}\right) = \frac{\pi}{4}\ \text{rad}$$

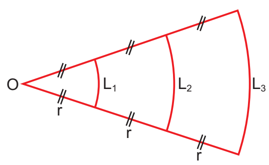
Se cumple: $L = \theta\, r$

$$L = \left(\frac{\pi}{4}\right)(20) = 5\pi$$

$\therefore L = 5\pi$ m

Clave D

2.



Por propiedad:

$$\frac{L_1}{r} = \frac{L_2}{2r} = \frac{L_3}{3r}$$

$$\frac{L_1}{1} = \frac{L_2}{2} = \frac{L_3}{3} = k$$

$\Rightarrow L_1 = k;\ L_2 = 2k;\ L_3 = 3k$

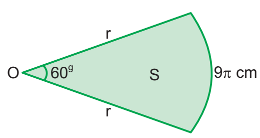
Piden:

$$E = \frac{L_1 + L_2}{L_2 + L_3} = \frac{(k) + (2k)}{(2k) + (3k)} = \frac{3k}{5k}$$

$$\therefore E = \frac{3}{5}$$

Clave C

3.



$$\theta = 60^g\left(\frac{\pi\,\text{rad}}{200^g}\right) = \frac{3\pi}{10}\ \text{rad}$$

$L = 9\pi$ cm

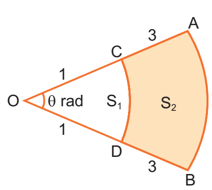
Piden el área del sector: S

$$\Rightarrow S = \frac{L^2}{2\theta} = \frac{(9\pi)^2}{2\left(\frac{3\pi}{10}\right)} = \frac{10\,.\,81\pi^2}{6\pi} = 135\pi$$

$\therefore S = 135\pi\ \text{cm}^2$

Clave A

4.



Entonces:

$$S_1 = \frac{\theta\,.\,r^2}{2} = \frac{\theta\,(1)^2}{2}$$

$$\Rightarrow S_1 = \frac{\theta}{2} \qquad \text{...(1)}$$

$$S_1 + S_2 = \frac{\theta\,.\,r^2}{2} = \frac{\theta\,(1+3)^2}{2}$$

$$S_1 + S_2 = 8\theta \qquad \text{...(2)}$$

De (1) y (2):

$$S_2 = \frac{15\theta}{2}$$

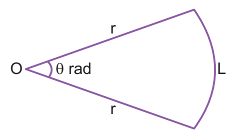
Piden:

$$\frac{S_2}{S_1} = \frac{\left(\frac{15\theta}{2}\right)}{\left(\frac{\theta}{2}\right)} = 15$$

$$\therefore \frac{S_2}{S_1} = 15$$

Clave C

5.



El radio y el arco son dos números pares consecutivos, entonces se pueden plantear:

$L - r = 2 \ \vee\ r - L = 2$

Por dato: $2r + L = 10$ ...(I)

Si: $L - r = 2 \Rightarrow L = 2 + r$

Reemplazando en (I):

$2r + (2 + r) = 10$

$3r = 8$

$\Rightarrow r = \frac{8}{3}$ (no cumple la condición)

Si: $r - L = 2 \Rightarrow r = 2 + L$

Reemplazando en (I):

$2(2 + L) + L = 10$

$4 + 3L = 10$

$L = 2$

$\Rightarrow r = 4$

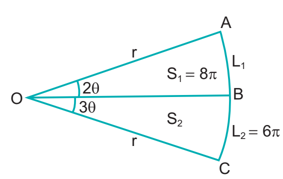
Piden el área del sector: S

$$\Rightarrow S = \frac{L\,.\,r}{2} = \frac{(2)(4)}{2} = 4$$

$\therefore S = 4\ \text{cm}^2$

Clave E

6.



Como el radio es constante, por propiedad:

$$\frac{S_1}{S_2} = \frac{2\theta}{3\theta} = \frac{L_1}{L_2}$$

$$\Rightarrow \frac{8\pi}{S_2} = \frac{2}{3} \quad \wedge \quad \frac{L_1}{6\pi} = \frac{2}{3}$$

$$\Rightarrow S_2 = 12\pi \quad \wedge \quad L_1 = 4\pi$$

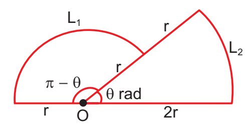
Piden:

$$\frac{L_1}{S_2} = \frac{4\pi}{12\pi} = \frac{1}{3}$$

$$\therefore \frac{L_1}{S_2} = \frac{1}{3}$$

Clave B

7.



Entonces:

$L_1 = (\pi - \theta)\,.\,r$

$L_2 = \theta\,(2r)$

Por dato: $2L_1 = 3L_2$

$\Rightarrow 2\,(\pi - \theta)r = 3\,\theta(2r)$

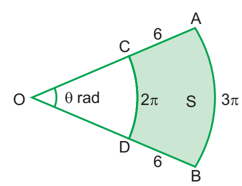
$2\pi - 2\theta = 6\theta$

$2\pi = 8\theta$

$\therefore \theta = \frac{\pi}{4}$ rad

Clave C

8.



Por las propiedades del trapecio circular:

$$S = \left(\frac{3\pi + 2\pi}{2}\right)(6) = 15\pi$$

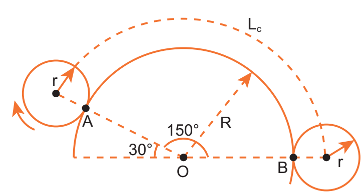
$$\theta = \left(\frac{3\pi - 2\pi}{6}\right) = \frac{\pi}{6}$$

Piden: $\dfrac{S}{\theta} = \dfrac{15\pi}{\left(\frac{\pi}{6}\right)} = 15\,.\,6 = 90$

$$\therefore \frac{S}{\theta} = 90$$

Clave D

9.



Por dato: $R = 7{,}6$ m $\wedge$ $r = 2$ m

$$150° = 150°\left(\frac{\pi\,\text{rad}}{180°}\right) = \frac{5\pi}{6}\ \text{rad}$$

Piden:
n: el número de vueltas que da la rueda al ir de A hasta B.

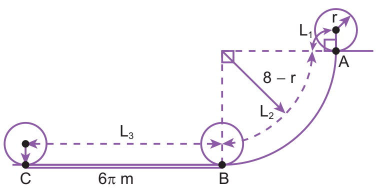
$$n = \frac{L_c}{2\pi r} = \frac{\frac{5\pi}{6}(R+r)}{2\pi r} = \frac{\frac{5\pi}{6}(7,6+2)}{2\pi(2)}$$

$$n = \frac{\frac{5\pi}{6}\left(\frac{48}{5}\right)}{4\pi} = \frac{48\pi}{24\pi} = 2$$

$\therefore n = 2$

Clave B

10.



La longitud que recorre el centro:
$L_c = L_1 + L_2 + L_3$
Luego:

- $L_1 = \theta_1 \cdot r = \frac{\pi}{2}r$
- $L_2 = \theta_2(8-r) = \frac{\pi}{2}(8-r)$
- $L_3 = BC = 6\pi$

Entonces:
$$L_c = \frac{\pi}{2}r + \frac{\pi}{2}(8-r) + 6\pi = 10\pi$$

Por dato: n.° de vueltas (n) de A hasta C es 5.

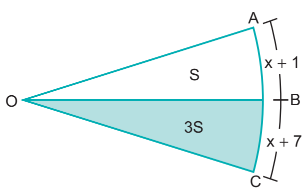
n.° de vueltas: $n = \frac{L_c}{2\pi r}$

$$5 = \frac{10\pi}{2\pi r} \Rightarrow 10\pi r = 10\pi$$

$\therefore r = 1$ m

Clave A

11.



Del gráfico
Sea OA = r, radio del sector circular AOC:

$$S = \frac{(x+1)R}{2} \qquad \ldots (1)$$

$$3S = \frac{(x+7)R}{2} \qquad \ldots (2)$$

(1) en (2):

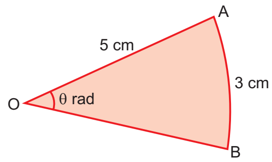
$$3\left[\frac{(x+1)R}{2}\right] = \frac{(x+7)R}{2}$$

$$3(x+1) = x+7$$
$$3x + 3 = x + 7$$
$$2x = 4$$
$$x = 2$$

Nos pide $L_{\widehat{AB}}$:
$L_{\widehat{AB}} = x + 1$
$L_{\widehat{AB}} = 2 + 1$
$\therefore L_{\widehat{AB}} = 3$

Clave C

12. Del enunciado:
Al inicio

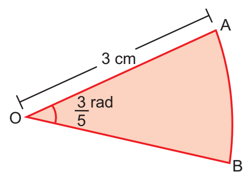


Del gráfico:
$\theta \cdot R = L_{\widehat{AB}}$, también $S_{\sphericalangle AOB} = \frac{L.R}{2} = \frac{3.5}{2}$

$\theta \cdot 5 = 3$

$\theta = \frac{3}{5} \qquad S_{\sphericalangle AOB} = \frac{15}{2}\,cm^2$

Luego, el radio disminuye 2 cm y el ángulo no varía.



Del gráfico:
$$S_{\sphericalangle AOB} = \frac{1}{2}\cdot\theta R^2 = \frac{1}{2}\cdot\frac{3}{5}\cdot(3)^2$$

$$S_{\sphericalangle AOB} = \frac{27}{10}\,cm^2$$

Variación de área (V):
$$V = \frac{15}{2} - \frac{27}{10} = \frac{48}{10}$$
$V = 4,8\ cm^2$

$\therefore$ El área varía 4,8 cm^2

Clave A

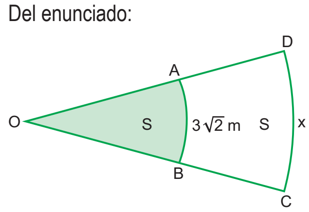
13. De la expresión $n_v = \frac{L_C}{2\pi r}$

Datos: $L_c = 110$ m , $r = \frac{1}{2}$m

Reemplazando
$$n_v = \frac{110}{2\pi.\frac{1}{2}} = \frac{110}{\frac{22}{7}} = 7.5$$
$n_v = 35$
$\therefore$ La rueda da 35 vueltas

Clave D

14. Del enunciado:



Sea θ rad la medida del ángulo AOC:

$$S = \frac{L^2_{\widehat{AB}}}{2\theta};\quad 2S = \frac{L^2_{\widehat{DC}}}{2\theta}$$

Entonces:
$$2\left(\frac{L^2_{\widehat{AB}}}{2\theta}\right) = \frac{L^2_{\widehat{DC}}}{2\theta}$$
$$2L^2_{\widehat{AB}} = L^2_{\widehat{DC}}$$

$$L_{\widehat{AB}}\sqrt{2} = L_{\widehat{DC}} \qquad \ldots(1)$$

Del gráfico:
$L_{\widehat{AB}} = 3\sqrt{2}$ m, $\quad L_{\widehat{DC}} = x$

En (1):
$(3\sqrt{2})(\sqrt{2}) = x$
$\therefore x = 6$ m

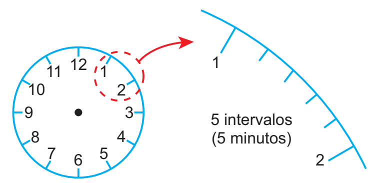
Clave B

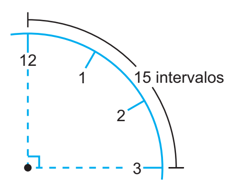
PRACTIQUEMOS

Nivel 1 (página 13) Unidad 1

Comunicación matemática

1. Para un reloj analógico:





Regla de tres simple:
15 intervalos ——— 90°
intervalo ——— x

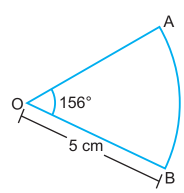
$$x = \frac{90°}{15} = 6°$$

Se concluye que por cada minuto el minutero barre un ángulo de 6°.

I. A las 12:26, han pasado 26 minutos; por regla de tres simple:
1 minuto ——— 6°
26 minutos ——— a°

$a° = 6° \cdot 26 = 156°$

El minutero barre 156° en 26 minutos, entonces:



Ángulo central:

$156° = 156° \times \dfrac{\pi \text{ rad}}{180°} = \dfrac{13\pi}{15}$ rad

(Falsa)

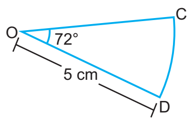
II. A las 12:12 el minutero avanza 12 minutos, entonces:

1 minuto —— 6°

12 minutos —— b°

$b° = 6° \,.\, 12 = 72°$

El minutero barre un sector circular de ángulo central 72°, luego:



$72° = 72° \,.\, \dfrac{\pi \text{ rad}}{180°} = \dfrac{72\pi}{180} = \dfrac{2\pi}{5}$ rad

$S_{\triangleleft COD} = \dfrac{1}{2}\left(\dfrac{2\pi}{5}\right)(5)^2 = 5\pi$

$S_{\triangleleft COD} = 5\pi \text{ cm}^2$

(Verdadera)

III. A las 12:17 el minutero avanza 17 minutos, luego:

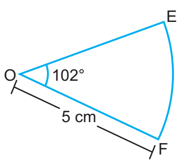
1 minuto —— 6°

17 minutos —— C°

$C° = 6° \,.\, 17 = 102°$

El minutero barre un sector circular de ángulo central 102°.

$102° = 102° \,.\, \dfrac{\pi \text{ rad}}{180°} = \dfrac{102\pi}{180} = \dfrac{17\pi}{30}$ rad



Luego:

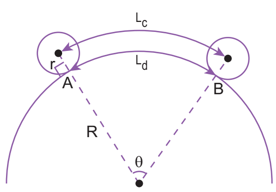
$L_{\widehat{EF}} = \dfrac{17\pi}{30} \,.\, 5 = \dfrac{17\pi}{6}$

$L_{\widehat{EF}} = \dfrac{17\pi}{6}$ cm

(Falsa)

Clave E

2. En el gráfico:



Cálculo de la longitud que recorre el centro (L_c):

$L_c = \theta(r + R)$

(Falsa)

Longitud recorrida por la rueda sobre el camino circular (L_d):

$L_d = \theta R$

(Falsa)

Cálculo del n.° de vueltas que da la rueda desde A hasta B (n_v):

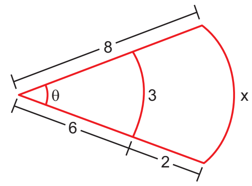
$n_v = \dfrac{L_c}{2\pi r} = \dfrac{\theta(r+R)}{2\pi r}$ $\quad\therefore n_v = \dfrac{\theta(r+R)}{2\pi r}$

(Verdadera)

Clave C

Razonamiento y demostración

3.



$\theta \,.\, 6 = 3$

$\theta = \dfrac{1}{2}$ rad

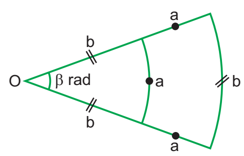
$\theta \,.\, 8 = x$

$\left(\dfrac{1}{2}\right) . \, 8 = x$

$\therefore x = 4$

Clave B

4.



$\beta \,.\, b = a$ ...(1)

Además:

$\beta(a + b) = b$

$\beta a + \beta b = b$

$\beta a + a = b$

$(\beta + 1)a = b$ ...(2)

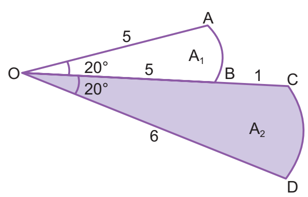
Multiplicando (1) y (2) tenemos:

$\beta(\beta + 1)ab = ab$

$\therefore \beta(\beta + 1) = 1$

Clave B

5.



Se tiene: $20° = \dfrac{\pi}{9}$ rad

Del gráfico:

$A_1 = \dfrac{\left(\frac{\pi}{9}\right)(5)^2}{2} = \dfrac{25\pi}{18}$

$A_2 = \dfrac{\left(\frac{\pi}{9}\right)(6)^2}{2} = \dfrac{36\pi}{18}$

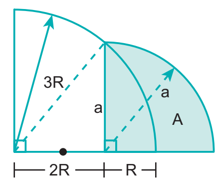
Piden:

$J = \dfrac{A_1}{A_2} = \dfrac{\frac{25\pi}{18}}{\frac{36\pi}{18}} = \dfrac{25}{36}$

$\therefore J = \dfrac{25}{36}$

Clave D

6.



Por el teorema de Pitágoras:

$a^2 + (2R)^2 = (3R)^2$

$a^2 + 4R^2 = 9R^2$

$\Rightarrow a^2 = 5R^2$

Piden el área de la región sombreada (A).

$A = \dfrac{\left(\frac{\pi}{2}\right).a^2}{2} = \dfrac{\pi}{4}a^2 = \dfrac{\pi}{4}(5R^2)$

$\therefore A = \dfrac{5\pi}{4}R^2$

Clave C

7. En el sector circular AOB: $L_{\widehat{AB}} = \theta \,.\, R$

Luego:

$27 + x = x(x^2 + 1)$

$27 + x = x^3 + x$

$27 = x^3$

$\therefore x = 3$

Entonces:

$S_{\triangleleft AOB} = \dfrac{1}{2}\theta R^2 = \dfrac{1}{2}(x)(x^2 + 1)^2$

Reemplazando:

$S_{\triangleleft AOB} = \dfrac{1}{2}(3)(3^2 + 1)^2 = 150$

$\therefore S_{\triangleleft AOB} = 150$

Clave D

Resolución de problemas

8. Del problema:

$\theta_1 = 25°$

$\theta_1 = 25° \times \dfrac{\pi \text{ rad}}{180°}$

$\theta_1 = \dfrac{5\pi}{36}$ rad

$R_1 = 20$ m

$\theta_2 = 25° - 9° = 16°$

$\theta_2 = 16° \times \dfrac{\pi \text{ rad}}{180°}$

$\theta_2 = \dfrac{4\pi}{45}$ rad

$R_2 = 20 \text{ m} + x$

Como el área no varía, entonces:

$\dfrac{R_1^2\theta_1}{2} = \dfrac{R_2^2\theta_2}{2}$

$R_1^2\theta_1 = R_2^2\theta_2$

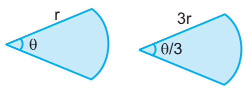
$(20)^2\dfrac{5\pi}{36} = (20 + x)^2\dfrac{4\pi}{45}$

Simplificando: x = 5 m

Para que el área no varíe, hay que aumentar el radio inicial en 5 m.

Clave B

9.



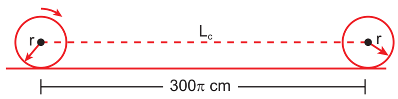
$S = \dfrac{\theta \,.\, r^2}{2}$ $\qquad S_2 = \dfrac{\theta}{3} \,.\, \dfrac{(3r)^2}{2}$

$\Rightarrow S_2 = \dfrac{\theta}{3} \,.\, \dfrac{9r^2}{2} = 3 \,.\, \dfrac{\theta \,.\, r^2}{2}$

$\therefore S_2 = 3S$

Clave B

10.



Por dato: r = 30 cm

Piden:

$n_v = \dfrac{L_C}{2\pi r} = \dfrac{300\pi}{2\pi(30)} = 5 \quad \Rightarrow n_v = 5$

Además: $L_C = \theta_g \,.\, r$

$300\pi = \theta_g \,.\, (30)$

$\Rightarrow \theta_g = 10\pi$ rad

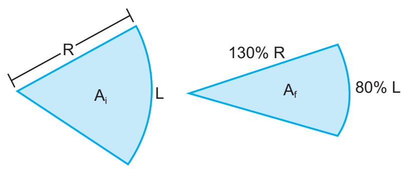
$\therefore n_v = 5 \ \wedge \ \theta_g = 10\pi$ rad

Clave E

Nivel 2 (página 14) Unidad 1

Comunicación matemática

11. Del enunciado:



A_i: Área inicial

A_f: Área final

$A_i = \dfrac{1}{2}(R)(L) = \dfrac{RL}{2}$

$A_i = \dfrac{RL}{2}$ ...(1)

Luego:

$A_f = \dfrac{1}{2}(80\%L)(130\%R)$

$A_f = \dfrac{1}{2}\left(\dfrac{80L}{100}\right)\left(\dfrac{130R}{100}\right)$

$A_f = LR\left(\dfrac{52}{100}\right) = 0{,}52$ RL

$A_f = 0{,}52$ ...(2)

La variación del área (ΔA) será: de (1) y (2)

$\Delta A = A_f - A_i$

$\Delta A = 0{,}52RL - \dfrac{RL}{2}$

$\Delta A = RL(0{,}52 - 0{,}5)$

$\Delta A = 0{,}02\ RL = 2\% \dfrac{RL}{2} \,.\, 2$

$\Delta A = 4\% \dfrac{RL}{2}$

$\Delta A = 4\%\ A_i$

$\therefore$ El área aumenta en %4.

Clave C

12.

I. Para el tramo de A hasta B:

$n_v = \dfrac{L_c}{2\pi r}$; del gráfico: $L_c = d$

Luego: $n_v = \dfrac{d}{2\pi r}$

$\therefore$ Desde A hasta B da $\dfrac{d}{2\pi r}$ vueltas

II. Para el tramo B a D, consideramos los tramos:

De B a C

$n_1 = \dfrac{\theta_1(R+r)}{2\pi r}$; desde $\theta_1 = \dfrac{\pi}{2}$

$n_1 = \dfrac{\pi}{2} . \dfrac{(R+r)}{2\pi r} = \dfrac{(R+r)}{4r}$

$n_1 = \dfrac{R+r}{4r}$

De C a D

$n_2 = \dfrac{\theta_2(R-r)}{2\pi r}$; $\theta_2 = \dfrac{\pi}{2}$

$n_2 = \dfrac{\pi}{2} . \dfrac{(R-r)}{2\pi r} = \dfrac{(R-r)}{4r}$

$n_2 = \dfrac{R-r}{4r}$

Finalmente:

De B a D

$n_v = n_1 + n_2$

$n_v = \dfrac{(R+r)}{4r} + \dfrac{(R-r)}{4r} = \dfrac{2R}{4r}$

$n_v = \dfrac{R}{2r}$

$\therefore$ Desde B hasta D da $\dfrac{R}{2r}$ vueltas.

III. Desde C hasta E

$n_v = \dfrac{\theta_2(R-r)}{2\pi r}$; desde $\theta = \pi$

$n_v = \dfrac{\pi(R-r)}{2\pi r}$

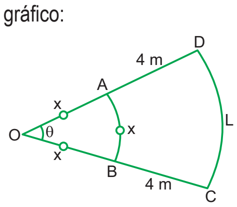
$n_v = \dfrac{(R-r)}{2r}$

$\therefore$ Desde C hasta E da $\dfrac{R-r}{2r}$ vueltas.

Clave D

Razonamiento y demostración

13. Del gráfico:



Dato:

$A_{\square ABCD} = 20\ m^2$

Del gráfico:

$L_{\overset{\frown}{AB}} = x = \theta \,.\, x \Rightarrow \theta = 1$ rad

$L_{\overset{\frown}{DC}} = L = \theta(4 + x) = 1(4 + x) = 4 + x$

$A_{\square ABCD} = \left(\dfrac{B+b}{2}\right)h$

$20 = \dfrac{(4 + x + x)4}{2}$

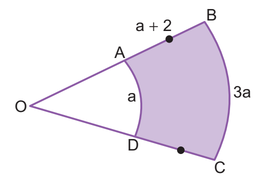
$20 = 8 + 4x \Rightarrow x = 3$ m

Piden:

$L_{\overset{\frown}{DC}} = L = 4 + x = 4 + 3 = 7$ m

Clave D

14. Del gráfico:



$A_{somb.} = 16 = \dfrac{(a + 3a)(a + 2)}{2}$

$32 = 4a^2 + 8a$

Entonces:

$4a^2 + 8a - 32 = 0$

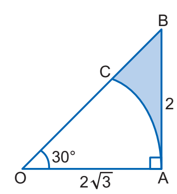
2a $\quad -4 \Rightarrow a = 2$

2a $\quad +8 \Rightarrow a = -4$

Piden: 2a = 2(2) = 4 m

Clave E

15.



$S_{\triangle} = \dfrac{2 \,.\, 2\sqrt{3}}{2} = 2\sqrt{3}\ m^2$

30° a radianes:

$\dfrac{S}{9} = \dfrac{20R}{\pi}$

$\dfrac{30}{9} = \dfrac{20R}{\pi} = \theta$

$R = \dfrac{\pi}{6} \Rightarrow \theta = \dfrac{\pi}{6}$ rad

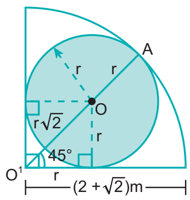
$S_{\triangleleft} = \dfrac{\pi}{6} \dfrac{(2\sqrt{3})^2}{2} = \dfrac{12\pi}{12} = \pi\ m^2$

$\Rightarrow S_{sombreada} = S_{\triangle} - S_{\triangleleft}$

$= (2\sqrt{3} - \pi)\ m^2$

Clave E

16.



Sea r el radio del círculo, del gráfico:

$O'A = r + r\sqrt{2} = 2 + \sqrt{2}$

$r(1 + \sqrt{2}) = 2 + \sqrt{2}$

$$r = \frac{(2+\sqrt{2})}{1+\sqrt{2}} \cdot \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}$$

$$r = \frac{2+\sqrt{2}-2}{2-1}$$

$$r = \sqrt{2}$$

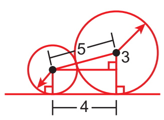
Finalmente:

$S = \pi r^2 = \pi(\sqrt{2})^2$

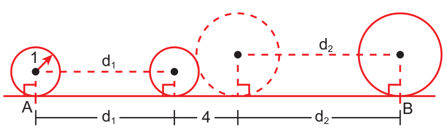
$\therefore S = 2\pi\ m^2$

Clave B

17. Al inicio:



Luego



Se sabe:

$$n = \frac{L_c}{2\pi r}$$

Para la rueda de radio 1:

$$7 = \frac{d_1}{2\pi(1)}$$

$d_1 = 14\pi$

Para la rueda de radio 4:

$$3 = \frac{d_2}{2\pi(4)}$$

$d_2 = 24\pi$

Finalmente:

$AB = 4 + d_1 + d_2$

$AB = 4 + 14\pi + 24\pi$

$\therefore AB = 4 + 38\pi$

Clave E

Resolución de problemas

18. $\theta = 36° \Rightarrow \theta = \frac{\pi}{5}$ rad

1.er caso: ángulo θ y radio R

2.° caso: ángulo α y radio $\frac{3}{4}R$

Por dato el área no varía:

$S_{1er.\ caso} = S_{2°.\ caso}$

$$\theta \cdot R^2 = \alpha\left(\frac{3}{4}R\right)^2 \Rightarrow \theta \cdot R^2 = \alpha \cdot \frac{9}{16}R^2$$

$$\Rightarrow \alpha = \frac{16}{9}\theta$$

Como $\theta = \frac{\pi}{5}$ rad, entonces:

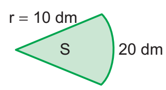
$$\alpha = \frac{16}{9}\left(\frac{\pi}{5}\right) = \frac{16\pi}{45} \text{ rad} = 64°$$

$\therefore$ Lo que hay que aumentar es:

$64° - 36° = 28°$

Clave A

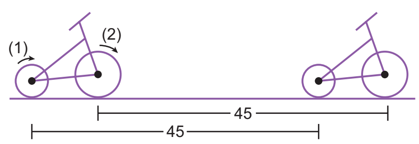
19.



$$S = \frac{L \cdot r}{2} = \frac{20.10}{2} = 100\ dm^2$$

Clave B

20.



Por dato: $n_{v(1)} + n_{v(2)} = \frac{15}{r}$

$$\Rightarrow \frac{L_C}{2\pi r} + \frac{L_C}{2\pi R} = \frac{15}{r}$$

$$\frac{45}{2\pi r} + \frac{45}{2\pi R} = \frac{15}{r}$$

$$\frac{3}{2\pi}\left(\frac{R+r}{Rr}\right) = \frac{1}{r}$$

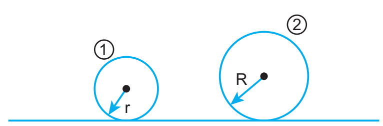
$$3r = R(2\pi - 3)$$

$$\Rightarrow r = \frac{R(2\pi-3)}{3}$$

$$\therefore \frac{r}{R} = \frac{2\pi - 3}{3}$$

Clave A

21. Del enunciado:



La distancia que recorre la rueda (1) es igual a la recorrida por la rueda (2)

Luego:

Para (1):

$$n_1 = \frac{L_1}{2\pi r}$$

$L_1 = n_1 2\pi r$ ...(1)

Para (2):

$$n_2 = \frac{L_2}{2\pi r}$$

$L_2 = n_2 2\pi R$ ...(2)

$L_1 = L_2$, de (1) y (2):

$2\pi n_1 r = 2\pi n_2 R$

$$\frac{n_1}{n_2} = \frac{R}{r}$$

Por dato: $\frac{R}{r} = \frac{11}{10}$; $n_2 = 20$

Luego: $\frac{n_1}{20} = \frac{11}{10}$

$\therefore n_1 = 22$

Clave D

Nivel 3 (página 15) Unidad 1

Comunicación matemática

22.

I. De la condición:

$\theta = \alpha$

Por dato. $S_1 = S_2 + S_3$

Por propiedad

$S_3 = 5S_2 \Rightarrow S_1 = S_2 + 5S_2$

$S_1 = 6S_2$ ...(1)

De la fórmula $S = \frac{1}{2}\theta R^2$; en (1):

$$\frac{1}{2}\theta R_1^2 = 6\ \frac{1}{2}\alpha R_2^2$$

$$\theta R_1^2 = 6\alpha R_2^2;\ \theta = \alpha$$

$$R_1^2 = 6R_2^2$$

$$R_1 = R_2\sqrt{6}$$

$\therefore R_1$ y R_2 no son equivalentes.

(Falsa)

II. Por lo anterior:

$$\frac{1}{2}\theta R_1^2 = \frac{6}{2}\alpha R_2^2$$

Condición: $R_1 = R_2$, entonces:

$$\frac{1}{2}\theta R_1^2 = \frac{6}{2}\alpha R_1^2$$

$\theta = 6\alpha$

$\therefore \theta$ es igual a 6 veces α.

(Verdadera)

III. De la relación:

$$\frac{1}{2}\theta R_1^2 = \frac{6}{2}\alpha R_2^2$$

$$\frac{\theta}{\alpha} = \frac{6R_2^2}{R_1^2}$$

De la condición: $\frac{\theta}{\alpha} = \frac{24}{49}$

$$\frac{6R_2^2}{R_1^2} = \frac{24}{49}$$

$$\frac{R_2^2}{R_1^2} = \frac{4}{49}$$

$$\frac{R_2}{R_1} = \frac{2}{7}$$

$\therefore R_2$ y R_1 están en razón de 2 a 7. (Falsa)

Clave A

23.

I. De la igualdad:

$2\pi \cdot n_v = \theta_g \Rightarrow n_v = \frac{\theta_g}{2\pi}$ ...(1)

De la condición: $\theta_g = 39\pi$

En (1):

$n_v = \frac{39\pi}{2\pi} = \frac{39}{2}$

$n_v = \frac{39}{2}$

Para que se cumpla la igualdad $n_v = \frac{L_c}{2\pi r}$

$n_v = \frac{39}{2} = \frac{L_c}{2\pi(3)} \Rightarrow L_c = 117\pi$ m

Para que la igualdad se cumpla:

$\theta = 39\pi \wedge L_c = 117\pi$ m

II. Para que se cumpla la igualdad; si $l_c = 210\pi$ m, entonces:

$n_v = \frac{L_c}{2\pi r} = \frac{210\pi}{2\pi(3)} = 35$

$n_v = 35$

Además:

$\theta_g = 2\pi \cdot n_v$

$\theta_g = 2\pi \cdot 35$

$\theta_g = 70\pi$

Para que la igualdad se cumpla:

$L_c = 210\pi$ m $\theta_g = 70\pi$

III. Para que se cumpla la igualdad;

Si $L_c = 186\pi$ m, entonces:

$n_v = \frac{L_c}{2\pi r} = \frac{186\pi}{2\pi(3)} = 31$

$n_v = 31$

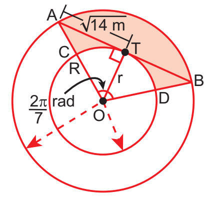
Para que se cumpla la igualdad:

$L_c = 186\pi$ m $\wedge n_v = 31$

Clave D

Razonamiento y demostración

24. En el gráfico



Sea S área del trapecio circular ACDB:

$$S = \frac{1}{2}\left(\frac{2\pi}{7}\right)R^2 - \frac{1}{2}\left(\frac{2\pi}{7}\right)r^2$$

$$S = \frac{1}{2}\left(\frac{2\pi}{7}\right)(R^2 - r^2) \quad ... (1)$$

En el triángulo rectángulo ATO:

$R^2 = r^2 + (\sqrt{14})^2$

$R^2 - r^2 = 14$ ... (2)

(2) en (1): $S = \frac{1}{2}\left(\frac{2\pi}{7}\right)(14)$

$\therefore S = 2\pi\ m^2$

Clave E

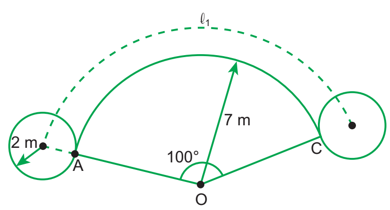
25. Sabemos:

$n_v = \frac{L_c}{2\pi r}$;

L_c: longitud que recorre el centro de la rueda.

r = 2 m

Del gráfico, tramo AC:



$m\angle AOC = 100° = 100° \cdot \frac{\pi}{180°}$rad $= \frac{5\pi}{9}$ rad

$m\angle AOC = \frac{5\pi}{9}$ rad

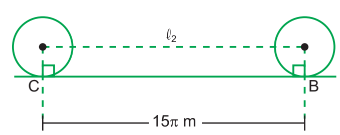
Entonces:

$L_1 = \theta \cdot R$

$L_1 = \frac{5\pi}{9} \cdot (2 + 7)$

$L_1 = 5\pi$ m ... (1)

Tramo CB:



$L_2 = 15\pi$ m ... (2)

De (1) y (2)

$L_c = L_1 + L_2 = 5\pi$ m $+ 15\pi$ m

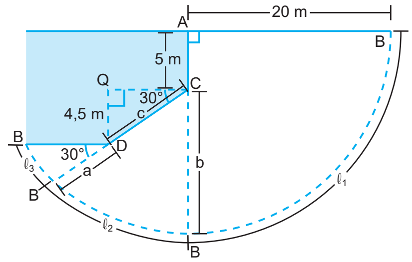
$L_c = 20\pi$ m

Finalmente: $n_v = \frac{20\pi}{2\pi(2)} = 5$

$\therefore n_v = 5$

Clave B

26. Resolvemos la trayectoria que realiza la punta al envolver la región:



Sea L, la longitud que recorre la punta B de la cuerda:

$L = L_1 + L_2 + L_3$ ...(1)

Del gráfico:

$L_1 = \frac{\pi}{2} \cdot (20) = 10\pi m$ ...(2)

$L_2 = \theta b$; donde $\theta = 60 \cdot \frac{\pi}{180} = \frac{\pi}{3}$

$L_2 = \frac{\pi}{3} b$

Además; $20 = 5 + b \Rightarrow b = 15$ m

$L_2 = \frac{\pi}{3}(15)$

$L_2 = 5\pi$ m ...(3)

$L_3 = \alpha \cdot a$; donde $\alpha = \frac{30\pi}{180} = \frac{\pi}{6}$

$L_3 = \frac{\pi}{6} \cdot a$; además $b = a + c$

$a = b - c$

En el ◺DQC(30°, 60°):

$c = (4,5)(2)$

$c = 9$ m

$\Rightarrow a = 15 - 9 = 6$

$a = 6$ m

$L_3 = \frac{\pi}{6} \cdot 6$

$L_3 = \pi$ m ...(4)

(2), (3), (4) en (1):

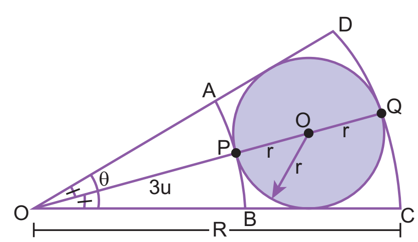
$L = 10\pi$ m $+ 5\pi$ m $+ \pi$m

$\therefore L = 16\pi$ m

Clave A

27. En el gráfico:

- Sea O centro del círculo.
- P y Q puntos de tangencia.



Se observa:

$R = 3 + 2r$ ...(1)

Por dato.
Área del trapecio circular ABCD (S) es igual a 48 u^2, luego:

$$S = \frac{1}{2}\theta R^2 - \frac{1}{2}\theta 3^2$$

$$S = \frac{1}{2}\theta(R^2 - 3^2)$$

De (1), reemplazando valores:

$$48 = \frac{1}{2}\ \frac{4}{3}(R^2 - 9)$$

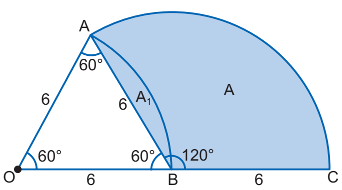
$R^2 - 9 = 72$
$R^2 = 81$
$R = 9$
$3 + 2r = 9$
$2r = 6$
$r = 3$ u

Luego, área de la región sombreada (círculo):
$A_{\odot} = \pi r^2$
$A_{\odot} = \pi(3)^2$
$\therefore A_{\odot} = 9\pi$ u^2

Clave C

Resolución de problemas

28.



Sabemos: $60° = \frac{\pi}{3}$ rad $\wedge$ $120° = \frac{2\pi}{3}$ rad

Del gráfico:

$$A_{\Delta AOB} + A_1 = \frac{\left(\frac{\pi}{3}\right).(6)^2}{2}$$

$$\frac{6^2 . \sqrt{3}}{4} + A_1 = 6\pi$$

$$\Rightarrow A_1 = 6\pi - 9\sqrt{3}$$

Además:

$$A_1 + A = \frac{\left(\frac{2\pi}{3}\right).(6)^2}{2}$$

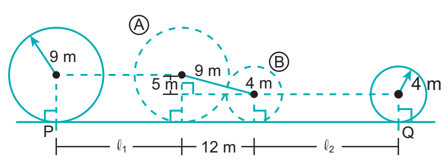
$A_1 + A = 12\pi$

$\Rightarrow A = 12\pi - (6\pi - 9\sqrt{3})$

$\therefore A = 3(2\pi + 3\sqrt{3})$ m^2

Clave D

29. Sean P y Q las proyecciones de los centros:



Sabemos:

$n_v = \frac{L_c}{2\pi r}$ r: radio de la rueda
l: longitud que recorre el centro

Luego:

Para la rueda A:

$n_{v_A} = \frac{L_1}{2\pi(9)}$; dato: $n_{v_A} = 14$

$14 = \frac{L_1}{18\pi}$

$L_1 = 14 . 18\pi = 14 . 18 . \frac{22}{7} = 36 . 22$

$L_1 = 792$ m

Para la rueda B: $n_{v_B} = \frac{L_2}{2\pi(4)}$; dato: $n_{v_B} = 7$

$7 = \frac{L_2}{8\pi}$

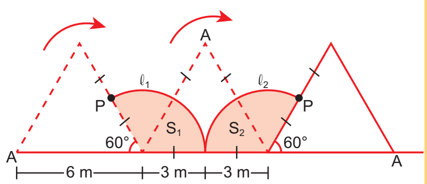
$L_2 = 7 . 8\pi = 7 . 8\frac{22}{7} = 176$

$L_2 = 176$ m

Finalmente:
$PQ = L_1 + 12 + L_2$
$PQ = 792 + 12 + 176$ $\therefore PQ = 980$ m

Clave E

30. Del enunciado:



Se observa que la trayectoria del punto P cuando el triángulo gira es la de un arco de circunferencia.

De la región S_1:
$L_1 = \theta_1 R$; donde: θ_1 rad $= 120°$
$R = 3$ m

Luego:

θ_1 rad $= 120° . \frac{\pi \text{ rad}}{180°} = \frac{2\pi}{3}$ rad

θ_1 rad $= \frac{2\pi}{3}$ rad

$\theta_1 = \frac{2\pi}{3}$

Entonces: $L_1 = \frac{2\pi}{3}$ (3)

$L_1 = 2\pi$ m

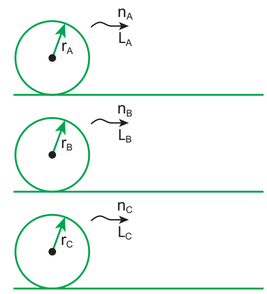
Además: $S_1 = S_2 \Rightarrow \frac{3l_1}{2} = \frac{l_2 . 3}{2}$

$L_1 = L_2 = 2\pi$ m

Finalmente, sea L la longitud de la trayectoria de P:
$L = L_1 + L_2$
$L = 2\pi$ m $+ 2\pi$ m
$\therefore L = 4\pi$ m

Clave D

31. Del enunciado:



n_A, n_B, n_C: número de vueltas
L_A, L_B, L_C: longitudes recorridas

Datos:

$$\frac{2}{\frac{1}{R_A} + \frac{1}{R_C}} = R_B \Rightarrow \frac{2}{R_B} = \frac{1}{R_A} + \frac{1}{R_B} \quad ...(1)$$

$L_A = \frac{L_B}{3} = \frac{L_C}{2} \Rightarrow$ $L_A = k$
$L_B = 3k$
$L_C = 2k$ ...(2)

Sabemos: $n_v = \frac{L}{2\pi r} \Rightarrow r = \frac{L}{2\pi n_v}$

En (1):

$$\frac{2}{\frac{L_B}{2\pi n_B}} = \frac{1}{\frac{L_A}{2\pi n_A}} + \frac{1}{\frac{L_C}{2\pi n_c}}$$

$$\frac{4\pi n_B}{L_B} = \frac{2\pi n_A}{L_A} + \frac{2\pi n_C}{L_C}$$

$$\frac{2n_B}{L_B} = \frac{n_A}{L_A} + \frac{n_C}{L_C}$$

De (2):

$$\frac{2n_B}{3k} = \frac{n_A}{k} + \frac{n_C}{2k}$$

$$n_B = \frac{3}{2}n_A + \frac{3}{4}n_C$$

Por dato: $n_A = 7$; $n_C = 2$

Luego:

$n_B = \frac{3}{2}(7) + \frac{3}{4}(2)$

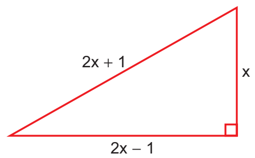
$n_B = \frac{21}{2} + \frac{3}{2}$

$n_B = \frac{24}{2}$ $\therefore n_B = 12$

Clave C

APLICAMOS LO APRENDIDO (página 17) Unidad 1

1.



Por el teorema de Pitágoras:

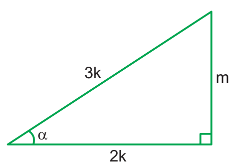
$(2x-1)^2+(x)^2=(2x+1)^2$

$4x^2-4x+1+x^2=4x^2+4x+1$

$x^2=8x \qquad \therefore x=8$

Clave C

2. $\cos\alpha=\dfrac{2}{3}$; α es agudo.



Por el teorema de Pitágoras: $m=\sqrt{5}\,k$

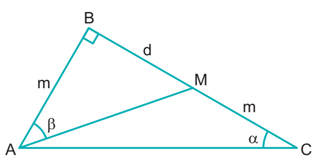
Piden: $\tan\alpha$

$\tan\alpha=\dfrac{m}{2k}=\dfrac{\sqrt{5}\,k}{2k}=\dfrac{\sqrt{5}}{2}$

$\therefore \tan\alpha=\dfrac{\sqrt{5}}{2}$

Clave C

3.



Del gráfico: $\cot\alpha=\dfrac{m+d}{m} \wedge \tan\beta=\dfrac{d}{m}$

Piden:

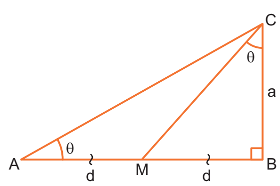
$R=\cot\alpha-\tan\beta$

$R=\dfrac{m+d}{m}-\dfrac{d}{m}=\dfrac{m+d-d}{m}=\dfrac{m}{m}=1$

$\therefore R=1$

Clave A

4.



Del gráfico:

$\tan\theta=\dfrac{a}{2d} \qquad$ ...(I)

$\tan\theta=\dfrac{d}{a} \qquad$ ...(II)

Multiplicamos (I) y (II):

$\Rightarrow \tan^2\theta=\dfrac{a}{2d}\cdot\dfrac{d}{a}$

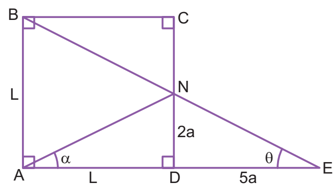
$\tan^2\theta=\dfrac{1}{2}$

$\tan\theta=\dfrac{1}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}$

$\therefore \tan\theta=\dfrac{\sqrt{2}}{2}$

Clave B

5.



Por dato: $\tan\theta=\dfrac{2}{5}$

Del gráfico: $\tan\theta=\dfrac{L}{L+5a}$

Entonces:

$\dfrac{2}{5}=\dfrac{L}{L+5a}$

$2L+10a=5L$

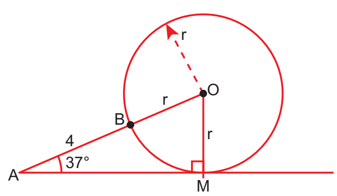
$10a=3L \Rightarrow L=\dfrac{10a}{3}$

Piden: $\tan\alpha=\dfrac{ND}{AD}=\dfrac{2a}{L}=\dfrac{2a}{\left(\dfrac{10a}{3}\right)}=\dfrac{6}{10}$

$\therefore \tan\alpha=0{,}6$

Clave D

6.



M punto de tangencia: $\overline{AM}\perp\overline{OM} \wedge OM=r$

⊿AMO notable de 37° y 53°

$\text{sen}37°=\dfrac{r}{4+r}$

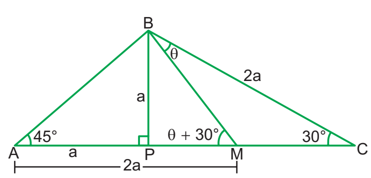
$\dfrac{3}{5}=\dfrac{r}{4+r}$

$12+3r=5r$

$\therefore r=6$ cm

Clave B

7.



Trazamos: $\overline{BP}\perp\overline{AM}$

Por dato $BC=AM$

⊿CPB notable de 30° y 60°.

Sea $BC=2a \wedge BP=a$

⊿APB notable de 45°.

$AP=PB=a$

Luego:

$AM=AP+PM$

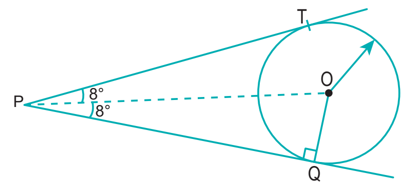
$2a=a+PM$

$PM=a$

$\therefore \tan(\theta+30°)=\dfrac{BP}{PM}=\dfrac{a}{a}=1$

Clave A

8.



$\overline{PO}$: bisectriz del ∠TPQ

m∠OPQ = 8°, ⊿OQP notable de 8° y 82°.

$PQ=4k \wedge PT=5\sqrt{2}\,k$

$21=7k$

$k=3 \quad \therefore PT=15\sqrt{2}$ cm

Clave D

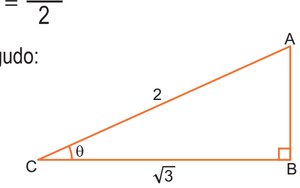
9. Dato:

$\cos\theta=\sqrt{3}\,\text{sen}^2 45°$

$\cos\theta=\sqrt{3}\left(\dfrac{1}{\sqrt{2}}\right)^2$

$\cos\theta=\dfrac{\sqrt{3}}{2}$

θ: agudo:



⊿ABC notable de 30° y 60°

$\Rightarrow \theta=30°$

$\therefore 2\theta=60°$

Clave C

10. $\cos\alpha=\dfrac{1}{\sec\alpha}$, dato $\cos\alpha=\dfrac{53}{28}$

$\dfrac{53}{28}=\dfrac{1}{\sec\alpha}$

$\sec\alpha=\dfrac{28}{53}$

En M:

$M=\dfrac{1}{\sqrt{\dfrac{28}{53}+1}}=\dfrac{1}{\sqrt{\dfrac{81}{53}}}$

$\therefore M=\dfrac{\sqrt{53}}{9}$

Clave E

11. Datos:

sena csc41 = 1; a agudo, de razones recíprocas.

$a = 41°$

tanb = cot57°; b agudo, por propiedad de ángulos complementarios:

$b + 57° = 90°$

$b = 33°$

En R:

$R = \cot(a - b) + 7\tan(a + b)$

$R = \cot(41° - 33°) + 7\tan(41° + 33°)$

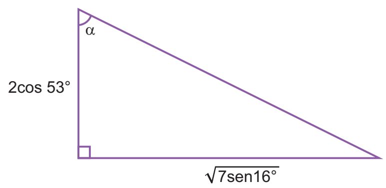
$R = \cot 8° + 7\tan 74°$

$R = 7 + 7 \cdot \frac{24}{7}$

$\therefore R = 31$

Clave A

12.



$$\tan\alpha = \frac{\sqrt{7\text{sen}16°}}{2\cos 53°} = \frac{\sqrt{7 \cdot \frac{7}{25}}}{2 \cdot \frac{3}{5}}$$

$$\tan\alpha = \frac{\frac{7}{5}}{\frac{6}{5}}$$

$$\therefore \tan\alpha = \frac{7}{6}$$

Clave D

13. Datos:

tana tan6° = 1

cot(90° – a) tan6° = 1

Por razones trigonométricas recíprocas:

$6° = 90° - a \Rightarrow a = 84°$

sec(26° – x) = csca

sec(26° – x) = csc84°

Por razones complementarias:

$26° - x + 84° = 90° \Rightarrow x = 20°$

Luego:

$\cot(10° + x) = \cot(10° + 20°)$

$\cot(10° + x) = \cot 30°$

$\cot(10° + x) = \sqrt{3}$

Clave C

14. Sabemos: senx cscy = 1 ⇒ x = y

Entonces en la expresión se cumplirá que:

$(2x - 10°) = (50° - x)$

$3x = 60°$

$x = 20°$

Clave B

PRACTIQUEMOS

Nivel 1 (página 19) Unidad 1

Comunicación matemática

1.

A) $\sec\alpha = \frac{b}{c}$

B) $\tan\theta = \frac{c}{a}$

C) $\text{sen}\alpha = \frac{a}{b}$

D) $\cot\theta = \frac{a}{c}$

∴ Ninguna es correcta.

Clave E

2.

I. Para un mismo ángulo, dos razones son recíprocas si el producto de ellas es igual a la unidad, luego: $\text{sen}\theta \cos\theta \neq 1$ (Falsa)

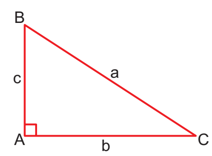
II. El teorema de Pitágoras se cumple en los triángulos rectángulos. (Falsa)

III. Para α y θ complementarios se cumple: $\alpha + \theta = 90° \Rightarrow \text{sen}\alpha = \cos\theta$ (Verdadera)

Clave E

Razonamiento y demostración

3. Por dato:



Piden:

$M = \text{sen}B \cdot \text{sen}C \cdot \tan B \cdot a^2$

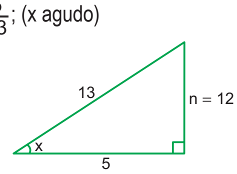
$\Rightarrow M = \left(\frac{b}{a}\right) \cdot \left(\frac{c}{a}\right) \cdot \left(\frac{b}{c}\right) \cdot a^2$

$\therefore M = b^2$

Clave C

4. Por dato:

$\cos x = \frac{5}{13}$; (x agudo)



Por el teorema de Pitágoras: n = 12

Piden:

$M = 4(\cot x + \csc x)$

$M = 4\left(\frac{5}{12} + \frac{13}{12}\right)$

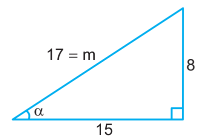
$M = 4\left(\frac{18}{12}\right) = 6$

$\therefore M = 6$

Clave D

5. Por dato:

$\tan\alpha = \frac{8}{15}$; (α agudo)



Por el teorema de Pitágoras: m = 17

Piden:

$R = 60(\tan\alpha + \sec\alpha)$

$R = 60\left(\frac{8}{15} + \frac{17}{15}\right)$

$R = 60\left(\frac{25}{15}\right) = 100$

$\therefore R = 100$

Clave C

6. Por dato:

$\tan\theta^{\tan\theta} = \cos 45°$

Sabemos: $\cos 45° = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

Entonces:

$\tan\theta^{\tan\theta} = \frac{1}{\sqrt{2}} = \left(\frac{1}{2}\right)^{\left(\frac{1}{2}\right)}$

$\Rightarrow \tan\theta^{\tan\theta} = \left(\frac{1}{2}\right)^{\left(\frac{1}{2}\right)}$

Por comparación: $\tan\theta = \frac{1}{2}$

$\Rightarrow \cot\theta = 2$

Pero también:

$\left(\frac{1}{2}\right)^{\left(\frac{1}{2}\right)} <> \left(\frac{1}{4}\right)^{\left(\frac{1}{4}\right)}$

$\Rightarrow \tan\theta^{\tan\theta} = \left(\frac{1}{4}\right)^{\left(\frac{1}{4}\right)}$

Por comparación: $\tan\theta = \frac{1}{4}$

$\Rightarrow \cot\theta = 4$

Por lo tanto, el mayor valor para cotθ es 4.

Clave E

7. Por dato:

$\sec(3x + 43°) - \csc(8x - 30°) = 0$

Entonces: $\sec(3x + 43°) = \csc(8x - 30°)$

Sabemos: $\sec\theta = \csc\beta \Rightarrow \theta + \beta = 90°$

$\Rightarrow (3x + 43°) + (8x - 30°) = 90°$

$11x + 13° = 90°$

$11x = 77°$

$\therefore x = 7°$

Clave D

8. $E = \sec x \tan 2x - 2\cot\left(\frac{3x}{2}\right)$

Para $x = 30°$

$E = \sec 30° \tan 60° - 2\cot 45°$

$E = \frac{2}{\sqrt{3}}\sqrt{3} - 2 \times 1$

$E = 2 - 2$

$\therefore E = 0$

Clave C

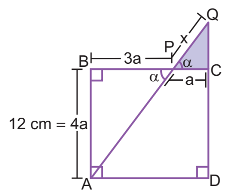
9. $k = (\tan 5° - 1)(\tan 15° - 1) \ldots (\tan 45° - 1) \ldots (\tan 85° - 1)$

$\tan 45° = 1 \Rightarrow \tan 45° - 1 = 0$

$\therefore k = 0$

Clave B

10.



De los datos; ABCD cuadrado, sea PC = a:

$BP = 3PC = 3a$

$AB = BC = 4a$

$12 = 4a$

$a = 3$

$\Rightarrow PC = 3$

⊿ABP y ⊿PCQ notables de 37° y 53°.

$\Rightarrow a = 53°$

Luego: $\cos 53° = \frac{PC}{PQ}$

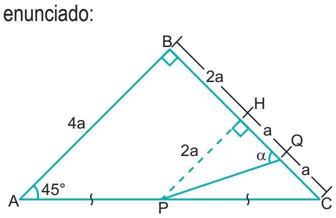
$\frac{3}{5} = \frac{3}{x}$

$\therefore x = 5$ cm

Clave C

Resolución de problemas

11. Del enunciado:



Sea QC = a

$BQ = 3QC = 3a$

Trazamos $\overline{PH} \perp \overline{BC}$, H punto medio de $\overline{BC}$:

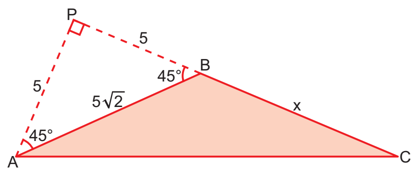
$BH = HC = 2a \wedge PH = \frac{AB}{2} = 2a$

Luego, ⊿PHQ notable de $\frac{53°}{2}$ y $\frac{127°}{2}$:

$\therefore m\angle PQB = \frac{127°}{2}$

Clave D

12.



De los datos; trazamos $\overline{AP} \perp \overleftrightarrow{CB}$

$m\angle A + m\angle C = 45°$

$\Rightarrow m\angle ABP = 45°$

⊿APB notable de 45°

$AP = PB = 5$

Luego en el ⊿APC:

$\tan C = \frac{5}{5 + x}$

$\frac{5}{16} = \frac{5}{5 + x}$

$5 + x = 16$

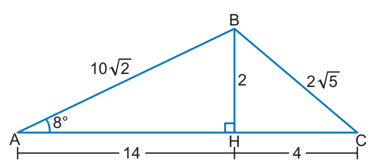
$\therefore x = 11$

Clave B

Nivel 2 (página 19) Unidad 1

Comunicación matemática

13.



$\overline{BH}$ altura relativa a $\overline{AC}$.

⊿AHB notable de 8° y 82°:

$BH = 2 \wedge AH = 14 \wedge HC = 4$

Luego:

⊿BHC notable de $\frac{53°}{2}$ y $\frac{127°}{2}$.

$\Rightarrow m\angle C = \frac{53°}{2} \wedge BC = 2\sqrt{5}$

Además:

$\frac{AC}{BH} = \frac{18}{2} = 9$

$\therefore$ Se puede afirmar I y II.

Clave C

14. La razón trigonométrica de un ángulo es igual a la co-razón de su complemento, luego

$\text{sen} 45° = \cos(90° - 45°)$

$\text{sen} 45° = \cos 45°$

$\therefore$ Solo C es correcta.

Clave C

15. Se tiene:

$\tan\alpha \tan\theta = 1$

Por RT de ángulos complementarios:

$\tan\theta = \cot(90° - \theta)$

Luego:

$\tan\alpha \cot(90° - \theta) = 1$

α y $90° - \theta$ agudos, se cumple:

$\alpha = 90° - \theta$ ...(F)

$\Rightarrow \alpha + \theta = 90° \frac{\pi}{180°}$

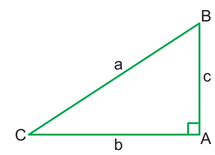
$\Rightarrow \alpha + \theta = \frac{\pi}{2}$ rad $= 90°$ ...(V)

$\Rightarrow \tan\frac{\alpha + \theta}{2} = \tan 45° = 1$ ...(V)

Clave B

Razonamiento y demostración

16. Por dato:



Piden:

$E = \sqrt{(a + b)^2 - 2bc\sqrt{\frac{1 + \cos C}{1 - \cos C}}}$

Primero hallamos el equivalente de:

$\sqrt{\frac{1 + \cos C}{1 - \cos C}} = \sqrt{\frac{1 + \left(\frac{b}{a}\right)}{1 - \left(\frac{b}{a}\right)}}$

$\sqrt{\frac{1 + \cos C}{1 - \cos C}} = \sqrt{\frac{1 + \left(\frac{b}{a}\right)}{1 - \left(\frac{b}{a}\right)}}$

$\sqrt{\frac{1 + \cos C}{1 - \cos C}} = \sqrt{\frac{(a + b)^2}{a^2 - b^2}}$

Por el teorema de Pitágoras:

$a^2 = b^2 + c^2 \Rightarrow a^2 - b^2 = c^2$

$\Rightarrow \sqrt{\frac{1 + \cos C}{1 - \cos C}} = \sqrt{\frac{(a + b)^2}{(c^2)}}$

$\Rightarrow \sqrt{\frac{1 + \cos C}{1 - \cos C}} = \frac{a + b}{c}$

Reemplazando en la expresión E:

$E = \sqrt{(a + b)^2 - 2bc\left(\frac{a + b}{c}\right)}$

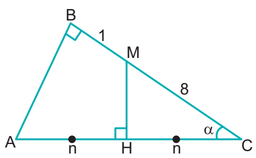
$E = \sqrt{a^2 + 2ab + b^2 - 2ab - 2b^2}$

$\Rightarrow E = \sqrt{a^2 - b^2} = \sqrt{c^2}$

$\therefore E = c$

Clave C

17.



En el ⊿ABC: $\cos\alpha = \dfrac{9}{2n}$

En el ⊿CHM: $\cos\alpha = \dfrac{n}{8}$

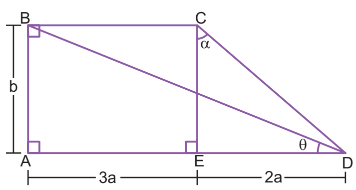
$\Rightarrow \dfrac{9}{2n} = \dfrac{n}{8} \Rightarrow n^2 = 36 \Rightarrow n = 6$

Piden: $\cos\alpha = \dfrac{n}{8} = \dfrac{6}{8}$

$\therefore \cos\alpha = \dfrac{3}{4}$

Clave C

18.



Sea: AB = EC = b

Dato: $\tan\alpha = \dfrac{5}{4}$

$\dfrac{ED}{EC} = \dfrac{5}{4}$

$\dfrac{2a}{b} = \dfrac{5}{4}$

$\dfrac{a}{b} = \dfrac{5}{8}$

En ⊿BAD: $\cot\theta = \dfrac{5a}{b}$

$\cot\theta = 5\left(\dfrac{5}{8}\right)$

$\therefore \cot\theta = \dfrac{25}{8}$

Clave A

19. $49\tan^2\alpha + 1 = 7\tan\alpha + 7\cot(90° - \alpha)$

Por propiedad de ángulos complementarios:

$49\tan^2\alpha + 1 = 7\tan\alpha + 7\tan\alpha$

$49\tan^2\alpha + 1 = 14\tan\alpha$

$\Rightarrow 49\tan^2\alpha - 14\tan\alpha + 1 = 0$

$(7\tan\alpha)^2 - 2 . (1)(7\tan\alpha) + 1 = 0$

$(7\tan\alpha - 1)^2 = 0$

$7\tan\alpha - 1 = 0$

$\tan\alpha = \dfrac{1}{7}$

α agudo:

$\Rightarrow \alpha = 8°$

$\therefore 2\alpha = 16°$

Clave D

20. $P = \dfrac{\tan 17°\left(\tan 73° \cot\dfrac{37°}{2} - \cot\dfrac{53°}{2}\cot 17°\right)}{\sec 85°(\tan 45° \operatorname{sen} 5° + \cos 85°)}$

$P = \dfrac{\tan 17°\ (\tan 73° \times 3 - 2\cot 17°)}{\sec 85° \times 1 \times \operatorname{sen} 5° + \sec 85° \cos 85°}$

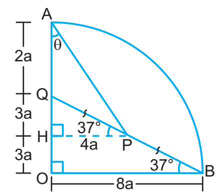
$P = \dfrac{\tan 17°(3\cot 17° - 2\cot 18°)}{\csc 5° \operatorname{sen} 5° + 1}$

$P = \dfrac{\tan 17° \cot 17°}{1 + 1}$

$\therefore P = \dfrac{1}{2}$

Clave D

21.



Trazamos $\overline{PH} \perp \overline{AO}$, ⊿QHP notable de 37° y 53°:

HP = 4a ∧ QH = 3a

H punto medio de $\overline{QO}$:

QH = HO = 3a ∧ OB = AO = 2HP = 8a

En el ⊿AHP:

AH = AO − HO

AH = 8a − 3a

AH = 5a

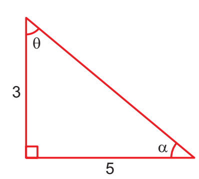
$\tan\theta = \dfrac{HP}{AM} = \dfrac{4a}{5a}$

$\therefore \tan\theta = \dfrac{4}{5}$

Clave E

Resolución de problemas

22.



Del gráfico: $\alpha < \theta$

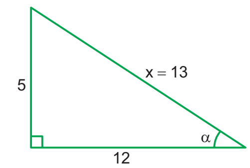
Piden:

$\tan\alpha = \dfrac{3}{5}$

$\therefore \tan\alpha = \dfrac{3}{5}$

Clave D

23.



Por el teorema de Pitágoras:

$x^2 = 5^2 + 12^2$

$x^2 = 25 + 144$

$x^2 = 169$

$\Rightarrow x = 13$

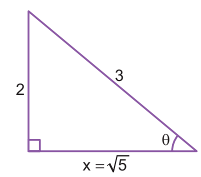
Piden: $P = \csc\alpha + \cot\alpha$

$P = \dfrac{13}{5} + \dfrac{12}{5} = \dfrac{25}{5} = 5$

$\therefore P = 5$

Clave D

24.



Por el teorema de Pitágoras:

$3^2 = x^2 + 2^2$

$9 = x^2 + 4$

$\sqrt{5} = x$

Piden:

$K = \sqrt{5}\ \tan\theta + \dfrac{1}{\sqrt{5}}\ \cos\theta$

$\Rightarrow K = \sqrt{5}\left(\dfrac{2}{\sqrt{5}}\right) + \dfrac{1}{\sqrt{5}}\left(\dfrac{\sqrt{5}}{3}\right)$

$K = 2 + \dfrac{1}{3}$

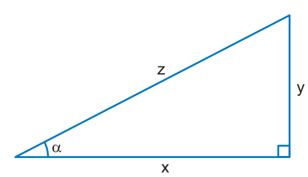
$\therefore K = \dfrac{7}{3}$

Clave D

Nivel 3 (página 20) Unidad 1

Comunicación matemática

25.



I. Para $\alpha = 30°$

$\dfrac{y}{z} = \dfrac{1}{2}$

II. Para $\alpha = 37°$

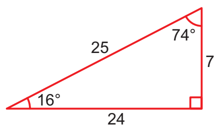
$\dfrac{x}{y} = \dfrac{4}{3}$

III. Para $\alpha = \dfrac{53°}{2}$

$\dfrac{z}{x} = \dfrac{\sqrt{5}}{2}$

Clave C

26. Del ⊿ notable de 16° y 74°:

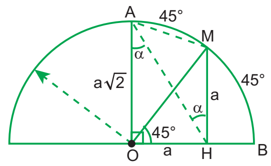


$\therefore \cot 16° = \dfrac{24}{7}$

Clave E

Razonamiento y demostración

27.



Por dato: M es punto medio del arco AB.

$\Rightarrow m\overset{\frown}{AM} = m\overset{\frown}{MB} = 45°$

Por ángulo central: $m\angle MOB = 45°$

En el ⊿OHM notable de 45°:

$OH = HM = a \;\wedge\; OM = a\sqrt{2}$

Pero: $OM = OA \Rightarrow OA = a\sqrt{2}$

En el ⊿AOH por el teorema de Pitágoras:

$AH = a\sqrt{3}$

Como: $\overline{MH} \,//\, \overline{AO} \Rightarrow m\angle MHA = m\angle HAO = \alpha$

Piden:

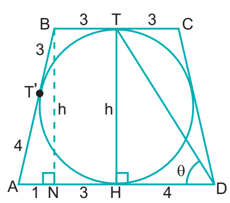
$$\sec^3\alpha = \left(\frac{AH}{OA}\right)^3 = \left(\frac{a\sqrt{3}}{a\sqrt{2}}\right)^3$$

$$\Rightarrow \sec^3\alpha = \left(\frac{\sqrt{3}}{\sqrt{2}}\right)^3 = \left(\frac{\sqrt{6}}{2}\right)^3 = \frac{6\sqrt{6}}{8}$$

$$\therefore \sec^3\alpha = \frac{3\sqrt{6}}{4}$$

Clave C

28.



Por dato: ABCD es un trapecio isósceles.

En el ⊿BNA por el teorema de Pitágoras:

$(BN)^2 + (AN)^2 = (BA)^2$

$h^2 + 1^2 = 7^2$

$h^2 = 48 \Rightarrow h = 4\sqrt{3}$

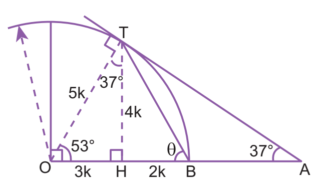
Piden:

$$\tan\theta = \frac{HT}{HD} = \frac{h}{4}$$

$$\Rightarrow \tan\theta = \frac{4\sqrt{3}}{4} = \sqrt{3} \qquad \therefore \tan\theta = \sqrt{3}$$

Clave A

29.



Del ⊿OHT notable de 37° y 53°:

$OT = 5k$; $TH = 4k$; $OH = 3k$

Del gráfico: $OT = OB$

$\Rightarrow OT = OH + HB$

$5k = 3k + HB$

$\Rightarrow HB = 2k$

Piden:

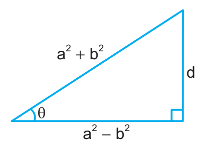
$$\cot\theta = \frac{HB}{TH} = \frac{2k}{4k} = \frac{1}{2}$$

$$\therefore \cot\theta = \frac{1}{2}$$

Clave C

30. Por dato:

$$\sec\theta = \frac{a^2 + b^2}{a^2 - b^2};\ (\theta \text{ agudo})$$



Por el teorema de Pitágoras:

$d^2 + (a^2 - b^2)^2 = (a^2 + b^2)^2$

$d^2 = (a^2 + b^2)^2 - (a^2 - b^2)^2$

$d^2 = 4a^2b^2$

$\Rightarrow d = 2ab$

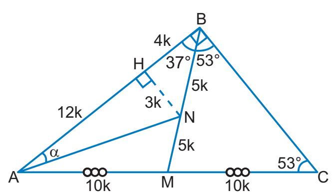
Piden:

$$\tan\theta = \frac{d}{a^2 - b^2} = \frac{(2ab)}{a^2 - b^2}$$

$$\therefore \tan\theta = \frac{2ab}{a^2 - b^2}$$

Clave B

31.



Por dato: $BN = NM \;\wedge\; AM = MC$

Por propiedad: $BM = AM = MC = 10k$

Además: $m\angle MBC = m\angle MCB = 53°$

$\Rightarrow m\angle NBH = 37°$

Del ⊿NHB notable de 37° y 53°:

$NH = 3k \;\wedge\; HB = 4k$

Del ⊿ABC notable de 37° y 53°:

$AB = 16k$

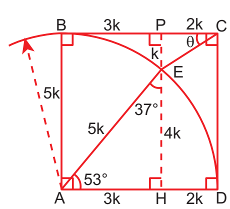
$\Rightarrow AH = 12k$

Piden:

$$\cot\alpha = \frac{AH}{NH} = \frac{12k}{3k} = 4 \qquad \therefore \cot\alpha = 4$$

Clave A

32.



Del gráfico:

$AB = AE = AD = 5k$

Del ⊿AHE notable de 37° y 53°:

$AH = 3k \;\wedge\; HE = 4k$

Luego: $AB = HP$

$\Rightarrow HE + EP = HP$

$4k + EP = 5k \Rightarrow EP = k$

También: $AD = AH + HD$

$\Rightarrow 5k = 3k + HD$

$\Rightarrow HD = 2k \Rightarrow PC = 2k$

Piden:

$$\tan\theta + \cot\theta = \frac{EP}{PC} + \frac{PC}{EP}$$

$$\tan\theta + \cot\theta = \frac{k}{2k} + \frac{2k}{k}$$

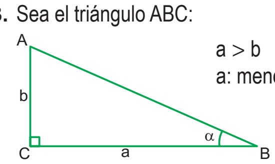
$$\Rightarrow \tan\theta + \cot\theta = \frac{1}{2} + 2$$

$$\therefore \tan\theta + \cot\theta = \frac{5}{2}$$

Clave C

Resolución de problemas

33. Sea el triángulo ABC:



$a > b$

a: menor ángulo agudo

Por dato:

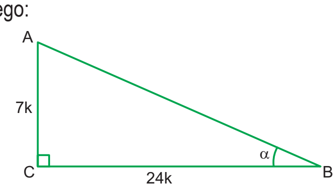
$$\frac{a + b}{a - b} = \frac{31}{17}$$

$17a + 17b = 31a - 31b$

$48b = 14a$

$$\frac{b}{a} = \frac{7}{24}$$

Luego:



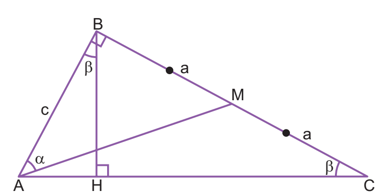
⊿ABC notable de 16° y 74°

$\Rightarrow \alpha = 16°$

$$\therefore \csc 16° = \frac{25}{7}$$

Clave E

34.



Del gráfico: $m\angle ABH = m\angle BCH = \beta$

Piden:

$J = \tan\alpha \,.\, \tan\beta$

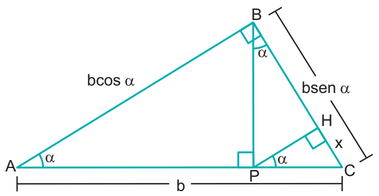
$$J = \left(\frac{a}{c}\right) . \left(\frac{c}{2a}\right) = \frac{1}{2}$$

$$\therefore J = \frac{1}{2}$$

Clave D

APLICAMOS LO APRENDIDO (página 22) Unidad 1

1.



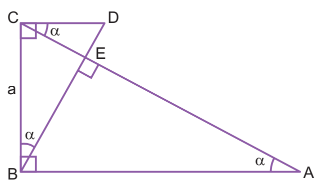
En el ◺BPC:
PC = (bsenα)senα $\Rightarrow$ PC = bsen$^2\alpha$

En el ◺PHC:
HC = PC . senα
HC = (bsen$^2\alpha$) senα $\Rightarrow$ HC = bsen$^3\alpha$

Como: HC = x
$\therefore$ x = bsen$^3\alpha$

Clave D

2. Considera que piden AE en el gráfico siguiente:

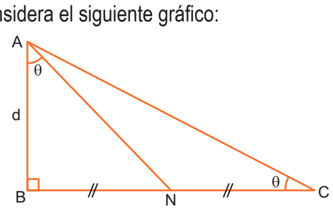


En el ◺BEC: BE = acosα $\wedge$ CE = asenα

En el ◺BEA:
AE = BE cotα
AE = (acosα) cotα
$\therefore$ AE = acosαcotα

Clave E

3. Considera el siguiente gráfico:



Sea: AB = d
Entonces: BN = dtanθ
Por dato: BN = NC
En el ◺ABC:
BC = AB . cotθ
2BN = dcotθ
$\Rightarrow$ 2dtanθ = d . cotθ

$$2\tan\theta = \frac{1}{\tan\theta} \Rightarrow \tan^2\theta = \frac{1}{2}$$

$$\tan\theta = \frac{1}{\sqrt{2}}$$

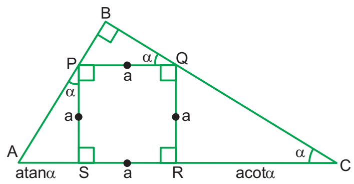
Piden:
M = $\sqrt{2}$ tanθ + 1

$$M = \sqrt{2}\left(\frac{1}{\sqrt{2}}\right) + 1 = 1 + 1 = 2$$

$\therefore$ M = 2

Clave C

4.



Del gráfico:
RC = acotα $\wedge$ AS = atanα

Por dato: AC = 5PQ

$\Rightarrow$ atanα + a + acotα = 5(a)

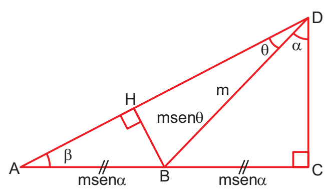
a(tanα + cotα + 1) = 5a

tanα + cotα + 1 = 5

$\therefore$ J = tanα + cotα = 4

Clave B

5.



Sea: BD = m
Trazamos: $\overline{BH} \perp \overline{AD}$

Del gráfico:
BC = msenα $\wedge$ BH = msenθ

En el ◺BHA:

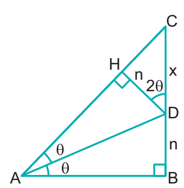
$$\text{sen}\beta = \frac{\text{msen}\theta}{\text{msen}\alpha}$$

$\Rightarrow$ senα senβ = senθ

$$\therefore R = \frac{\text{sen}\alpha\,.\,\text{sen}\beta}{\text{sen}\theta} = 1$$

Clave E

6.

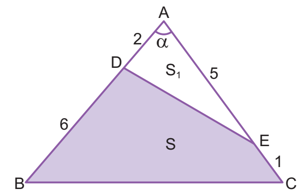


Trazamos: $\overline{DH} \perp \overline{AC}$
Por el teorema de la bisectriz interior:
BD = DH

En el ◺DHC:
DC = HD sec2θ
x = (n) sec2θ
$\therefore$ x = nsec2θ

Clave B

7.



Del gráfico:

$$S_1 = \frac{DA\,AE}{2}\text{sen}\alpha$$

$$S_1 = \frac{(2)(5)}{2}\text{sen}\alpha \Rightarrow S_1 = 5\text{sen}\alpha$$

Además:

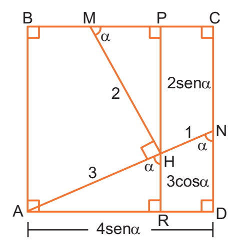
$$S_1 + S = \frac{AB\,AC}{2}\text{sen}\alpha$$

$$S_1 + S = \frac{(8)(6)}{2}\text{sen}\alpha = 24\text{sen}\alpha$$

Entonces:
(5senα) + S = 24senα
$\therefore$ S = 19senα

Clave D

8.



Por dato: ABCD es un cuadrado.
Entonces: PR = AD
2senα + 3cosα = 4senα
3cosα = 2senα

$$\Rightarrow \frac{\text{sen}\alpha}{\cos\alpha} = \frac{3}{2}$$

$$\Rightarrow \tan\alpha = \frac{3}{2}$$

Como: tanα cotα = 1

$$\frac{3}{2}\cot\alpha = 1 \Rightarrow \cot\alpha = \frac{2}{3}$$

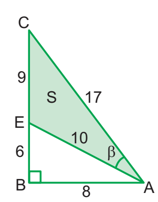
Piden:

$$P = \tan\alpha + \cot\alpha = \frac{3}{2} + \frac{2}{3} = \frac{13}{6}$$

$$\therefore P = \frac{13}{6}$$

Clave C

9.



Por el teorema de Pitágoras:
AE = 10 $\wedge$ AC = 17

Por áreas:

$$S = \frac{(\text{base})(\text{altura})}{2} = \frac{9 \cdot 8}{2} \quad ...(I)$$

$$S = \frac{(AC)(AE)}{2} \cdot \text{sen}\beta = \frac{17 \cdot 10}{2}\text{sen}\beta \quad ...(II)$$

Igualando (I) y (II): $36 = 85\text{sen}\beta \Rightarrow \text{sen}\beta$

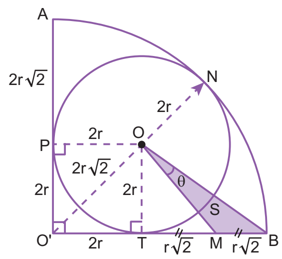
Piden:

$T = 85\text{sen}\beta + 2 = (36) + 2 = 38$

$\therefore T = 38$

Clave E

10.



Por el teorema de Pitágoras: $OM = \sqrt{6}\,r \ \wedge \ OB = 2\sqrt{3}\,r$

Por áreas:

$$S = \frac{(\text{base})(\text{altura})}{2} = \frac{(r\sqrt{2})(2r)}{2}$$

$$\Rightarrow S = \sqrt{2}\,r^2 \quad ...(I)$$

$$S = \frac{(OB)(OM)}{2}\text{sen}\theta = \frac{(2\sqrt{3}\,r)(\sqrt{6}\,r)}{2}\text{sen}\theta$$

$$\Rightarrow S = 3\sqrt{2}\,r^2\text{sen}\theta \quad ...(II)$$

Igualando (I) y (II): $\sqrt{2}\,r^2 = 3\sqrt{2}\,r^2\text{sen}\theta$

$$\frac{1}{3} = \text{sen}\theta \Rightarrow \frac{1}{\text{sen}\theta} = 3$$

$\therefore \csc\theta = 3$

Clave B

11.

- ⊿AND: $DN = AD\text{sen}\alpha$
  $DN = L\text{sen}\alpha$
  $AN = AD\cos\alpha$
  $AN = L\cos\alpha$

- ⊿AMB: $m\angle MBA = \alpha$
  $MA = BA\text{sen}\alpha$
  $MA = L\text{sen}\alpha$
  $BM = BA\cos\alpha$
  $BM = L\cos\alpha$

Luego: $MN = L\text{sen}\alpha + L\cos\alpha$

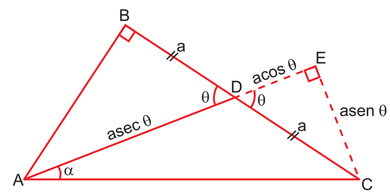
$$= \left(\frac{BM + DN}{2}\right)$$

$$= \left(\frac{L\cos\alpha + L\text{sen}\alpha}{2}\right)$$

$$= \frac{L}{2}(\cos\alpha + \text{sen}\alpha)$$

Clave A

12.



- ⊿DEC: $EC = DC\text{sen}\theta$
  $= a\text{sen}\theta$
  $DE = DC\cos\theta$
  $= a\cos\theta$

- ⊿ABD: $AD = BD\sec\theta$
  $AD = a\sec\theta$

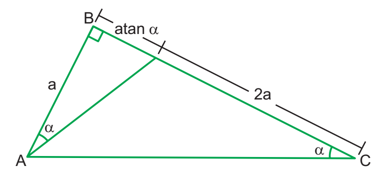
Luego:

$$\cot\alpha = \frac{a\sec\theta + a\cos\theta}{a\text{sen}\theta}$$

$$\therefore \cot\alpha = \frac{\sec\theta + \cos\theta}{\text{sen}\theta}$$

Clave C

13.



Piden: $P = \cot\alpha - \tan\alpha$

Del ⊿ABC:

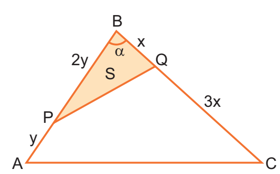
$$\cot\alpha = \frac{a\tan\alpha + 2a}{a} = \tan\alpha + 2$$

$\Rightarrow \cot\alpha - \tan\alpha = 2$

$\therefore P = 2$

Clave B

14.



$$S = \frac{2y\,x}{2}\text{sen}\alpha = xy\text{sen}\alpha$$

$$S_{TOTAL} = \frac{3y\,4x}{2}\text{sen}\alpha = 6xy\text{sen}\alpha$$

$\therefore S_{TOTAL} = 6S$

Clave E

PRACTIQUEMOS

Nivel 1 (página 24) Unidad 1

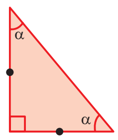
Comunicación matemática

1.

$$\text{sen}(\alpha) = \frac{\text{cateto opuesto}}{\text{hipotenusa}} \qquad (F)$$

$$S = \frac{a.b}{2}\text{sen}\alpha \qquad (F)$$

El teorema de Pitágoras se aplica solo a triángulos rectángulos. (F)



$\alpha + \alpha = 90$

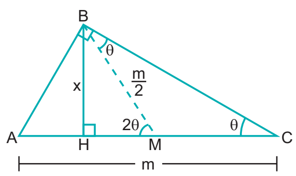
$2\alpha = 90°$

$\therefore \alpha = 45°$ (V)

2.

Razonamiento y demostración

3.



Trazamos la mediana relativa a la hipotenusa.

Por propiedad: $BM = \dfrac{AC}{2}$

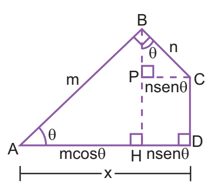
Además: $m\measuredangle MCB = m\measuredangle MBC = \theta$

En el ⊿BHM: $BH = BM\ \text{sen}2\theta$

$\therefore x = \dfrac{m}{2}\text{sen}2\theta$

Clave D

4.



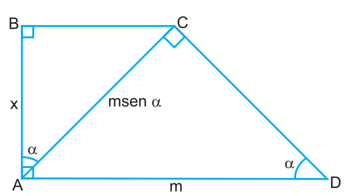
Piden: x

Del gráfico: $x = AH + HD$

$\therefore x = m\cos\theta + n\text{sen}\theta$

Clave B

5.



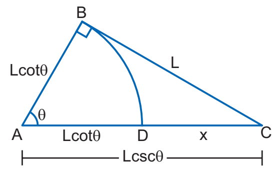
En el ⊿ABC: $AB = AC\cos\alpha$

$\Rightarrow x = (m\text{sen}\alpha)\cos\alpha$

$\therefore x = m\text{sen}\alpha\cos\alpha$

Clave C

6.



Del ⊿ABC: $AB = L\cot\theta \ \wedge\ AC = L\csc\theta$

Como BAD es un sector circular, entonces:

$AB = AD = L\cot\theta$

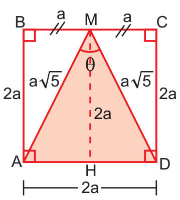
Luego: $AD + DC = AC$

$\Rightarrow L\cot\theta + x = L\csc\theta$

$\therefore x = L(\csc\theta - \cot\theta)$

Clave B

7.



En los triángulos rectángulos ABM y DCM, por el teorema de Pitágoras: $AM = MD = a\sqrt{5}$

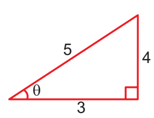
Luego por áreas:

$$A_{\triangle AMD} = \frac{(AD)(MH)}{2} = \frac{(AM)(MD)}{2}\text{sen}\theta$$

Entonces:

$$\frac{(2a)(2a)}{2} = \frac{(a\sqrt{5})(a\sqrt{5})}{2}\text{sen}\theta$$

$$\Rightarrow 2a^2 = \frac{5a^2}{2}\text{sen}\theta \Rightarrow \text{sen}\theta = \frac{4}{5}$$



Piden:

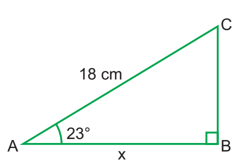
$A = \text{sen}\theta + 2\cos\theta$

$$\Rightarrow A = \frac{4}{5} + 2\left(\frac{3}{5}\right) = \frac{4+6}{5}$$

$\therefore A = 2$

Clave E

8. Por dato: $\cos 23° \approx 0{,}920506$



Del ⊿ABC: $AB = AC\cos 23°$

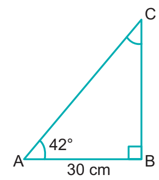
$\Rightarrow x = (18)(0{,}920506)$

$\therefore x = 16{,}5691$ cm

Clave A

Resolución de problemas

9.



$AC = \dfrac{30}{\cos 42°}$

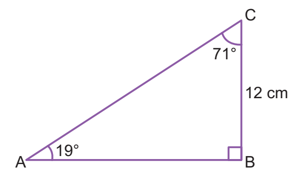
Dato:

$\cos 42° = 0{,}743145$

$$\therefore A = \frac{30}{0{,}743145} = 40{,}36897 \text{ cm}$$

Clave C

10.



$AB = 12\tan 71°$

$AB = 12\ (2{,}904208)$

$\therefore AB = 34{,}8505$ cm

Clave B

Nivel 2 (página 25) Unidad 1

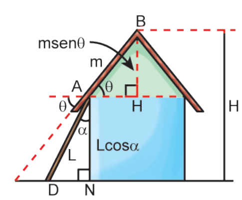
Comunicación matemática

11.

12.

Razonamiento y demostración

13.



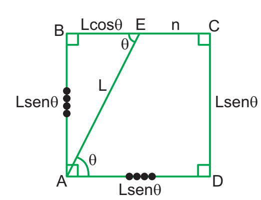
Del gráfico:

$H = BH + AN$

$\therefore H = m\text{sen}\theta + L\cos\alpha$

Clave B

14.



Por dato: ABCD es un cuadrado.

Entonces: $BC = AD$

$L\cos\theta + n = L\text{sen}\theta$

$\Rightarrow n = L\text{sen}\theta - L\cos\theta$

Piden el perímetro (2p) del trapecio AECD.

$2p = L + n + L\text{sen}\theta + L\text{sen}\theta$

$2p = L + 2L\text{sen}\theta + n$

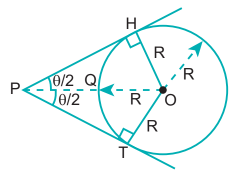
$2p = L + 2L\text{sen}\theta + (L\text{sen}\theta - L\cos\theta)$

$\Rightarrow 2p = L + 3L\text{sen}\theta - L\cos\theta$

$\therefore 2p = L(1 + 3\text{sen}\theta - \cos\theta)$

Clave B

15.



La mínima distancia de P a la circunferencia es PQ.

Del ◺PHO: $PO = OH \csc\frac{\theta}{2}$

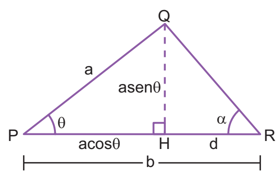
$\Rightarrow PQ + QO = (R)\csc\frac{\theta}{2}$

$\Rightarrow PQ + (R) = R\csc\frac{\theta}{2}$

$\therefore PQ = R\left(\csc\frac{\theta}{2} - 1\right)$

Clave D

16.



Del gráfico: $PH + HR = PR$

$\Rightarrow a\cos\theta + d = b$

$\Rightarrow d = b - a\cos\theta$

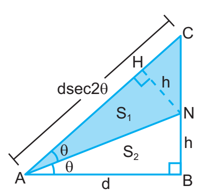
Piden:

$\tan\alpha = \frac{QH}{HR} = \frac{a\text{sen}\theta}{d}$

$\therefore \tan\alpha = \frac{a\text{sen}\theta}{b - a\cos\theta}$

Clave D

17.



Por el teorema de la bisectriz:

$BN = NH = h$

Piden:

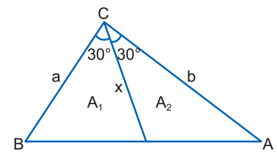
$$\frac{S_1}{S_2} = \frac{\frac{(AC)(NH)}{2}}{\frac{(AB)(NB)}{2}} = \frac{(AC)(h)}{(AB)(h)}$$

$$\Rightarrow \frac{S_1}{S_2} = \frac{AC}{AB} = \frac{d\sec 2\theta}{d}$$

$$\therefore \frac{S_1}{S_2} = \sec 2\theta$$

Clave D

18.



Por áreas: $A_{\Delta ABC} = A_1 + A_2$

Entonces:

$\frac{ab}{2}\text{sen }60° = \frac{ax}{2}\text{sen}30° + \frac{xb}{2}\text{sen}30°$

$ab\left(\frac{\sqrt{3}}{2}\right) = ax\left(\frac{1}{2}\right) + xb\left(\frac{1}{2}\right)$

$ab\sqrt{3} = (a + b)x$

$\Rightarrow x = \frac{ab\sqrt{3}}{a + b}$

Por dato: $a + b = ab$

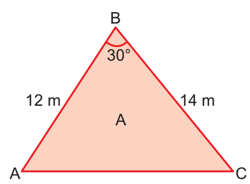
$\Rightarrow x = \frac{ab\sqrt{3}}{(ab)} = \sqrt{3}$

$\therefore x = \sqrt{3}$

Clave A

Resolución de problemas

19.



Piden:

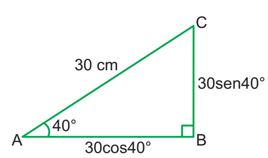
$A_{\Delta ABC} = \frac{(AB)(BC)}{2}\text{sen}30°$

$\Rightarrow A_{\Delta ABC} = \frac{(12)(14)}{2}\left(\frac{1}{2}\right) = 42$

$\therefore A_{\Delta ABC} = 42 \text{ m}^2$

Clave C

20.



Piden el perímetro del triángulo (2p).

$2p = AC + CB + AB$

$\Rightarrow 2p = 30 + 30\text{sen}40° + 30\cos 40°$

$\Rightarrow 2p = 30(1 + \text{sen}40° + \cos 40°)$

Además:

$\text{sen}40° \approx 0{,}64279 \wedge \cos 40° \approx 0{,}76604$

$\Rightarrow 2p = 30(1 + 0{,}64279 + 0{,}76604)$

$2p = 30(2{,}40883)$

$\therefore 2p = 72{,}2649$ cm

Clave C

Nivel 3 (página 26) Unidad 1

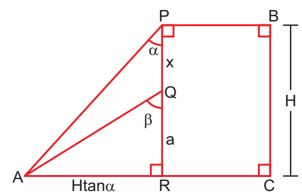
Comunicación matemática

21.

22.

Razonamiento y demostración

23.



Del gráfico: $BC = PR = H$

En el ◺ARP: $AR = PR \tan\alpha$

$\Rightarrow AR = H\tan\alpha$

En el ◺ARQ: $QR = AR \cot\beta$

$\Rightarrow a = H\tan\alpha\cot\beta$

Piden: $PQ = x$

Luego: $PQ + QR = PR$

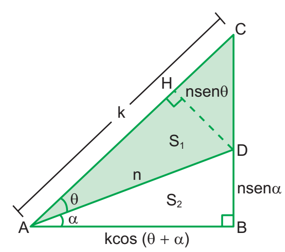
$\Rightarrow x + a = H$

$x + H\tan\alpha\cot\beta = H$

$\therefore x = H(1 - \tan\alpha\cot\beta)$

Clave C

24.



Sea: $AD = n$

Por dato: $\frac{S_1}{S_2} = \frac{k\text{sen}\theta}{\text{sen}\alpha}$

$$\Rightarrow \frac{\frac{(AC)(HD)}{2}}{\frac{(AB)(DB)}{2}} = \frac{k\text{sen}\theta}{\text{sen}\alpha}$$

$$\Rightarrow \frac{(AC)(HD)}{(AB)(DB)} = \frac{k\text{sen}\theta}{\text{sen}\alpha}$$

Entonces:

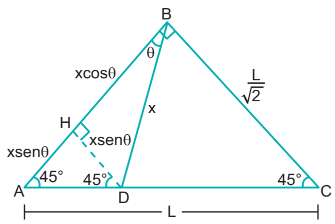
$$\frac{(k)(n\,\text{sen}\theta)}{(k\cos(\theta+\alpha))(n\,\text{sen}\alpha)} = \frac{k\,\text{sen}\theta}{\text{sen}\alpha}$$

$$\Rightarrow \frac{1}{\cos(\theta+\alpha)} = k$$

$\therefore k = \sec(\theta+\alpha)$

Clave C

25.



Del ◺ABC notable de 45°:

$BC = AB = \dfrac{L}{\sqrt{2}}$

Del ◺BHD: $BH = x\cos\theta \ \wedge\ HD = H\text{sen}\theta$

Del ◺AHD notable de 45°:

$AH = HD = x\text{sen}\theta$

Luego: $AH + HB = AB$

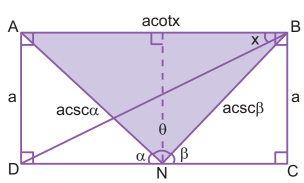
$\Rightarrow x\text{sen}\theta + x\cos\theta = \dfrac{L}{\sqrt{2}}$

$\Rightarrow x(\text{sen}\theta + \cos\theta) = \dfrac{L}{\sqrt{2}}$

$\therefore x = \dfrac{L}{\sqrt{2}}(\text{sen}\theta + \cos\theta)^{-1}$

Clave D

26.



Por áreas:

$$A_{\Delta ANB} = \frac{(\text{base})(\text{altura})}{2} = \frac{(a\cot x)(a)}{2}$$

$$\Rightarrow A_{\Delta ANB} = \frac{a^2\cot x}{2} \quad ...(I)$$

Además:

$$A_{\Delta ANB} = \frac{(AN)(NB)}{2}\,\text{sen}\theta$$

$$\Rightarrow A_{\Delta ANB} = \frac{(a\csc\alpha)\,(a\csc\beta)}{2}\,\text{sen}\theta$$

$$\Rightarrow A_{\Delta ANB} = \frac{a^2\csc\alpha\,\csc\beta}{2}\,\text{sen}\theta \quad ...(II)$$

Igualando (I) y (II):

$$\frac{a^2\csc\alpha\,\csc\beta}{2}\,\text{sen}\theta = \frac{a^2\cot x}{2}$$

Entonces:

$$\csc\alpha\csc\beta = \frac{\cot x}{\text{sen}\theta} = \cot x\left(\frac{1}{\text{sen}\theta}\right)$$

$\csc\alpha\csc\beta = \cot x(\csc\theta)$

Piden:

$M = \csc\alpha\ \csc\beta\ \sec x$

$M = (\cot x\csc\theta)\sec x$

Recordar: $\cot x = \dfrac{\cos x}{\text{sen}x} \ \wedge\ \sec x = \dfrac{1}{\cos x}$

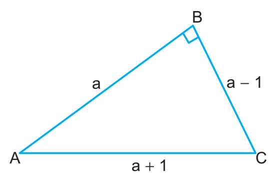
$\Rightarrow M = \dfrac{\cos x}{\text{sen}x}\csc\theta\,.\,\dfrac{1}{\cos x}$

$\Rightarrow M = \left(\dfrac{1}{\text{sen}x}\right)\csc\theta = (\csc x)\csc\theta$

$\therefore M = \csc x\csc\theta$

Clave B

27. Primero:

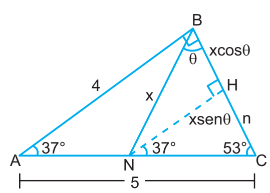


Por el teorema de Pitágoras:

$(a+1)^2 = a^2 + (a-1)^2$

Resolviendo: $a = 4$

Luego el triángulo rectángulo ABC resulta ser notable de 37° y 53°.



Del ◺NHC:

$n = (x\text{sen}\theta)\cot 53° = x\text{sen}\theta\left(\dfrac{3}{4}\right)$

$\Rightarrow n = \dfrac{3x}{4}\text{sen}\theta$

Como: $BC = 3 \Rightarrow BH + HC = 3$

Entonces:

$x\cos\theta + n = 3$

$x\cos\theta + \dfrac{3x}{4}\text{sen}\theta = 3$

$x(4\cos\theta + 3\text{sen}\theta) = 12$

$\Rightarrow 4\cos\theta + 3\text{sen}\theta = \dfrac{12}{x}$

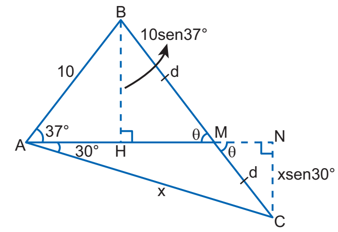
Piden:

$\left(\dfrac{x}{12}\right)^{-1} = \dfrac{12}{x}$

$\therefore \left(\dfrac{x}{12}\right)^{-1} = 4\cos\theta + 3\text{sen}\theta$

Clave C

28.



Del ◺BHM: $\text{sen}\theta = \dfrac{10\text{sen}37°}{d}$ ...(I)

Del ◺MNC: $\text{sen}\theta = \dfrac{x\text{sen}30°}{d}$ ...(II)

Igualando (I) y (II): $\dfrac{10\text{sen}37°}{d} = \dfrac{x\text{sen}30°}{d}$

$\Rightarrow 10\left(\dfrac{3}{5}\right) = x\left(\dfrac{1}{2}\right)$

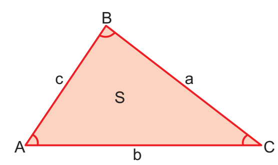
$6 = \dfrac{x}{2}$

$\therefore x = 12$

Clave E

Resolución de problemas

29.



Por dato: $S = 0{,}5\ m^2$

Sabemos:

- $S = \dfrac{bc}{2}\text{sen}A \Rightarrow \dfrac{1}{\text{sen}A} = \dfrac{bc}{2S}$

  $\Rightarrow \csc A = \dfrac{bc}{2S}$

- $S = \dfrac{ab}{2}\text{sen}C \Rightarrow \dfrac{1}{\text{sen}C} = \dfrac{ab}{2S}$

  $\Rightarrow \csc C = \dfrac{ab}{2S}$

- $S = \dfrac{a\,.\,c}{2}\text{sen}B \Rightarrow \dfrac{1}{\text{sen}B} = \dfrac{ac}{2S}$

  $\Rightarrow \csc B = \dfrac{ac}{2S}$

Piden:

$\csc A\ \csc B\ \csc C = \left(\dfrac{bc}{2S}\right)\left(\dfrac{ab}{2S}\right)\left(\dfrac{ac}{2S}\right)$

$\Rightarrow \csc A\ \csc B\ \csc C = \dfrac{a^2b^2c^2}{8S^3}$

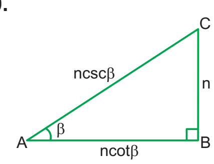
Como: $S = 0{,}5 = \dfrac{1}{2}$

$\Rightarrow S^3 = \dfrac{1}{8} \Rightarrow 8\,S^3 = 1$

$\therefore \csc A\ \csc B\ \csc C = a^2b^2c^2$

Clave B

30.



Piden: el perímetro (2p) del triángulo.

$2p = CB + AB + AC$

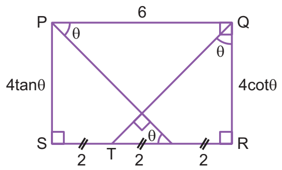
$\Rightarrow 2p = n + n\cot\beta + n\csc\beta$

$\therefore 2p = n(1 + \cot\beta + \csc\beta)$

Clave D

MARATÓN MATEMÁTICA
(página 27) Unidad 1

1. Del gráfico:



Como PQRS es rectángulo:
PS = QR
$4\tan\theta = 4\cot\theta$
$\tan\theta = \cot\theta \Rightarrow \theta = 45°$

Nos piden:
$M = \cot^2\theta - 1$
$M = (1)^2 - 1 = 0$
$\therefore M = 0$

Clave A

2. Sabemos:

$\frac{S}{C} = \frac{9}{10}$

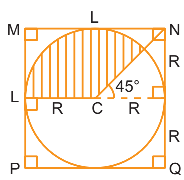
$\Rightarrow 10.\sqrt{2x+3} = 9\sqrt{\frac{5x+5}{2}}$

$100(2x+3) = 81 \times 5 \times \frac{(x+1)}{2}$

$200(2x+3) = 81 . 5(x+1)$
$40(2x+3) = 81(x+1)$
$80x + 120 = 81x + 81$
$\therefore 39 = x$

Clave B

3. Del gráfico:



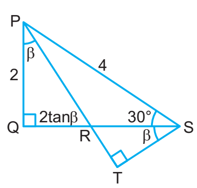
$S = \frac{\theta r^2}{2}$

$2S = (180° - 45°) . R^2$
$27\pi\ m^2 = (\pi - \pi/4)R^2$
$27\pi\ m^2 = \frac{3\pi}{4}R^2$
$36\ m^2 = R^2 \Rightarrow R = 6\ m$

Nos piden:
$L = 2R$
$L = 2(6\ m) \Rightarrow L = 12\ m$

Clave C

4. Del gráfico:



$\frac{ST}{RS} = \cos\beta$

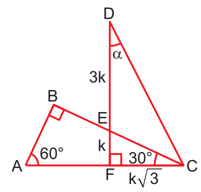
$ST = RS\cos\beta$

$ST = (2\sqrt{3} - 2\tan\beta)\cos\beta$

$\therefore ST = 2\cos\beta(\sqrt{3} - \tan\beta)$

Clave B

5. Del gráfico tenemos:



Entonces:

$\tan\alpha = \frac{FC}{DF}$

$\tan\alpha = \frac{k\sqrt{3}}{3k+k} = \frac{k\sqrt{3}}{4k}$

$\therefore \tan\alpha = \frac{\sqrt{3}}{4}$

Clave D

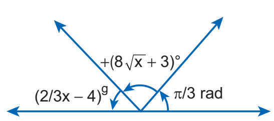
6.
- 1° ———— 360"
  $(\overline{a3}_{(b)})°$ ———— 4320"
  a < b
  $\Rightarrow \overline{a3}_{(b)} = \frac{4320}{360} = 12$

- ab + 3 = 12
  ab = 9
  1 × 9
  3 × 3 ⇒ a = 1, b = 9
  9 × 1

- 1° ———— 60'
  19° ———— x ⇒ x = 60' × 19, x = 1140'

Clave A

7.



$\left(\frac{2}{3}x - 4\right)^g\left(\frac{9°}{10^g}\right) + (8\sqrt{x}+3)° + \frac{\pi}{3}rad\left(\frac{180°}{\pi rad}\right) = 180°$

$\left(\frac{2x-12}{3}\right) \times \frac{9}{10} + (8\sqrt{x}+3)° + 60° = 180°$

$3x - 18 + (8\sqrt{x}+3)\,5 = 600$

$3x - 18 + 40\sqrt{x} + 15 = 600$

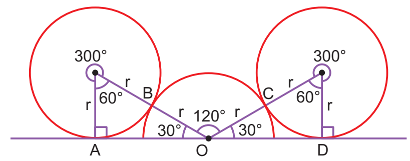
Si: $x = k^2 \Rightarrow 3k^2 + 40k = 603$

$k = 9$

$\Rightarrow x = 81$

Clave E

8.



Luego la longitud total de la curva es:

$L = \overset{\frown}{AB} + \overset{\frown}{BC} + \overset{\frown}{CD}$

$L = 300°\left(\frac{\pi}{180°}\right)r + 120°\left(\frac{\pi}{180°}\right)r + 300°\left(\frac{\pi}{180°}\right)r$

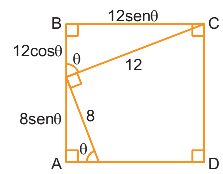
$L = r\left(\frac{5}{3}\pi + \frac{2}{3}\pi + \frac{5}{3}\pi\right) \Rightarrow L = r(4\pi)$

$L = 14\ m \times 4 \times \left(\frac{22}{7}\right)$

$\therefore L = 176\ m$

Clave B

9. Tenemos:



AB = BC

$8\text{sen}\theta + 12\cos\theta = 12\text{sen}\theta$

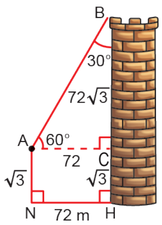
$12\cos\theta = 4\text{sen}\theta$

$\therefore \cot\theta = 1/3$

Clave B

APLICAMOS LO APRENDIDO (página 30) Unidad 2

1.



Del gráfico:

El ⊿ACB es notable de 30° y 60°.

$\Rightarrow BC = AC\sqrt{3} \Rightarrow BC = 72\sqrt{3}$

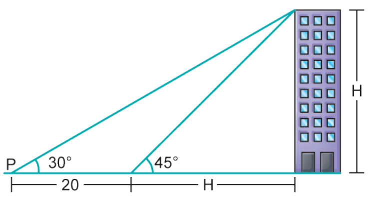
Piden: la altura de la torre.

$BH = BC + CH \Rightarrow BH = 72\sqrt{3} + \sqrt{3}$

$\therefore\ BH = 73\sqrt{3}$ m

Clave C

2. Sea H: la altura del edificio.



Del gráfico: $\dfrac{H+20}{H} = \cot 30°$

$\dfrac{H+20}{H} = \sqrt{3} \Rightarrow H + 20 = \sqrt{3}H$

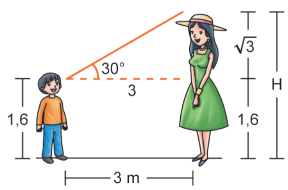
$20 = H(\sqrt{3} - 1)$

$H = \dfrac{20(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$\therefore\ H = 10(\sqrt{3} + 1)$ m

Clave A

3. Sea la altura de la mamá: h.



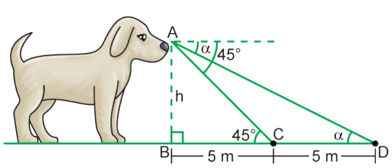
Entonces:

$H = \sqrt{3} + 1{,}6$

$\therefore H = (\sqrt{3} + 1{,}6)$ m

Clave C

4.



El ⊿ABC es notable de 45° $\Rightarrow$ n = 5 m

Del gráfico:

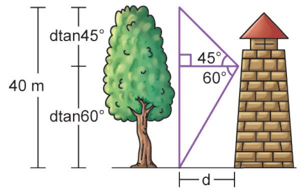
$\tan\alpha = \dfrac{h}{10} = \dfrac{5}{10} = \dfrac{1}{2} \Rightarrow \tan\alpha = \dfrac{1}{2}$

$\dfrac{1}{2} = \tan\dfrac{53°}{2}$

$\therefore \alpha = \dfrac{53°}{2}$

Clave B

5.



Del gráfico:

$d\tan 45° + d\tan 60° = 40$

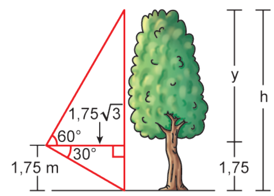
$d(1) + d(\sqrt{3}) = 40$

$d = \dfrac{40}{\sqrt{3}+1}$

$\therefore d = 20(\sqrt{3} - 1)$ m

Clave A

6. Sea la altura del árbol: h.



Del gráfico:

$y = (1{,}75\sqrt{3})\sqrt{3}$

Además:

$h = 1{,}75 + y$

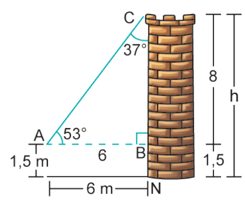
$h = 1{,}75 + (1{,}75\sqrt{3})\sqrt{3}$

$h = 1{,}75 + (1{,}75)(3) = 1{,}75 + 5{,}25$

$\therefore h = 7$ m

Clave C

7.



Del ⊿ABC notable de 37° y 53°:

CB = 8

Sea h: la altura de la torre.

Del gráfico:

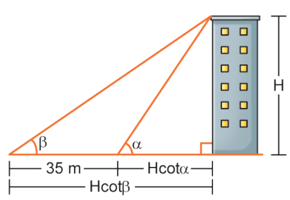
h = CB + BN

h = 8 + 1,5 = 9,5

$\therefore$ h = 9,5 m

Clave B

8.



Sea H: la altura del edificio.

Por dato: $\cot\beta - \cot\alpha = 0{,}7$

Del gráfico:

$H\cot\beta = 35 + H\cot\alpha$

$H\cot\beta - H\cot\alpha = 35$

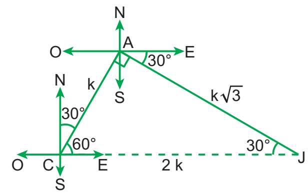
$H(\cot\beta - \cot\alpha) = 35$

$H(0{,}7) = 35$

$\therefore H = 50$ m

Clave B

9.



Del gráfico: el triángulo CAJ resulta ser rectángulo y notable de 30° y 60°.

Por dato: CA + AJ = 1 $\Rightarrow k + k\sqrt{3} = 1$

$k(1 + \sqrt{3}) = 1 \Rightarrow k = \dfrac{\sqrt{3}-1}{2}$

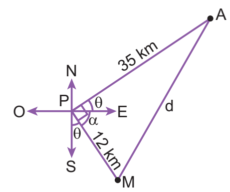
Piden:

$JC = 2k = 2\left(\dfrac{\sqrt{3}-1}{2}\right)$

$\therefore JC = (\sqrt{3} - 1)$ km

Clave A

10.



Del gráfico: $\theta + \alpha = 90°$

$\Rightarrow m\angle APM = 90°$

Entonces, el triángulo MPA es rectángulo.

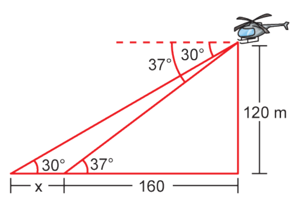
Por el teorema de Pitágoras: $d^2 = 12^2 + 35^2$

$\Rightarrow d^2 = 1369$

$\therefore\ d = 37$ km

Clave B

11.



Del gráfico: $x + 160 = 120\cot 30°$

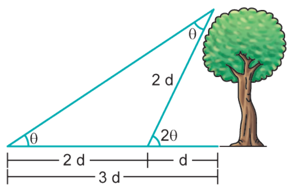
$x + 160 = 120\sqrt{3}$

$x = 120\sqrt{3} - 160$

$\Rightarrow x = 40(3\sqrt{3} - 4)$

Clave D

12.

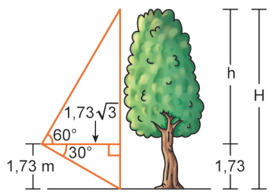


Del gráfico: $\cos 2\theta = \dfrac{d}{2d} = \dfrac{1}{2} \Rightarrow 2\theta = 60°$

$\theta = 30°$

Clave E

13.



Del gráfico: $h = (1{,}73\sqrt{3})\sqrt{3}$

$h = 3(1{,}73)$

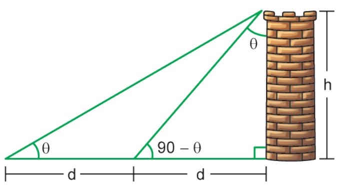
Además: $H = h + 1{,}73$

$H = 3(1{,}73) + 1{,}73$

$H = 4(1{,}73) = 6{,}92$

Clave A

14.



Del gráfico: $\tan\theta = \dfrac{d}{h} = \dfrac{h}{2d} \Rightarrow 2d^2 = h^2$

$$\frac{d^2}{h^2} = \frac{1}{2} \Rightarrow \underbrace{\frac{d}{h}}_{} = \frac{1}{\sqrt{2}}$$

$$\tan\theta = \frac{\sqrt{2}}{2}$$

Clave B

PRACTIQUEMOS

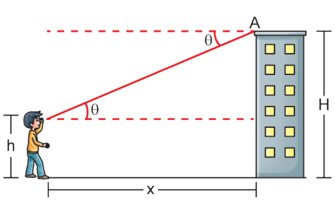
Nivel 1 (página 32) Unidad 2

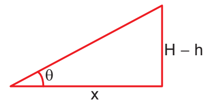
Comunicación matemática

1. Pierre Simon Laplace (1749–1827): matemático francés que publicó un libro de 5 volúmenes titulado Mecánica celeste.
2.

Razonamiento y demostración

3.

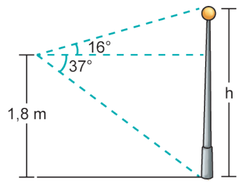


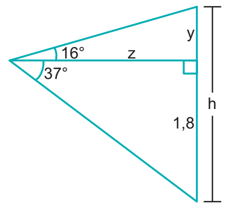


$\therefore\ x = (H - h)\cot\theta$

Clave B

4.





$z = 1{,}8\cot 37° \ \Rightarrow z = 2{,}4$ m

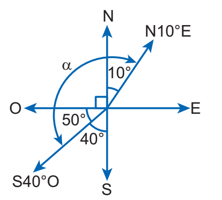
$y = 2{,}4\tan 16° \ \Rightarrow \ y = 0{,}7$ m

$\therefore\ h = 0{,}7 + 1{,}8 = 2{,}5$ m

Clave A

Resolución de problemas

5.



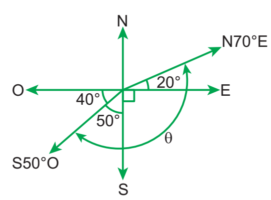
El menor ángulo que forman estas direcciones será:

$\alpha = 50° + 90° + 10°$

$\therefore \alpha = 150°$

Clave D

6.

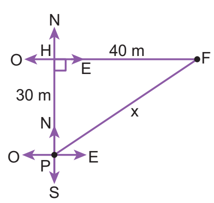


El menor ángulo que forman estas direcciones será:

$\theta = 50° + 90° + 20° \qquad \therefore \theta = 160°$

Clave E

7.



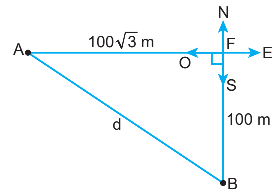
En el ⊿PHF por el teorema de Pitágoras:

$x^2 = 30^2 + 40^2 \Rightarrow x^2 = 2500$

$\therefore x = 50$ m

Clave B

8.



En el ⊿AFB por el teorema de Pitágoras:

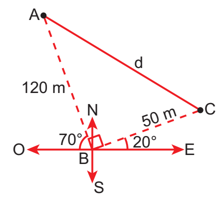
$d^2 = 100^2 + (100\sqrt{3})^2$

$d^2 = 40\ 000$

$\therefore d = 200$ m

Clave B

9.



En el ⊿ABC por el teorema de Pitágoras:

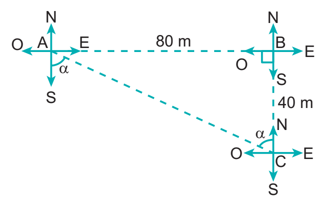
$d^2 = 120^2 + 50^2$

$d^2 = 16\ 900$

$\therefore d = 130$ m

Clave A

10.



Del ⊿ABC: $\tan\alpha = \dfrac{AB}{BC}$

$\tan\alpha = \dfrac{80}{40}$

$\therefore \tan\alpha = 2$

Clave C

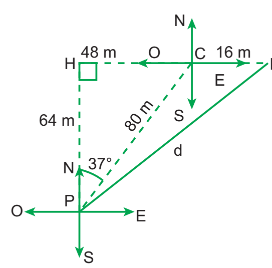
Nivel 2 (página 33) Unidad 2

Comunicación matemática

11.
12.

Razonamiento y demostración

13.



En el ⊿PHC notable de 37° y 53°:

HP = 64 ∧ HC = 48 m

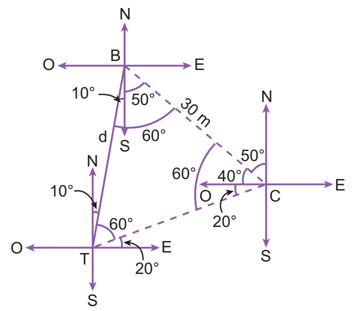
En el ⊿PHF por el teorema de Pitágoras

$d^2 = (PH)^2 + (HF)^2 \Rightarrow d^2 = 64^2 + (48 + 16)^2$

$\therefore\ d = 64\sqrt{2}$ m

Clave D

14.



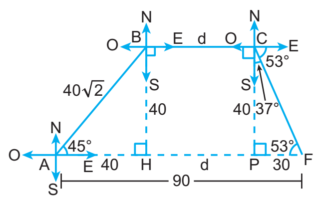
Del gráfico: el ΔTBC es equilátero

$\Rightarrow$ TB = BC = CT $\qquad \therefore\ d = 30$ m

Clave E

Resolución de problemas

15.



Del ⊿AHB notable de 45°: AH = 40

Del ⊿CPF notable de 37° y 53°: PF = 30

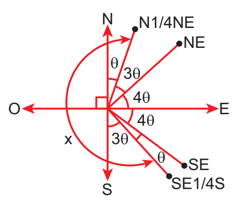
Luego:

$40 + d + 30 = 90 \Rightarrow d + 70 = 90$

$\therefore\ d = 20$ km

Clave B

16.



Del gráfico: $8\theta = 90° \Rightarrow 4\theta = 45°$

Además:

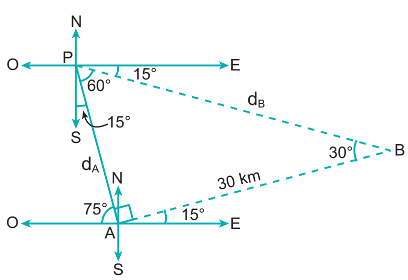
$x = \theta + 180° + 3\theta$

$x = 180° + 4\theta = 180° + 45°$

$\therefore\ x = 225°$

Clave C

17.



Del gráfico:

$d_B = 30\csc 60° = 30\left(\frac{2\sqrt{3}}{3}\right)$

$\Rightarrow d_B = 20\sqrt{3}$

$d_A = 30\cot 60° = 30\left(\frac{\sqrt{3}}{3}\right)$

$\Rightarrow d_A = 10\sqrt{3}$

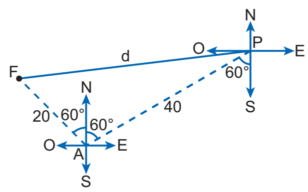
Piden:

$d_B - d_A = 20\sqrt{3} - 10\sqrt{3}$

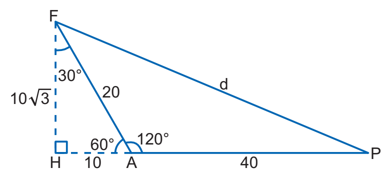
$\therefore\ d_B - d_A = 10\sqrt{3}$ km

Clave B

18.



Luego:



Del ⊿FHA notable de 30° y 60°:

$FH = 10\sqrt{3} \quad \wedge \quad HA = 10$

En el ⊿FHP por el teorema de Pitágoras:

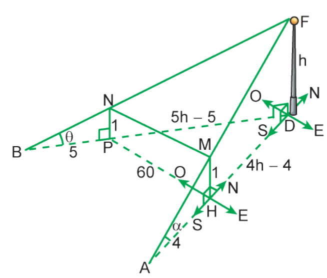
$d^2 = (10\sqrt{3})^2 + (50)^2$

$d^2 = 2800$

$\therefore\ d = 20\sqrt{7}$ km

Clave C

19.



Sea h la altura del faro.

Del gráfico:

$\cot\alpha = \frac{AH}{MH} = \frac{AD}{FD}$

$\Rightarrow \frac{4}{1} = \frac{AD}{h} \Rightarrow AD = 4h$

$\cot\theta = \frac{BP}{NP} = \frac{BD}{FD}$

$\Rightarrow \frac{5}{1} = \frac{BD}{h} \Rightarrow BD = 5h$

En el ⊿PHD por el teorema de Pitágoras:

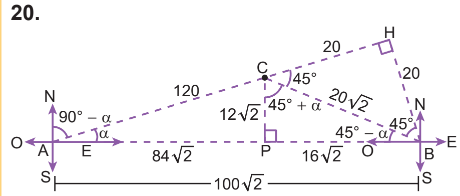
$(4h - 4)^2 + 60^2 = (5h - 5)^2$

$(h - 1)^2 = 400 \Rightarrow h - 1 = 20$

$\therefore\ h = 21$ m

Clave D

20.



En el ⊿AHB por el teorema de Pitágoras:

$AB = 100\sqrt{2}$

Luego: $\operatorname{sen}\alpha = \frac{CP}{AC} = \frac{HB}{AB}$

$\Rightarrow \frac{CP}{120} = \frac{20}{100\sqrt{2}} \Rightarrow CP = 12\sqrt{2}$

En el ⊿APC por el teorema de Pitágoras: $AP = 84\sqrt{2}$

Como: AP + PB = AB

$\Rightarrow 84\sqrt{2} + PB = 100\sqrt{2}$

$PB = 16\sqrt{2}$

En el ⊿CPB:

$\cot(45° + \alpha) = \frac{CP}{PB} = \frac{12\sqrt{2}}{16\sqrt{2}} = \frac{3}{4}$

$\therefore\ \cot(45° + \alpha) = \frac{3}{4}$

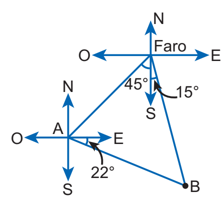
Clave B

Nivel 3 (página 33) Unidad 2

Comunicación matemática

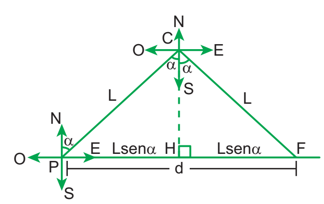
21.

22.



Razonamiento y demostración

23.



Del gráfico: el ΔPCF resulta isósceles

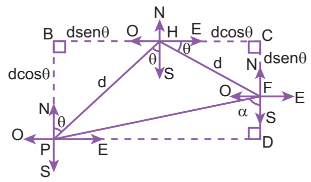
$\Rightarrow$ PC = FC = L

Además: PH = HF = Lsenα

Piden: d = 2Lsenα

Clave E

24.



Del gráfico:
PD = dcosθ + dsenθ

Además:
FD = BP – CF = dcosθ – dsenθ

Piden:

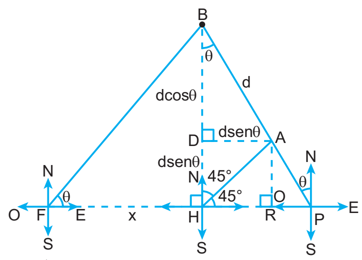
$$\tan\alpha = \frac{PD}{FD} = \frac{d(\operatorname{sen}\theta + \cos\theta)}{d(\cos\theta - \operatorname{sen}\theta)}$$

$$\therefore\ \tan\alpha = \left(\frac{\operatorname{sen}\theta + \cos\theta}{\cos\theta - \operatorname{sen}\theta}\right)$$

Clave E

Resolución de problemas

25.



Del ◺ADH notable de 45°: AD = HD = dsenθ

Del ◺FHB: $\cot\theta = \frac{FH}{BH}$

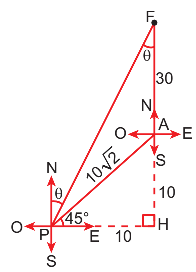
Entonces:

$$\frac{x}{\operatorname{dsen}\theta + \operatorname{dcos}\theta} = \cot\theta$$

$\therefore$ x = dcotθ(senθ + cosθ)

Clave C

26.



Por dato: el tiempo para ir de P a A es $\sqrt{2}$ s y para ir de A a F es 3 s, además la velocidad en todo el trayecto es de 10 m/s.

$\Rightarrow PA = v\,.\,t_{PA} = (10)(\sqrt{2}) \Rightarrow PA = 10\sqrt{2}$ m

$\Rightarrow AF = v\,.\,t_{AF} = (10)(3) \Rightarrow AF = 30$ m

Del ◺PHA notable de 45°: PH = HA = 10

Del ◺PHF: $\tan\theta = \frac{PH}{HF}$

$$\Rightarrow \tan\theta = \frac{10}{40} \Rightarrow \tan\theta = \frac{1}{4}$$

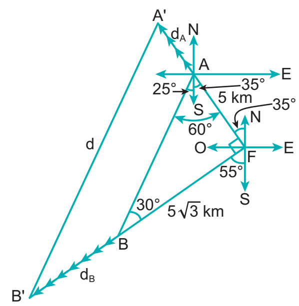
$$\Rightarrow \theta = \arctan\frac{1}{4}$$

Luego la dirección del barco al final respecto al punto de partida será:

NθE <> $\text{Narctan}\frac{1}{4}$E

Clave E

27.



Del gráfico: el ◺AFB resulta notable de 30° y 60°.

$\Rightarrow BF = 5\sqrt{3}$ km

Luego:

$d_A = v_A\,.\,t$

$d_A = (24)(1{,}25) = 30$

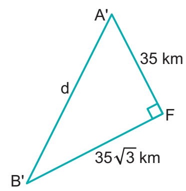
$d_A = 30$ km

$d_B = v_B\,.\,t$

$d_B = (24\sqrt{3})(1{,}25) = 30\sqrt{3}$

$d_B = 30\sqrt{3}$ km

Entonces:



Por el teorema de Pitágoras:

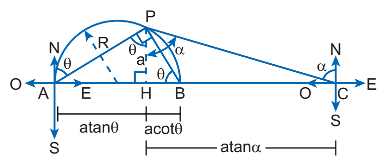
$d^2 = (35)^2 + (35\sqrt{3})^2$

$d^2 = 4900$

$\therefore d = 70$ km

Clave C

28.



Por dato: AB = BC = 2R

Del gráfico:

AB = atanθ + acotθ

$\Rightarrow$ 2R = a(tanθ + cotθ) ...(I)

AC = atanθ + atanα

$\Rightarrow$ AB + BC = atanθ + atanα

2R + 2R = a(tanθ + tanα)

4R = a(tanθ + tanα) ...(II)

De (II) y (I):

$$\frac{4R}{2R} = \frac{a(\tan\theta + \tan\alpha)}{a(\tan\theta + \cot\theta)}$$

$$2 = \frac{\tan\theta + \tan\alpha}{\tan\theta + \cot\theta}$$

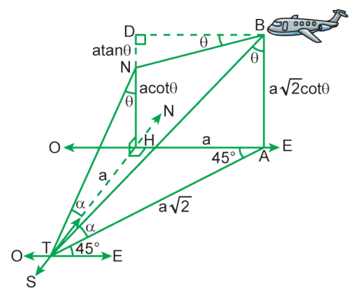
Entonces:

2tanθ + 2cotθ = tanθ + tanα

$\therefore$ tanα = tanθ + 2cotθ

Clave D

29.



Por dato: α = 90° – θ

Del gráfico: AB = HD

$\Rightarrow$ AB = HN + ND

$a\sqrt{2}\cot\theta = a\cot\theta + a\tan\theta$

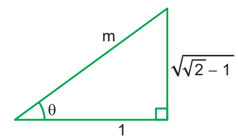
$(\sqrt{2} - 1)\cot\theta = \tan\theta$

$$(\sqrt{2} - 1)\left(\frac{1}{\tan\theta}\right) = \tan\theta$$

Entonces:

$\tan^2\theta = \sqrt{2} - 1 \Rightarrow \tan\theta = \sqrt{\sqrt{2} - 1}$

Luego:



Por el teorema de Pitágoras:

$m^2 = 1^2 + (\sqrt{\sqrt{2} - 1})^2$

$m^2 = 1 + \sqrt{2} - 1$

$m^2 = \sqrt{2} \Rightarrow m = \sqrt[4]{2}$

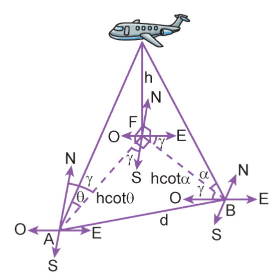
Piden:

$\sec\theta = \frac{m}{1} = m$

$\therefore \sec\theta = \sqrt[4]{2}$

Clave C

30.



Sea h: la altura a la que vuela el avión.
Se plantea la equivalencia:

$N\frac{1}{4}NE$ <> NγNE ∧ $O\frac{1}{4}NO$ <> OγNO

Luego se deduce que: m∠AFB = 90°

En el ◺BFA por el teorema de Pitágoras:

$(h\cot\theta)^2 + (h\cot\alpha)^2 = d^2$

$h^2(\cot^2\theta + \cot^2\alpha) = d^2$

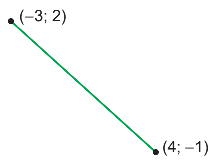
$h\sqrt{\cot^2\theta + \cot^2\alpha} = d$

$$\therefore\ h = \frac{d}{\sqrt{\cot^2\theta + \cot^2\alpha}}$$

Clave C

APLICAMOS LO APRENDIDO (página 35) Unidad 2

1.



Pendiente $\Rightarrow$ $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Reemplazamos:

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m = \dfrac{(-1) - 2}{4 - (-3)} = \dfrac{-3}{7}$

$m = \dfrac{-3}{7}$

Clave B

2. Hallamos la pendiente

$m = \dfrac{y_1 - y_2}{x_1 - x_2} \Rightarrow m = \dfrac{6 - 2}{2 - (-4)} = \dfrac{4}{6} = \dfrac{2}{3}$

Tomamos un punto de paso, en este caso P.

La ecuación de la recta:

$y - y_0 = m(x - x_0)$

$y - 2 = \dfrac{2}{3}(x - (-4))$

$3y - 6 = 2x + 8$

$\therefore\ \overleftrightarrow{L}: 2x - 3y + 14 = 0$

Clave D

3.

Para un punto $(x_1; y_1)$ y:

$\overleftrightarrow{L}: Ax + By + C = 0$

$d(P; \overleftrightarrow{L}) = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

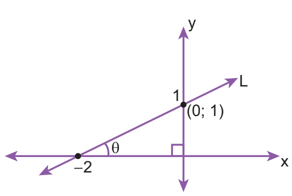
Reemplazamos para (3; 0) y $\overleftrightarrow{L}: 3x + 2y + 4 = 0$

$d(P; \overleftrightarrow{L}) = \dfrac{|3(3) + 2(0) + 4|}{\sqrt{3^2 + 2^2}}$

$d(P; \overleftrightarrow{L}) = \dfrac{|13|}{\sqrt{13}} \Rightarrow d(P; \overleftrightarrow{L}) = \sqrt{13}$

Clave A

4.



Del gráfico:

$\tan\theta = \dfrac{1}{2}$

Entonces, la pendiente (m):

$m = \tan\theta = \dfrac{1}{2}$

Empleando la ecuación ordinaria de la recta:

$y = mx + b$

$y = \dfrac{1}{2}x + b$

Evaluando el punto de paso (0; 1):

$1 = \dfrac{1}{2}(0) + b \Rightarrow b = 1$

Entonces: $y = \dfrac{1}{2}x + 1$

$0 = x - 2y + 2$

$\therefore$ La ecuación de la recta será: $x - 2y + 2 = 0$

Clave B

5. Si $\overleftrightarrow{L}$ es perpendicular $\overleftrightarrow{L}_1$; se cumple:

$m_L \cdot m_{L_1} = -1$ ... (I)

Como: $3x + y - 8 = 0$

La pendiente de $\overleftrightarrow{L}$ es:

$m_L = \dfrac{-3}{1} \Rightarrow m_L = -3$

Reemplazamos en (I):

$\therefore\ (-3) \cdot m_{L_1} = -1$

$m_{L_1} = \dfrac{1}{3}$

La ecuación de $\overleftrightarrow{L}_1$ en el punto M(1; –3) y pendiente 1/3 es:

$(y - y_0) = m(x - x_0)$

$(y - (-3)) = m(x - 1)$

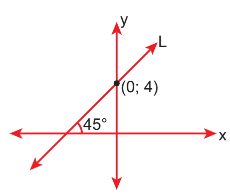
$y + 3 = \dfrac{1}{3}(x - 1)$

$\overleftrightarrow{L}_1$: $3y + 9 = x - 1$

$\therefore\ \overleftrightarrow{L}_1: x - 3y - 10 = 0$

Clave E

6.



Sea m: pendiente de la recta L.

$m = \tan 45° = 1$

$\Rightarrow m = 1$

Empleando la ecuación ordinaria de la recta:

$y = mx + b$

$y = 1x + b$

Evaluando el punto de paso (0; 4):

$4 = 1(0) + b \Rightarrow b = 4$

Entonces:

$y = x + 4$

$0 = x - y + 4$

$\therefore$ La ecuación de la recta será: $x - y + 4 = 0$

Clave E

7.

Hallamos la pendiente de $\overleftrightarrow{L}_1$:

$m_1 = \dfrac{3 - 0}{0 - 4} \Rightarrow m_1 = -3/4$

Como $L_1 \perp L_2$, entonces se cumple:

$m_1 \cdot m_2 = -1$

$\left(\dfrac{-3}{4}\right)m_2 = -1 \Rightarrow m_2 = 4/3$

Luego, $\overleftrightarrow{L}_2$ tiene pendiente 4/3 y pasa por el punto (2; 3/2).

$\overleftrightarrow{L}_2: y - y_2 = m(x - x_2)$

$\overleftrightarrow{L}_2: y - \dfrac{3}{2} = \dfrac{4}{3}(x - 2)$

$\overleftrightarrow{L}_2: 3y - \dfrac{9}{2} = 4x - 8$

$\overleftrightarrow{L}_2: 6y - 9 = 8x - 16$

$\therefore\ \overleftrightarrow{L}_2: 8x - 6y - 7 = 0$

Clave B

8.

Hallamos el baricentro G(x; y):

$G(x; y) = \dfrac{(-3; 3) + (-3; -4) + (3; -2)}{3}$

$G(x; y) = \dfrac{(-3 - 3 + 3; 3 - 4 - 2)}{3}$

$G(x; y) = (-1; -1)$

Como tiene pendiente $m = 2/3$; entonces:

$\overleftrightarrow{L}: y - y_0 = m(x - x_0)$

$\overleftrightarrow{L}: y - (-1) = \dfrac{2}{3}(x - (-1))$

$\overleftrightarrow{L}: 3y + 3 = 2x + 2$

$\therefore\ \overleftrightarrow{L}: 2x - 3y - 1 = 0$

Clave A

9. El punto P(a; b) es el punto de intersección de:

$\overleftrightarrow{L}_1: 4x - 3y + 1 = 0$

$\overleftrightarrow{L}_2: 5x - 2y - 4 = 0$

Resolviendo el sistema:

$\left.\begin{array}{l} 4x - 3y = -1 \\ 5x - 2y = 4 \end{array}\right\} x = 2 \wedge y = 3$

Entonces el punto de intersección es:

$P(a; b) = P(2; 3) \Rightarrow a = 2 \wedge b = 3$

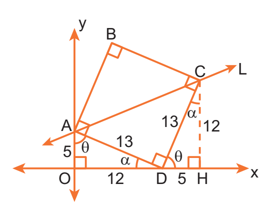
Piden:

$(3a - b)^2 = (3 \cdot 2 - 3)^2 = (3)^2 = 9$

$\therefore (3a - b)^2 = 9$

Clave C

10.



Del gráfico: $\triangle$AOD $\cong$ $\triangle$DHC (A-L-A)

Por el teorema de Pitágoras: AO = 5

Entonces se tiene:

A(0; 5) y C(17; 12)

Calculamos la pendiente de $\overleftrightarrow{L}$ con estos puntos:

$m = \dfrac{12-5}{17-0} = \dfrac{7}{17} \Rightarrow m = \dfrac{7}{17}$

Empleando la ecuación ordinaria de la recta:

$y = mx + b \Rightarrow y = \dfrac{7}{17}x + b$

El intercepto con el eje y es b; entonces: b = 5

Luego:

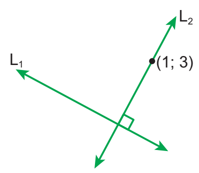
$y = \dfrac{7x}{17} + 5$

$0 = 7x - 17y + 85$

$\therefore$ La ecuación de la recta será: $7x - 17y + 85 = 0$

Clave E

11.



Del gráfico: $\overleftrightarrow{L_1} \perp \overleftrightarrow{L_2}$

Entonces: $m_1 \cdot m_2 = -1$

De $\overleftrightarrow{L_1}: y = \dfrac{3}{4}x - \dfrac{11}{4} \Rightarrow m_1 = \dfrac{3}{4}$

Luego:

$\left(\dfrac{3}{4}\right)m_2 = -1 \Rightarrow m_2 = -\dfrac{4}{3}$

Empleando la ecuación ordinaria de la recta:

$y = mx + b \Rightarrow y = \left(-\dfrac{4}{3}\right)x + b$

Evaluando el punto de paso (1; 3):

$3 = -\dfrac{4}{3}(1) + b \Rightarrow b = \dfrac{13}{3}$

Entonces:

$y = -\dfrac{4}{3}x + \dfrac{13}{3}$

$0 = 4x + 3y - 13$

$\therefore$ La ecuación de la recta L_2 será: $4x + 3y - 13 = 0$.

Clave B

12.

$\overleftrightarrow{L_2}: 3x - y + 5 = 0 \Rightarrow y = 3x + 5 \Rightarrow m_2 = 3$

Por dato: $\overleftrightarrow{L_1} \,//\, \overleftrightarrow{L_2}$

Entonces: $m_1 = m_2$

$m_1 = 3$

Además, M(4; –3) es un punto que pertenece a $\overleftrightarrow{L_1}$.

Empleando la ecuación ordinaria de la recta:

$y = mx + b \Rightarrow y = 3x + b$

Evaluando el punto de paso (4; –3):

$-3 = 3(4) + b \Rightarrow b = -15$

Luego:

$y = 3x - 15$

$0 = 3x - y - 15$

$\therefore$ La ecuación de la recta será: $3x - y - 15 = 0$.

Clave E

13. Calculamos la pendiente de L_1:

$m_1 = \dfrac{y_1 - y_2}{x_1 - x_2} \Rightarrow m_1 = \dfrac{2-(-1)}{-3-1} = \dfrac{3}{-4}$

$m_1 = -3/4$

Hallamos la $\tan\theta$:

$\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 + m_2}$

$\tan\theta = \dfrac{-\dfrac{3}{4} - \dfrac{1}{2}}{1 + \left(-\dfrac{3}{4}\right)\left(\dfrac{1}{2}\right)}$

$\tan\theta = \dfrac{-\dfrac{5}{4}}{\dfrac{5}{8}} \Rightarrow \tan\theta = -\dfrac{8}{4}$

$\therefore \tan\theta = -2$

Clave C

14. $L_1: ax + 2y - 6 + b = 0$

$\Rightarrow y = -\dfrac{ax}{2} + \dfrac{6-b}{2}$

$L_2: (b-2)x - 3y + a = 0$

$\Rightarrow y = \dfrac{b-2}{3}x + \dfrac{a}{3}$

Por dato: $\overleftrightarrow{L_1} \,//\, \overleftrightarrow{L_2} \Rightarrow m_1 = m_2$

$\left(-\dfrac{a}{2}\right) = \left(\dfrac{b-2}{3}\right)$

$\Rightarrow -3a = 2b - 4$ ... (I)

Además: $N(2; -3) \in \overleftrightarrow{L_1}$

Entonces:

$a(2) + 2(-3) - 6 + b = 0$

$2a + b = 12$ ... (II)

De (I) y (II): $a = 20 \wedge b = -28$

Piden: a + b

$a + b = 20 + (-28) = -8$

$\therefore a + b = -8$

Clave C

PRACTIQUEMOS

Nivel 1 (página 37) Unidad 2

Comunicación matemática

1.

2.

I. Si: $\overleftrightarrow{L_1} \perp \overleftrightarrow{L_2} \Rightarrow m_1 - m_2 = -1$ (F)

II. Si: $\overleftrightarrow{L_1} \,//\, \overleftrightarrow{L_2} \Rightarrow m_1 = m_2$ (V)

III. Si: $\overleftrightarrow{L_1} \,//\, \overleftrightarrow{x} \Rightarrow m_1 = 0$ (V)

IV. Si: $\overleftrightarrow{L_2} \perp \overleftrightarrow{y} \Rightarrow m_2 = 0$ (F)

$\therefore$ FVVF

Razonamiento y demostración

3. Si una recta tiene un ángulo de inclinación (α), el valor de su pendiente (m) será: $m = \tan\alpha$.

Por dato: $\alpha = 37°$

$\Rightarrow m = \tan 37° = \dfrac{3}{4}$

$\therefore m = \dfrac{3}{4}$

Clave D

4. Sabemos:

Pendiente: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Donde: $A(x_1; y_1)$ y $B(x_2; y_2)$ son puntos de paso de la recta.

Por dato: $A(x_1; y_1) = A(-2; -3)$

$B(x_2; y_2) = B(1; 1)$

$\Rightarrow m = \dfrac{(1)-(-3)}{(1)-(-2)} = \dfrac{1+3}{1+2} = \dfrac{4}{3}$

$\therefore m = \dfrac{4}{3}$

Clave C

5. Si una recta tiene un ángulo de inclinación (α), el valor de su pendiente (m) será: $m = \tan\alpha$.

Por dato: $\alpha = 150°$

$m = \tan 150° = -\tan 30°$

$\Rightarrow m = -\left(\dfrac{1}{\sqrt{3}}\right) = -\dfrac{\sqrt{3}}{3}$

$\therefore m = -\dfrac{\sqrt{3}}{3}$

Clave D

6. Por dato, la recta L tiene un ángulo de inclinación (α) de 37° y pasa por el punto $(x_0; y_0) = (1; 2)$.

Luego:

Pendiente: $m = \tan\alpha = \tan 37° \Rightarrow m = \dfrac{3}{4}$

Piden la ecuación de la recta L.

$y - y_0 = m(x - x_0)$

$y - 2 = \dfrac{3}{4}(x - 1)$

$4y - 8 = 3x - 3$

$\Rightarrow 3x - 4y + 5 = 0$

$\therefore \overleftrightarrow{L}: 3x - 4y + 5 = 0$

Clave A

7. Por dato:

$\overleftrightarrow{L}: 3x + y + 1 = 0$

La recta L tiene la forma general:

$L: Ax + By + C = 0$

Donde la pendiente es: $m = -\dfrac{A}{B}$

Comparando: $A = 3 \wedge B = 1$

Entonces: $m = -\dfrac{3}{1} = -3$

$\therefore m = -3$

Clave E

8. Si una recta tiene un ángulo de inclinación (α), el valor de su pendiente (m) será: $m = \tan\alpha$.

Por dato: $\alpha = 143°$

$\Rightarrow m = \tan 143° = \tan(180° - 37°)$

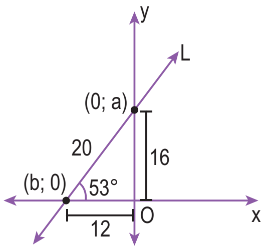
$\Rightarrow m = -\tan 37° = -\left(\dfrac{3}{4}\right)$

$\therefore m = -\dfrac{3}{4}$

Clave C

Resolución de problemas

9. Tenemos un triángulo notable.



Los interceptos son:

(b; 0) = (–12; 0)

(0; a) = (0; 16)

$\therefore$ a + b = –12 + 16 = 4

Clave B

10. Sea el punto P(a; b), su radio vector es:

$r = \sqrt{a^2 + b^2}$

$5^2 = a^2 + b^2$

$25 = a^2 + b^2$ ...(1)

Hallamos la pendiente de $\overleftrightarrow{PA}$:

$m = \dfrac{y_1 - y_2}{x_1 - x_2}$

$\dfrac{1}{2} = \dfrac{b-4}{a-3}$

a – 3 = 2b – 8

a = 2b – 5

Reemplazamos en I:

$25 = (2b - 5)^2 + b^2$

$25 = 4b^2 - 20b + 25 + b^2$

$25 = 5b^2 - 20b + 25$

$20b = 5b^2$

$\Rightarrow b = 4 \wedge a = 3$

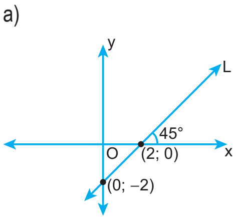
$\therefore$ P(a; b) = (3; 4)

Clave A

Nivel 2 (página 37) Unidad 2

Comunicación matemática

11. a)



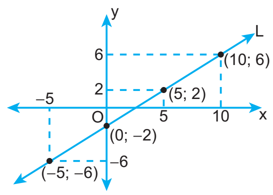
b) Despejamos y:

$y = \dfrac{4}{5}x - 2$

Comparamos dando valores:

x	y
–5	–6
0	–2
5	2
10	6

Dibujamos los puntos y los unimos:



12. A: $m_1 = -4$

$m_2 = 3/2$

$\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 \cdot m_2}$

$\tan\theta = \dfrac{-4 - \frac{3}{2}}{1 + (-4)\left(\frac{3}{2}\right)} = \dfrac{11}{10}$

$A = \dfrac{11}{10}$

B: $m_1 = -\dfrac{1}{4}$

$m_2 = \dfrac{2}{3}$

$\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 \cdot m_2}$

$\tan\theta = \dfrac{-\frac{1}{4} - \frac{2}{3}}{1 + \left(\frac{-1}{4}\right)\left(\frac{2}{3}\right)} = -\dfrac{11}{10}$

$B = -\dfrac{11}{10}$

$\therefore$ A = – B

Clave D

Razonamiento y demostración

13. Sabemos:

Pendiente: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Donde, $P(x_1; y_1)$ y $R(x_2; y_2)$ son puntos de paso de la recta.

Por dato: $P(x_1; y_1) = P(-1; 2)$

$R(x_2; y_2) = R(3; 1)$

$\Rightarrow m = \dfrac{(1) - (2)}{(3) - (-1)} = \dfrac{1-2}{3+1} = -\dfrac{1}{4}$

$\therefore m = -\dfrac{1}{4}$

Clave D

14. Por dato: la recta L tiene un ángulo de inclinación (α) de 135° y pasa por el punto $(x_0; y_0) = (1; 3)$.

Luego:

Pendiente: $m = \tan\alpha = \tan 135°$

$\Rightarrow m = -\tan 45° = -(1) \Rightarrow m = -1$

Piden la ecuación de la recta L:

$y - y_0 = m(x - x_0)$

$y - 3 = (-1)(x - 1)$

$y - 3 = -x + 1$

$\Rightarrow x + y - 4 = 0$

$\therefore \overleftrightarrow{L}: x + y - 4 = 0$

Clave A

15. Por dato:

$\overleftrightarrow{L}: 3x - 2y + 1 = 0$

La recta L tiene la forma general:

$\overleftrightarrow{L}: Ax + By + C = 0$

Donde la pendiente: $m = -\dfrac{A}{B}$

Comparando: $A = 3 \wedge B = -2$

Entonces: $m = -\dfrac{3}{(-2)} = \dfrac{3}{2}$

$\therefore m = \dfrac{3}{2}$

Clave D

16. Por dato:

$\overleftrightarrow{L}: 3x - 2y + 1 = 0$

Pasando a la forma ordinaria: y = mx + b

3x – 2y + 1 = 0

3x + 1 = 2y

$\Rightarrow y = \dfrac{3}{2}x + \dfrac{1}{2}$

Comparando: $m = \dfrac{3}{2}$

Piden: la pendiente (m_1) de la recta L_1.

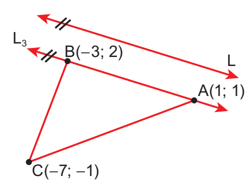
Luego, por ser rectas perpendiculares se cumple:

$m \cdot m_1 = -1$

$\Rightarrow \left(\dfrac{3}{2}\right) \cdot m_1 = -1$ $\therefore m_1 = -\dfrac{2}{3}$

Clave C

17.



Sea: la recta L de pendiente (m).

Por dato: $\overleftrightarrow{L} \parallel \overline{AB} \Rightarrow \overleftrightarrow{L} \parallel \overleftrightarrow{L_3}$

Luego:

Pendiente: $m_3 = \dfrac{2-1}{(-3)-1} = \dfrac{1}{-4}$

$\Rightarrow m_3 = -\dfrac{1}{4}$

Piden: la pendiente de la recta L.

Como las rectas L y L_3 son paralelas se cumple:

$m = m_3 = -\dfrac{1}{4}$

$\therefore m = -\dfrac{1}{4}$

Clave B

18. Tomamos las rectas como ecuaciones y resolvemos:

$6x - 5y + 27 = 0$ ...(I)
$8x + 7y - 5 = 0$ ...(II)

7(I) + 5(II):

$$\begin{array}{r} 42x - 35y + 189 = 0 \\ 40x + 35y - 25 = 0 \\ \hline 82x \quad + 164 = 0 \\ x = -2 \end{array}$$

En (I):

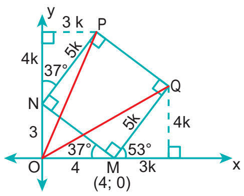
$6(-2) - 5y + 27 = 0$
$y = 3$

$\therefore$ El punto de intersección es $(-2; 3)$.

Clave A

Resolución de problemas

19.



De la figura tenemos:

$\Rightarrow OP^2 = (3k)^2 + (3 + 4k)^2$

$OP^2 = 9k^2 + 9 + 24k + 16k^2$

$OP^2 = 25k^2 + 24k + 9$

$\Rightarrow OQ^2 = (4k)^2 + (4 + 3k)^2$

$OQ^2 = 16k^2 + 16 + 24k + 9k^2$

$OQ^2 = 25k^2 + 24k + 16$

$R = OP^2 - OQ^2$

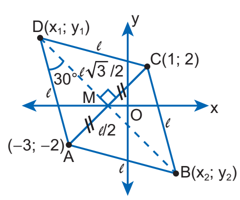
$R = 25k^2 + 24k + 9 - (25k^2 + 24k + 16)$

$R = 9 - 16$

$\therefore R = -7$

Clave C

20. $AD = AC$



Si ABCD es un rombo:

$\Rightarrow AD = AC = AB = BC = CD$

$AC = \sqrt{(-3-1)^2 + (-2-2)^2}$

$AC = 4\sqrt{2}$

Hallamos la pendiente de $\overline{AC}$ y su punto medio M.

$M_{(x;\,y)} = \left(\dfrac{1+(-3)}{2}; \dfrac{2+(-2)}{2}\right)$

$\Rightarrow M_{(x;\,y)} = (-1; 0)$ ... (I)

$m_{\overline{AC}} = \dfrac{y_A - y_C}{x_A - x_C}$

$m_{\overline{AC}} = \dfrac{-2-2}{-3-1} = \dfrac{-4}{-4}$ $\quad\Rightarrow m_{\overline{AC}} = 1$

Como $\overline{DB}$ y $\overline{AC}$ son perpendiculares, entonces:

$m_{\overline{DB}} \cdot m_{\overline{AC}} = -1$

$m_{\overline{DB}}(1) = -1$

$\therefore m_{\overline{DB}} = -1$

$y_1 - y_0 = (x_1 - x_0)m_{BD}$

$y_1 - y_0 = (x_1 - x_0)(-1)$

$y_1 + x_1 = y_0 + x_0$

$y_1 + x_1 = 0 + -1$

$\Rightarrow \quad y_1 = -(1 + x_1)$

En el segmento MD:

$(4\sqrt{2})\dfrac{\sqrt{3}}{2} = \sqrt{(x_1+1)^2 + (y_1-0)^2}$

$(2\sqrt{6})^2 = (x_1+1)^2 + (y_1)^2$

$24 = (x_1+1)^2 + (-(1+x_1))^2$

$12 = (x_1+1)^2$

$\Rightarrow x_1 = -1 - 2\sqrt{3} \quad \wedge \quad x_2 = 2\sqrt{3} - 1$

$\quad y_1 = 2\sqrt{3} \qquad\qquad y_2 = -2\sqrt{3}$

$\therefore B(2\sqrt{3} - 1; -2\sqrt{3}) \wedge D(-1 - 2\sqrt{3}; 2\sqrt{3})$

Clave B

Nivel 3 (página 38) Unidad 2

Comunicación matemática

21.

I. $\overleftrightarrow{L}_1: 3x - y + 2 = 0$

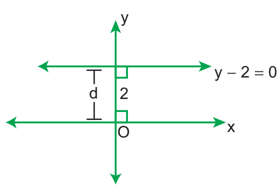
$m_1 = -\left(\dfrac{3}{-1}\right) = 3$

$\overleftrightarrow{L}_2: 4x - 2y + 3 = 0$

$m_2 = -\left(\dfrac{4}{-2}\right) = 2$

$\therefore m_1 > m_2$ (V)

II.



$\Rightarrow \quad d = 2$ (V)

III. Si: $90° < \theta < 180°$

$\Rightarrow \tan\theta = m < 0$ (F)

Clave C

22.

a) $d_a = \sqrt{(-2-2)^2 + (1-(-2))^2}$

$d_a = \sqrt{4^2 + 3^2}$ $\quad\Rightarrow d_a = 5$

b) $d_b = \sqrt{(-3-0)^2 + (1-4)^2}$

$d_b = \sqrt{(-3)^2 + (-3)^2}$ $\quad\Rightarrow d_b = 3\sqrt{2}$

c) $d_c = \sqrt{(1-2)^2 + (3-2)^2}$

$d_c = \sqrt{(-1)^2 + (1)^2}$ $\quad\Rightarrow d_c = \sqrt{2}$

d) $d_d = \sqrt{(2-2)^2 + (5-1)^2}$

$d_d = \sqrt{(0)^2 + (4)^2}$ $\quad\Rightarrow d_d = 4$

Ordenamos ascendentemente:

$\sqrt{2} < 4 < 3\sqrt{2} < 5$

$d_c < d_d < d_b < d_a$

$\therefore$ cdba

Clave D

Razonamiento y demostración

23. Por dato, la recta L pasa por los puntos:

$A(x_1; y_1) = A(1; 2)$

$B(x_2; y_2) = B(3; 1)$

Luego:

Pendiente: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-2}{3-1}$

$\Rightarrow m = -\dfrac{1}{2}$

Piden: la ecuación de la recta L.

$y - y_0 = m(x - x_0)$

Tomando como punto de paso $(x_0; y_0)$ al punto A(1; 2), tenemos:

$y - 2 = \left(-\dfrac{1}{2}\right)(x - 1)$

$2y - 4 = -x + 1$

$\Rightarrow x + 2y - 5 = 0$

$\therefore \overleftrightarrow{L}: x + 2y - 5 = 0$

Clave A

24. Sea la recta L de pendiente (m).

Por dato, la recta L pasa por el punto $(x_0; y_0) = (-2; 3)$ y es paralela a la recta L_1.

Además: $\overleftrightarrow{L}_1: 3x + y - 1 = 0$

Luego, la recta L_1 tiene la forma general:

$\overleftrightarrow{L}_1: Ax + By + C = 0$

Donde la pendiente: $m_1 = -\dfrac{A}{B}$

Comparando: $A = 3 \wedge B = 1$

$\Rightarrow m_1 = -\dfrac{3}{1} = -3$

Como las rectas L y L_1 son paralelas se cumple:

$m = m_1 = -3 \Rightarrow m = -3$

Piden la ecuación de la recta L.

$y - y_0 = m(x - x_0)$

$y - 3 = (-3)(x - (-2))$

$y - 3 = (-3)(x + 2)$

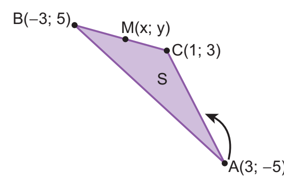
$y - 3 = -3x - 6$

$\Rightarrow 3x + y + 3 = 0$

$\therefore \overleftrightarrow{L}: 3x + y + 3 = 0$

Clave D

25.



Por dato: M es punto medio de $\overline{BC}$.

$\Rightarrow x = \dfrac{(-3)+(1)}{2} \Rightarrow x = -1$

$\Rightarrow y = \dfrac{(5)+(3)}{2} \Rightarrow y = 4$

$\therefore M(x; y) = M(-1; 4)$

Clave B

26. Sea la recta L de pendiente (m).
Por dato, la pendiente de la recta L es 2/5 y pasa por el punto $P(x_0; y_0) = P(-3; 1)$.

$\Rightarrow m = \dfrac{2}{5} \wedge (x_0; y_0) = (-3; 1)$

Piden, la ecuación de la recta L.

$y - y_0 = m(x - x_0)$

$y - 1 = \dfrac{2}{5}(x - (-3))$

$5y - 5 = 2(x + 3)$

$5y - 5 = 2x + 6$

$\Rightarrow 2x - 5y + 11 = 0$

$\therefore \overleftrightarrow{L}: 2x - 5y + 11 = 0$

Clave E

27. Sea la recta L de pendiente (m).
Por dato; la recta L pasa por el punto $(x_0; y_0) = (1; -2)$ y es perpendicular a la recta L_1.

Además: $\overleftrightarrow{L}_1: 2x - 3y + 1 = 0$

Luego, la recta L_1 tiene la forma general:

$\overleftrightarrow{L}_1: Ax + By + C = 0$

Donde la pendiente: $m_1 = -\dfrac{A}{B}$

Comparando: $A = 2 \wedge B = -3$

$\Rightarrow m_1 = -\dfrac{2}{(-3)} = \dfrac{2}{3}$

Como las rectas L y L_1 son perpendiculares se cumple:

$m \,.\, m_1 = -1 \Rightarrow m\left(\dfrac{2}{3}\right) = -1$

$\Rightarrow m = -\dfrac{3}{2}$

Piden: la ecuación de la recta L.

$y - y_0 = m(x - x_0)$

$y - (-2) = \left(-\dfrac{3}{2}\right)(x - 1)$

$y + 2 = \left(-\dfrac{3}{2}\right)(x - 1)$

$2y + 4 = -3x + 3$

$\Rightarrow 3x + 2y + 1 = 0$

$\therefore \overleftrightarrow{L}: 3x + 2y + 1 = 0$

Clave B

28. Por dato, la recta L pasa por los puntos:

$A(x_1; y_1) = A(-1; 2)$

$B(x_2; y_2) = B(5; -1)$

Luego:

Pendiente: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{(-1) - 2}{5 - (-1)}$

$\Rightarrow m = \dfrac{-1-2}{5+1} \Rightarrow m = -\dfrac{3}{6} = -\dfrac{1}{2}$

Piden la ecuación de la recta L.

$y - y_0 = m(x - x_0)$

Tomando como punto de paso $(x_0; y_0)$ al punto $A(-1; 2)$, tenemos:

$y - 2 = \left(-\dfrac{1}{2}\right)(x - (-1))$

$2y - 4 = (-1)(x + 1)$

$2y - 4 = -x - 1$

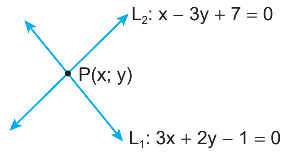
$\Rightarrow x + 2y - 3 = 0$

$\therefore \overleftrightarrow{L}: x + 2y - 3 = 0$

Clave A

Resolución de problemas

29.

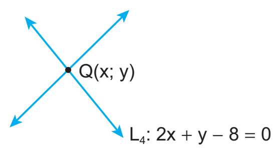


$\overleftrightarrow{L}_1: (3x + 2y - 1 = 0) \times 3$

$\overleftrightarrow{L}_2: (x - 3y + 7 = 0) \times 2$

$$\begin{array}{r} 9x + 6y - 3 = 0 \quad + \\ 2x - 6y + 14 = 0 \\ \hline 11x = -11 \\ x = -1 \\ y = 2 \end{array}$$

$\therefore P(x; y) = P(-1; 2)$



$\overleftrightarrow{L}_3: (4x - 5y - 2 = 0) \times 1$

$\overleftrightarrow{L}_4: (2x + y - 8 = 0) \times 5$

$$\begin{array}{r} 4x - 5y - 2 = 0 \quad + \\ 10x + 5y - 40 = 0 \\ \hline 14x - 42 = 0 \\ x = 3 \\ y = 2 \end{array}$$

$\therefore Q(x; y) = Q(3; 2)$

$d_{(P,\,Q)} = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2}$

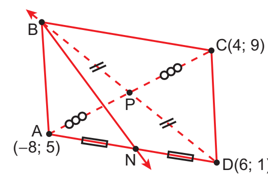
$d_{(P;\,Q)} = \sqrt{(-1-3)^2 + (2-2)^2}$

$d_{(P;\,Q)} = \sqrt{4^2 + 0^2} = 4$

$\therefore d_{(P;\,Q)} = 4$

Clave A

30.



Del gráfico: N es el punto medio de $\overline{AD}$.

$N = \left(\dfrac{-8+6}{2}; \dfrac{5+1}{2}\right)$

$N = (-1; 3)$

Trazamos los diagonales $\overline{AC}$ y $\overline{BD}$ que se cortan en su punto medio.

Entre A y C:

$P = \left(\dfrac{-8+4}{2}; \dfrac{5+9}{2}\right)$

$P = (-2; 7)$

Entre B y D:

$P = \left(\dfrac{B_x + 6}{2}; \dfrac{B_y + 1}{2}\right)$

$(-2; 7) = \left(\dfrac{B_x + 6}{2}; \dfrac{B_y + 1}{2}\right)$

$(B_x; B_y) = (-10; 13)$

Hallamos la ecuación de la recta:

$\dfrac{13 - 3}{-10 + 1} = \dfrac{y - 3}{x + 1}$

$-\dfrac{10}{9} = \dfrac{y - 3}{x + 1}$

$-10x - 10 = 9y - 27$

$10x + 9y - 17 = 0$

Clave C

RAZONES TRIGONOMÉTRICAS DE ÁNGULOS DE CUALQUIER MAGNITUD

APLICAMOS LO APRENDIDO (página 40) Unidad 2

1. $\cot\theta = x/y = -7/1 \;\wedge\; \text{sen}\theta = y/r < 0 \Rightarrow y < 0$

$y = -1 \;\wedge\; x = 7 \Rightarrow r = \sqrt{50}$

Reemplazamos en M:

$M = \sqrt{50}\cos\theta + 7\tan\theta = \sqrt{50}\left(\frac{x}{r}\right) + 7\left(\frac{y}{x}\right)$

$M = \sqrt{50}\left(\frac{7}{\sqrt{50}}\right) + 7\left(\frac{-1}{7}\right)$

$M = 7 - 1 = 6$

Clave B

2. $M = \frac{(-4;2)+(2;-4)}{2} = \frac{(-4+2;2-4)}{2}$

$M = \frac{(-2;-2)}{2} = (-1;-1)$

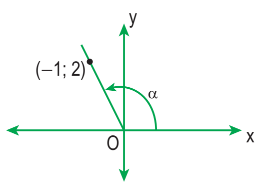
Hallamos el valor de:

$B = \tan\theta - \cot\theta = \frac{y}{x} - \frac{x}{y} = \frac{-1}{-1} - \frac{-1}{-1}$

$B = 1 - 1 = 0$

Clave D

3.



Calculamos el radio vector:

$r^2 = x^2 + y^2$

$r^2 = (-1)^2 + (2)^2 = 5$

$r = \sqrt{5}$

Piden:

$B = \sqrt{5}\,\text{sen}\alpha - \tan\alpha = \sqrt{5}\left(\frac{y}{r}\right) - \left(\frac{y}{x}\right)$

$B = \sqrt{5}\left(\frac{2}{\sqrt{5}}\right) - \left(\frac{2}{-1}\right) = 2 - (-2) = 4$

$\therefore B = 4$

Clave E

4. Sean los ángulos α y β, $\alpha > \beta$.

$\frac{\alpha}{\beta} = \frac{11}{3} = k \Rightarrow \begin{matrix}\alpha = 11k \\ \beta = 3k\end{matrix}$

$\alpha - \beta = 360°n, n \in \mathbb{Z} - \{0\}$

$11k - 3k = 360°n$

$8k = 360°n$

$k = 45n \Rightarrow \begin{matrix}\alpha = 495°n \\ \beta = 135°n\end{matrix}$

$450° < 495°n < 500°$

$0{,}9 < n < 1{,}01$

$n = 1 \Rightarrow \begin{matrix}\alpha = 495° \\ \beta = 135°\end{matrix}$

Clave A

5.

$195° \in \text{IIIC} \Rightarrow \text{sen}195° \quad (-)$

$230° \in \text{IIIC} \Rightarrow \cos230° \quad (-)$

$75° \in \text{IC} \Rightarrow \tan75° \quad (+)$

$140° \in \text{IIC} \Rightarrow \text{sen}140° \quad (+)$

$280° \in \text{IVC} \Rightarrow \cos280° \quad (+)$

$160° \in \text{IIC} \Rightarrow \tan160° \quad (-)$

$200° \in \text{IIIC} \Rightarrow \cos200° \quad (-)$

$340° \in \text{IVC} \Rightarrow \cos340° \quad (+)$

$145° \in \text{IIC} \Rightarrow \text{sen}145° \quad (+)$

$P = \frac{(-)(-)}{(+)} = (+)$

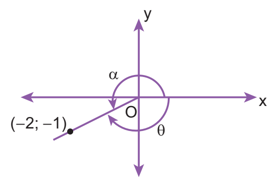
$Q = \frac{(+)(+)}{(-)} = (-)$

$R = ((-) - (+))(+) = (-)$

$\therefore$ Los signos serán: (+);(–);(–)

Clave C

6.



Del gráfico:

$\tan\alpha = \frac{y}{x} = \frac{-1}{-2} = \frac{1}{2}$

$\cot\theta = \frac{x}{y} = \frac{-2}{-1} = 2$

Piden:

$E = \tan\alpha - \cot\theta$

$E = \left(\frac{1}{2}\right) - (2) = \frac{1-4}{2} = -\frac{3}{2}$

$\therefore E = -\frac{3}{2}$

Clave D

7.

$H = \frac{\text{sen}110° - \cos215°}{\tan268°} = \frac{(+)-(-)}{(+)} = \frac{(+)}{(+)}$

$H = (+)$

$A = \sec295° \,.\, \csc152° \,.\, \cot302°$

$A = (+)(+)(-) = (+)(-) = (-)$

$A = (-)$

$P = \frac{\tan196°(1 - \text{sen}250°)}{\cos^2 100°}$

$P = \frac{(+).(1-(-))}{(-)^2}$

$P = \frac{(+).(+)}{(+)} = \frac{(+)}{(+)} = (+)$

$P = (+)$

Por lo tanto, los signos serán: (+); (–); (+)

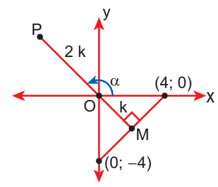
Clave B

8.

$\frac{2}{6} = \frac{-4}{a} \Rightarrow \begin{matrix}2a = (-4)(6) \\ a = -12\end{matrix}$

Clave D

9.



$M = \frac{(0;-4)+(4,0)}{2} = (2;-2)$

$O = \frac{2k(M)+k(P)}{2k+k} = \frac{2k(2;-2)+k(P)}{3k}$

$(0;0) = \frac{(4;-4)+P}{3} \Rightarrow P = (-4;4)$

Hallamos el valor de k:

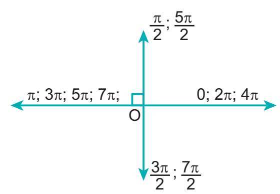
$k = \tan\alpha + \cot\alpha = \frac{y}{x} + \frac{x}{y} = \frac{4}{-4} + \frac{-4}{4}$

$k = (-1) + (-1) = -2$

Clave B

10.

$M = \frac{(a+1)\cos3\pi + (1-a)\text{sen}\frac{7\pi}{2}}{(a+1)\text{sen}\frac{5\pi}{2} - (1-a)\cos7\pi}$



Entonces:

$\cos3\pi = \cos\pi = -1$

$\text{sen}\frac{7\pi}{2} = \text{sen}\frac{3\pi}{2} = -1$

$\text{sen}\frac{5\pi}{2} = \text{sen}\frac{\pi}{2} = 1$

$\cos7\pi = \cos\pi = -1$

Reemplazando en la expresión tenemos:

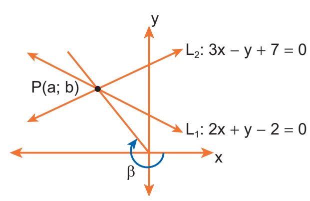
$M = \frac{(a+1)(-1)+(1-a)(-1)}{(a+1)(1)-(1-a)(-1)}$

$M = \frac{-a-1-1+a}{a+1+1-a} = \frac{-2}{2} = -1$

$\therefore M = -1$

Clave B

11.



El punto P(a; b) es donde se intersecan ambas rectas.

Resolviendo el sistema:

$$\left.\begin{array}{l}3x - y = -7\\ 2x + y = 2\end{array}\right\} x = -1 \ \wedge y = 4$$

Entonces: P(a; b) = P(– 1; 4)

Piden: $\tan\beta = \dfrac{y}{x} = \dfrac{b}{a} = \dfrac{4}{-1} = -4$

$\therefore \tan\beta = -4$

Clave D

12.

$$E = \frac{a^3 + b^3\cos^2\pi}{a^2\text{sen}^2\frac{\pi}{2} + ab\,\text{sen}\frac{3\pi}{2} - b^2\cos\pi}$$

$$E = \frac{a^3 + b^3(-1)^2}{a^2(1)^2 + ab(-1) - b^2(-1)}$$

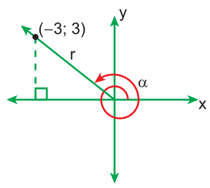
$$E = \frac{a^3 + b^3}{a^2 - ab + b^2}$$

$$E = \frac{(a+b)(a^2 - ab + b^2)}{a^2 - ab + b^2} = a + b$$

$\therefore E = a + b$

Clave A

13.



$$x^2 + y^2 = r^2$$
$$(-3)^2 + (3)^2 = r^2$$
$$3\sqrt{2} = r$$

$\text{sen}(-\alpha) = -\text{sen}\alpha$

$\cos(-\alpha) = \cos\alpha$

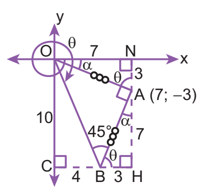
$M = -\text{sen}\alpha + \cos\alpha = \dfrac{-y}{r} + \dfrac{x}{r}$

$M = \dfrac{-3}{3\sqrt{2}} + \dfrac{-3}{3\sqrt{2}} = \dfrac{-2}{\sqrt{2}}$

$M = -\sqrt{2}$

Clave C

14.



Del gráfico:

◺ONA ≅ ◺AHB ⇒ AH = 7 ∧ BH = 3

Luego, las coordenadas del punto B serán:

B(x; y) = B(4; –10)

Piden:

$\tan\theta = \dfrac{y}{x} = \dfrac{-10}{4} = -\dfrac{5}{2}$

$\therefore \tan\theta = -\dfrac{5}{2}$

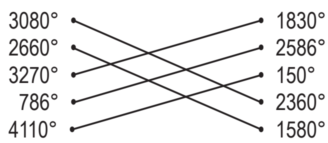
Clave C

PRACTIQUEMOS

Nivel 1 (página 42) Unidad 2

Comunicación matemática

1. Aplicamos la relación $a = b + 360°(n); n \in \mathbb{Z} - \{0\}$



2.

I. $\theta \in$ IC, IIC, IIIC o IVC

II. $\beta \in$ IIIC o IVC

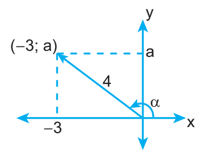
III. $\alpha \in$ IC o IIC

IV. $\gamma \in$ IIIC o IVC

V. $\psi \in$ IIC o IIIC

Razonamiento y demostración

3.



Por dato: $\cos\alpha = -\dfrac{3}{4} \wedge \alpha \in$ IIC

Luego, por radio vector:

$4^2 = (-3)^2 + a^2 \Rightarrow a^2 = 7$

$\Rightarrow a = \sqrt{7} \ \vee a = -\sqrt{7}$

Del gráfico: $a > 0 \Rightarrow a = \sqrt{7}$

Piden:

$P = 3\tan^2\alpha - 2\sec\alpha$

$P = 3\left(\dfrac{y}{x}\right)^2 - 2\left(\dfrac{r}{x}\right)$

Donde: $x = -3$; $y = a = \sqrt{7}$; $r = 4$

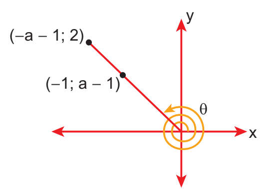
$P = 3\left(\dfrac{\sqrt{7}}{-3}\right)^2 - 2\left(\dfrac{4}{-3}\right)$

$P = 3\left(\dfrac{7}{9}\right) + \dfrac{8}{3} = 5$

$\therefore P = 5$

Clave C

4.



$\tan\theta = \dfrac{y}{x} = \dfrac{(2)}{(-a-1)} = \dfrac{(a-1)}{(-1)}$

$-2 = (a-1)(-a-1)$

$2 = (a-1)(a+1) \Rightarrow 2 = a^2 - 1$

$a^2 = 3 \Rightarrow a = \sqrt{3} \ \vee a = -\sqrt{3}$

Del gráfico: $a - 1 > 0 \Rightarrow a > 1$

$\Rightarrow a = \sqrt{3}$

Entonces: $\tan\theta = \dfrac{(a-1)}{(-1)}$

$\Rightarrow \tan\theta = 1 - a = 1 - (\sqrt{3})$

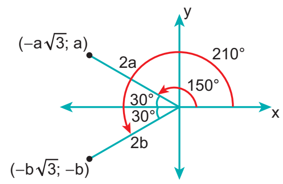
$\therefore \tan\theta = 1 - \sqrt{3}$

Clave B

5. Piden:

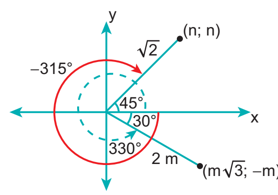
$$K = \frac{\text{sen}150°.\cos 210°.\tan 330°}{\cos 60°.\cot(-315°)}$$

Luego:



$\text{sen}150° = \dfrac{y}{r} = \dfrac{a}{2a} = \dfrac{1}{2}$

$\cos 210° = \dfrac{x}{r} = \dfrac{-b\sqrt{3}}{2b} = \dfrac{-\sqrt{3}}{2}$



$\tan 330° = \dfrac{y}{x} = \dfrac{-m}{m\sqrt{3}} = -\dfrac{1}{\sqrt{3}}$

$\cot(-315°) = \dfrac{x}{y} = \dfrac{n}{n} = 1$

Reemplazando en K, tenemos:

$$K = \frac{\left(\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{\sqrt{3}}\right)}{\left(\frac{1}{2}\right)(1)} = \frac{1}{2}$$

$\therefore K = \dfrac{1}{2}$

Clave C

6. Por dato: Q(x; y) = Q(8; 15) pertenece al lado final de un ángulo canónico β.

Por radio vector:

$r^2 = x^2 + y^2$

$\Rightarrow r^2 = 8^2 + 15^2 \Rightarrow r^2 = 289$

$\Rightarrow r = 17$

Piden:

$R = \csc\beta - \cot\beta = \left(\dfrac{r}{y}\right) - \left(\dfrac{x}{y}\right)$

$\Rightarrow R = \left(\dfrac{17}{15}\right) - \left(\dfrac{8}{15}\right) = \dfrac{9}{15}$

$\therefore R = \dfrac{3}{5} = 0{,}6$

Clave C

7. Sabemos:

$150° \in$ IIC $\Rightarrow$ sen150° es (+)
$230° \in$ IIIC $\Rightarrow$ cos230° es (–)
$315° \in$ IVC $\Rightarrow$ tan315° es (–)
$130° \in$ IIC $\Rightarrow$ sec130° es (–)
$242° \in$ IIIC $\Rightarrow$ cot242° es (+)
$300° \in$ IVC $\Rightarrow$ csc300° es (–)

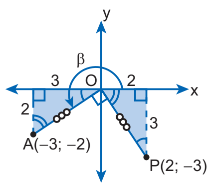
Piden, indicar el signo de:

$$M = \frac{\text{sen}150°.\cos 230°.\tan 315°}{\sec 130°.\cot 242°.\csc 300°}$$

$\therefore M = (+)$

Clave A

8.



Ubicamos un punto A tal que: OP = OA

Luego, los triángulos rectángulos sombreados resultan congruentes (A–L–A), entonces: A(–3; –2)

Piden:

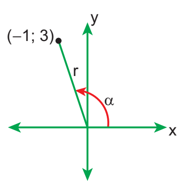
$$\tan\beta = \frac{y}{x} = \frac{-2}{-3} = \frac{2}{3}$$

$$\therefore \tan\beta = \frac{2}{3}$$

Clave B

9. Por dato:

$\tan\alpha = -3 \wedge \text{sen}\alpha > 0$
$\Rightarrow \tan\alpha < 0 \wedge \text{sen}\alpha > 0$
Entonces: $\alpha \in$ IIC



Por radio vector: $r = \sqrt{10}$
Piden:
$P = 2\cos\alpha + \text{sen}\alpha$

$$P = 2\left(\frac{x}{r}\right) + \left(\frac{y}{r}\right)$$

$$P = 2\left(\frac{-1}{\sqrt{10}}\right) + \left(\frac{3}{\sqrt{10}}\right) = \frac{1}{\sqrt{10}}$$

$$\therefore\ P = \frac{\sqrt{10}}{10}$$

Clave A

10.

$$E = \frac{\text{sen}270° + \cos 90° - \tan 0°}{\cos 450° + \cot 270° + \sec 180°}$$

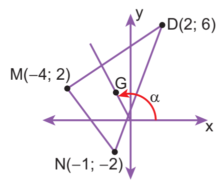
$$E = \frac{(-1)+(0)-(0)}{(0)+(0)+(-1)} = 1$$

$\therefore E = 1$

Clave B

Resolución de problemas

11.



Hallamos las coordenadas de baricentro.

$$G(x; y) = \frac{M+N+D}{3}$$

$$G(x; y) = \left(\frac{-4+(-1)+2}{3}; \frac{2+(-2)+6}{3}\right)$$

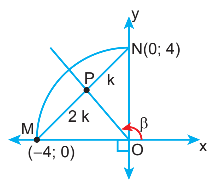
$G(x; y) = (-1; 2)$

Calculamos la cotα:

$$\cot\alpha = \frac{x}{y} \Rightarrow \cot\alpha = -1/2$$

Clave C

12.



$$P(x; y) = \frac{M(k) + N(2k)}{2k + k}$$

$$P(x; y) = \frac{(-4; 0)k + (0; 4)2k}{3k}$$

$$P(x; y) = \frac{(-4; 0) + (0; 8)}{3} = \left(\frac{-4}{3}; \frac{8}{3}\right)$$

Calculamos tanβ:

$$\tan\beta = \frac{y}{x} = \frac{\frac{8}{3}}{\frac{-4}{3}} = -2$$

Clave E

Nivel 2 (página 43) Unidad 2

Comunicación matemática

13.

14.

I. $\cos 780°\ .\ \sec 430° < 0$
$(+)\ (+)\ < 0$ (F)

II. $\cot 1134°\ .\ \csc 1630° > 0$
$(+)(-)\ > 0$ (F)

III. $2\text{sen}450° + 2\sec 1260° = 0$
$2(1) + 2\,(-1) = 0$ (V)

IV. $3\cos 2880° + 4\sec 2700° < 0$
$3(1) + 4\,(-1) < 0$ (V)

Clave B

Razonamiento y demostración

15. Por dato: $\alpha \in$ IIIC, además es positivo y menor que una vuelta.

Entonces: $180° < \alpha < 270°$
Luego:

$$90° < \frac{\alpha}{2} < 135° \Rightarrow \frac{\alpha}{2} \in \text{IIC}$$

$$\Rightarrow \text{sen}\frac{\alpha}{2} \text{ es } (+)$$

$$120° < \frac{2\alpha}{3} < 180° \Rightarrow \frac{2\alpha}{3} \in \text{IIC}$$

$$\Rightarrow \cos\frac{2\alpha}{3} \text{ es } (-)$$

$$108° < \frac{3\alpha}{5} < 162° \Rightarrow \frac{3\alpha}{5} \in \text{IIC}$$

$$\Rightarrow \tan\frac{3\alpha}{5} \text{ es } (-)$$

Piden, señalar el signo de:

$$Q = \left(\text{sen}\frac{\alpha}{2} - \cos\frac{2\alpha}{3}\right)\tan\frac{3\alpha}{5}$$

$Q = ((+) - (-))(-)$
$Q = (+)(-) = (-)$
$\therefore Q = (-)$

Clave B

16. Sabemos:

$\{116°; 140°; 160°\} \in$ IIC, entonces:
tan116°: (–); sen140°: (+); cos140°: (–); tan160°: (–)

$\{217°; 248°; 260°\} \in$ IIIC, entonces:
cos217°: (–); cos248°: (–); tan260°: (+)

$\{300°; 348°\} \in$ IVC, entonces:
tan300°: (–); sen348°: (–)

Piden, señalar los signos de:

$$M = \frac{\text{sen}140° - \cos 140°}{\tan 300°.\tan 260°}$$

$$M = \frac{(+)-(-)}{(-)(+)} = \frac{(+)}{(-)} = (-)$$

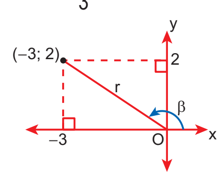
$\Rightarrow M = (-)$

$$R = \frac{\tan 160°.\cos 217° - \tan 116°}{\cos 248° + \text{sen}348°}$$

$$R = \frac{(-)(-)-(-)}{(-)+(-)} = \frac{(+)}{(-)} = (-) \Rightarrow R = (-)$$

Clave D

17. Por dato: $\tan\beta = -\frac{2}{3}$; $(\beta \in \text{IIC})$



Por radio vector: $r^2 = x^2 + y^2$
$r^2 = (-3)^2 + (2)^2 \Rightarrow r^2 = 13$
$r = \sqrt{13}$

$$\text{Piden: } H = \text{sen}\beta + \cos\beta = \left(\frac{y}{r}\right) + \left(\frac{x}{r}\right)$$

$$H = \left(\frac{2}{\sqrt{13}}\right) + \left(\frac{-3}{\sqrt{13}}\right) = -\frac{1}{\sqrt{13}}.\frac{\sqrt{13}}{\sqrt{13}}$$

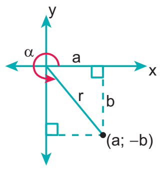
$$\therefore H = -\frac{\sqrt{13}}{13}$$

Clave B

18. Por dato: $\alpha \in$ IVC

Entonces; senα: (–); cosα: (+); tanα: (–)

Además:



$\cos\alpha = \frac{x}{r} = \frac{a}{r}$

Donde: a y r $\in \mathbb{R}^+$

Del triángulo rectángulo: $r > a \Rightarrow 1 > \frac{a}{r}$

$\Rightarrow 1 > \cos\alpha$

$\Rightarrow (1 - \cos\alpha) > 0$; es decir es: (+)

Piden, determinar el signo de:

$$E = \frac{\tan\alpha(1 - \cos\alpha)}{\text{sen}\alpha - \cos\alpha}$$

$$E = \frac{(-)(+)}{(-)-(+)} = \frac{(-)}{(-)} = (+)$$

$\therefore$ E = (+)

Clave A

19. Por dato: $\cos\theta\sqrt{-\tan\theta} > 0$

Sabemos que la raíz cuadrada de un número diferente de cero es siempre positiva.

$\Rightarrow \sqrt{-\tan\theta} > 0$

Además, el radicando debe ser un número real y positivo.

$\Rightarrow -\tan\theta > 0 \Rightarrow \tan\theta < 0$

$\Rightarrow \theta \in$ IIC $\vee\ \theta \in$ IVC ...(A)

Entonces: $\underbrace{\cos\theta}_{(+)}\underbrace{\sqrt{-\tan\theta}}_{(+)} > 0$

$\Rightarrow \cos\theta > 0 \Rightarrow \theta \in$ IC $\vee\ \theta \in$ IVC ...(B)

De (A) y (B) deducimos que: $\theta \in$ IVC

Clave D

20. Por dato:

$\text{sen}\theta + 1 - 3(5^{-1}) = -5^{-1}$

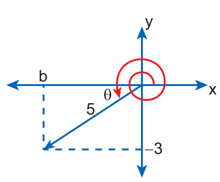
$\text{sen}\theta + 1 - 3\left(\frac{1}{5}\right) = -\frac{1}{5}$

$\text{sen}\theta + 1 = \frac{3}{5} - \frac{1}{5}$

$\text{sen}\theta + 1 = \frac{2}{5}$

$\text{sen}\theta = \frac{-3}{5}$

Además:



Luego por radio vector:

$5^2 = b^2 + (-3)^2 \Rightarrow b^2 = 16$

$\Rightarrow b = 4 \vee b = -4$

Del gráfico: $b < 0 \Rightarrow b = -4$

Piden:

$K = \text{sen}\theta + \cos\theta = \left(\frac{y}{r}\right) + \left(\frac{x}{r}\right)$

Donde: x = b = – 4; y = – 3; r = 5

$\Rightarrow K = \left(\frac{-3}{5}\right) + \left(\frac{-4}{5}\right) = \frac{-7}{5}$

$\therefore K = -\frac{7}{5}$

Clave B

21. Por dato: θ es un ángulo cuadrantal.

Además: $\theta \in \langle 250°; 320° \rangle$

Luego, los ángulos cuadrantales positivos son: 90°; 180°; 270°; 360°;...

Observamos que 270° es el único ángulo cuadrantal que pertenece al intervalo.

$\langle 250°; 320° \rangle$

$\Rightarrow \theta = 270°$

Piden:

$$P = \frac{\cot\frac{\theta}{3} + \cos\frac{\theta}{6}}{\csc\theta} = \frac{\cot\frac{270°}{3} + \cos\frac{270°}{6}}{\csc 270°}$$

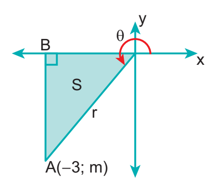
$$P = \frac{\cot 90° + \cos 45°}{\csc 270°} = \frac{(0) + \left(\frac{\sqrt{2}}{2}\right)}{(-1)}$$

$\therefore P = -\frac{\sqrt{2}}{2}$

Clave E

Resolución de problemas

22.



$S = 6 = \frac{b\,.\,h}{2} \quad \Rightarrow b\,.\,h = 12$

$|x\,.\,y| = 12$

$|-3\,.\,m| = 12$

$m = -4$

$(x^2 + y^2) = r^2 = (-3)^2 + (-4)^2$

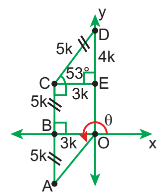
$r = 5$

$P = \tan\theta\,.\,\text{sen}\theta = \frac{y}{x} \cdot \frac{y}{r} = \frac{-4}{-3} \cdot \frac{-4}{5}$

$P = -\frac{16}{15}$

Clave D

23.



El punto A es: A = (–3k; –5k)

$\therefore \cot\theta = \frac{-3k}{-5k} = \frac{3}{5}$

Clave C

Nivel 3 (página 43) Unidad 2

Comunicación matemática

24.

I. $\text{sen}127°\,.\,\cos135° > 0$
$(+)(-) > 0$ (F)

II. $\sec0° + 1 = 0$
$1 + 1 = 0$ (F)

III. $\tan1880°\,.\,\cot2050° > 0$
$(+)(+) > 0$ (V)

IV. $\text{sen}760°\,.\,\cos870° < 0$
$(+)(-) < 0$ (V)

$\therefore$ I y II son incorrectas.

Clave E

25. Tenemos un punto (x; y) y radio vector r pertenecientes al lado final del ángulo θ:

$M = (\sec^2\theta - 1)(\csc^2\theta - 1)$

$M = \left(\left(\frac{r}{x}\right)^2 - 1\right)\left(\left(\frac{r}{y}\right)^2 - 1\right)$

$M = \left(\frac{r^2 - x^2}{x^2}\right)\left(\frac{r^2 - y^2}{y^2}\right)$

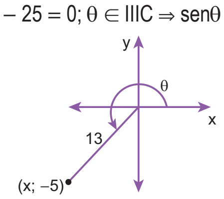
$M = \left(\frac{y^2}{x^2}\right)\left(\frac{x^2}{y^2}\right) = 1$

$\therefore$ No es necesario ningún dato.

Clave E

Razonamiento y demostración

26. $169\text{sen}^2\theta - 25 = 0$; $\theta \in$ IIIC $\Rightarrow \text{sen}\theta < 0$



$\Rightarrow \text{sen}^2\theta = \frac{25}{169} \quad \Rightarrow \text{sen}\theta = -\frac{5}{13}$

Empleando radio vector: x = –12

Piden:

$E = 12\tan\theta + 13\cos\theta$

$E = 12\left(\frac{y}{x}\right) + 13\left(\frac{x}{r}\right)$

$E = 12\left(\frac{-5}{-12}\right) + 13\left(\frac{-12}{13}\right)$

$\therefore$ E = –7

Clave E

27. La medida de un ángulo cuadrantal es de la forma: 90°n, donde $n \in \mathbb{Z}$.

Por dato:

$1000° < 90°n < 1500°$

$11{,}1 < n < 16{,}6$

$\Rightarrow n = \{12; 13; 14; 15; 16\}$

Por cada valor de n hay un ángulo cuadrantal.

$\therefore$ Hay 5 ángulos cuadrantales.

Clave C

28. Sabemos que la raíz cuadrada de un número diferente del cero es siempre positiva.

$\Rightarrow \sqrt{\cot\theta} > 0$

Además, el radicando debe ser un número real y positivo.

$\Rightarrow \cot\theta > 0 \Rightarrow \theta \in IC \vee \theta \in IIIC$ ...(A)

Entonces: $\underset{(+)}{\sqrt{\cot\theta}}\ \underset{(-)}{\operatorname{sen}\theta} < 0$

$\Rightarrow \operatorname{sen}\theta < 0 \Rightarrow \theta \in IIIC \vee \theta \in IVC$ ...(B)

De (A) y (B) deducimos que: $\theta \in IIIC$

Piden, hallar el signo de la expresión:

$$R = \frac{\csc\theta + \cos\theta}{\tan\theta}$$

Como $\theta \in IIIC$, entonces:

$\csc\theta$: (–); $\cos\theta$: (–); $\tan\theta$: (+)

Luego:

$$R = \frac{(-)+(-)}{(+)} = \frac{(-)}{(+)} = (-)$$

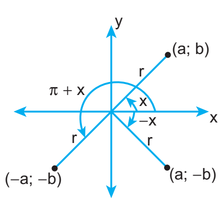
$\therefore R = (-)$

Clave A

29. Piden:

$$M = \frac{\operatorname{sen}(-x)}{\operatorname{sen}(\pi+x)} + \frac{\cos(-x)}{\cos(2\pi-x)} + \frac{\sec(-x)}{\sec(2\pi+x)}$$

Luego, tomamos como referencia un ángulo positivo $x \in IC$.



$$\operatorname{sen}(-x) = \frac{y}{r} = \frac{-b}{r} = -\frac{b}{r}$$

$$\operatorname{sen}(\pi + x) = \frac{y}{r} = \frac{-b}{r} = -\frac{b}{r}$$

$\Rightarrow \operatorname{sen}(-x) = \operatorname{sen}(\pi + x)$

Análogamente del gráfico se obtiene:

$\cos(-x) = \cos(2\pi - x) \wedge \sec(-x) = \sec(2\pi + x)$

Reemplazando en M, tenemos:

$$M = \frac{\operatorname{sen}(-x)}{\operatorname{sen}(-x)} + \frac{\cos(-x)}{\cos(-x)} + \frac{\sec(-x)}{\sec(-x)}$$

$\Rightarrow M = 1 + 1 + 1 = 3$

$\therefore M = 3$

Clave E

30. Por dato, el lado final de un ángulo canónico θ pasa por los puntos:

$A(x_1; y_1) = A(m + n; n)$

$B(x_2; y_2) = B(n; m - n)$

Sabemos: $\tan\theta = \dfrac{y_1}{x_1} = \dfrac{y_2}{x_2}$

$$\Rightarrow \frac{n}{m+n} = \frac{m-n}{n} \Rightarrow n^2 = m^2 - n^2$$

$\Rightarrow m^2 = 2n^2$

Piden:

$$K = \cot^2\theta + \tan^2\theta = \left(\frac{x_1}{y_1}\right)^2 + \left(\frac{y_2}{x_2}\right)^2$$

$$K = \left(\frac{m+n}{n}\right)^2 + \left(\frac{m-n}{n}\right)^2$$

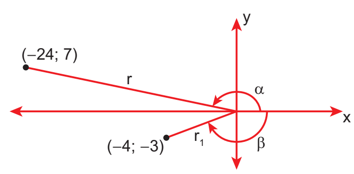
$$K = \frac{(m+n)^2 + (m-n)^2}{n^2} = \frac{2(m^2+n^2)}{n^2}$$

$$K = \frac{2(2n^2+n^2)}{n^2} = \frac{6n^2}{n^2}$$

$\therefore K = 6$

Clave C

31.



Empleando la propiedad del radio vector:

$r = 25 \wedge r_1 = 5$

Piden:

$$K = 5\cos\alpha - \cos\beta = 5\left(\frac{-24}{r}\right) - \left(\frac{-4}{r_1}\right)$$

$$K = 5\left(-\frac{24}{25}\right) - \left(-\frac{4}{5}\right) = -\frac{24}{5} + \frac{4}{5}$$

$$K = -\frac{20}{5} = -4$$

$\therefore K = -4$

Clave C

32.

$$L = \frac{(a+b)^2 \operatorname{sen}^3\frac{\pi}{2} + (a-b)^2\cos^5\pi}{a\operatorname{sen}\frac{3\pi}{2} + b\cos^2\frac{\pi}{2}}$$

$$L = \frac{(a+b)^2(1)^3 + (a-b)^2(-1)^5}{a(-1) + b(0)^2}$$

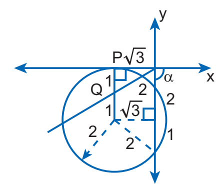
$$L = \frac{(a+b)^2 - (a-b)^2}{-a} = -\frac{4ab}{a}$$

$\therefore L = -4b$

Clave E

Resolución de problemas

33.



El punto $Q = (-\sqrt{3}; -1)$

$T = \cot^2\alpha + \csc^2\alpha$

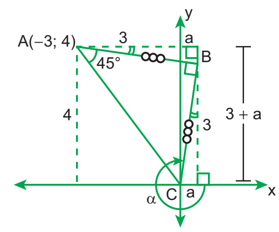
$$T = \left(\frac{x}{y}\right)^2 + \left(\frac{r}{y}\right)^2 = \left(\frac{-\sqrt{3}}{-1}\right)^2 + \left(\frac{2}{-1}\right)^2$$

$T = (\sqrt{3})^2 + (-2)^2$

$T = 3 + 4 = 7$

Clave C

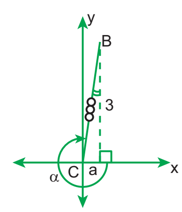
34.



$3 + a = 4$

$a = 1$

Del gráfico:



$$\therefore \cot\alpha = \frac{a}{3} = \frac{1}{3}$$

Clave B

PRACTIQUEMOS

Nivel 1 (página 47) Unidad 2

Comunicación matemática

1.

2.

Razonamiento y demostración

3. Piden: tan2933°

$\tan 2933° = \tan(8 \times 360° + 53°)$

$\tan 2933° = \tan 53° \qquad \therefore \tan 2933° = \frac{4}{3}$

Clave C

4. $L = \frac{\tan(-60°)}{\cos(-45°)}$

$L = \frac{(-\tan 60°)}{(\cos 45°)} = -\frac{\tan 60°}{\cos 45°}$

$L = -\frac{(\sqrt{3})}{\left(\frac{\sqrt{2}}{2}\right)} = -\frac{2\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{2\sqrt{6}}{2}$

$\therefore L = -\sqrt{6}$

Clave D

5. Piden: cos1741π

$\cos 1741\pi = \cos(870 \times 2\pi + \pi)$

$\cos 1741\pi = \cos\pi$

$\therefore \cos 1741\pi = -1$

Clave B

6. Piden: tan5520°

$\tan 5520° = \tan(15 \times 360° + 120°)$

$\tan 5520° = \tan 120°$

$\tan 5520° = \tan(90° + 30°)$

$\tan 5520° = -\cot 30° = -(\sqrt{3})$

$\therefore \tan 5520° = -\sqrt{3}$

Clave B

7. Piden: $\tan\frac{17\pi}{3}$

$\tan\frac{17\pi}{3} = \tan\left(2 \times 2\pi + \frac{5\pi}{3}\right)$

$\tan\frac{17\pi}{3} = \tan\frac{5\pi}{3}$

$\tan\frac{17\pi}{3} = \tan\left(2\pi - \frac{\pi}{3}\right)$

$\tan\frac{17\pi}{3} = -\tan\frac{\pi}{3} = -\tan 60°$

$\tan\frac{17\pi}{3} = -(\sqrt{3})$

$\therefore \tan\frac{17\pi}{3} = -\sqrt{3}$

Clave D

8. $C = (\mathrm{sen}330° + \cos 240°).\tan 210°$

Luego:

- $\mathrm{sen}330° = \mathrm{sen}(360° - 30°)$

  $\mathrm{sen}330° = -\mathrm{sen}30° = -\left(\frac{1}{2}\right)$

  $\mathrm{sen}330° = -\frac{1}{2}$

- $\cos 240° = \cos(180° + 60°)$

  $\cos 240° = -\cos 60° = -\left(\frac{1}{2}\right)$

  $\cos 240° = -\frac{1}{2}$

- $\tan 210° = \tan(270° - 60°)$

  $\tan 210° = \cot 60° = \left(\frac{\sqrt{3}}{3}\right)$

  $\tan 210° = \frac{\sqrt{3}}{3}$

Reemplazando en la expresión C:

$C = \left(\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\right)\left(\frac{\sqrt{3}}{3}\right)$

$C = (-1)\left(\frac{\sqrt{3}}{3}\right) = -\frac{\sqrt{3}}{3} \qquad \therefore C = -\frac{\sqrt{3}}{3}$

Clave D

9. $K = \frac{\mathrm{sen}120°.\cos 240°.\tan 300°}{\sec 225°}$

Luego:

- $\mathrm{sen}120° = \mathrm{sen}(90° + 30°)$

  $\mathrm{sen}120° = \cos 30° = \left(\frac{\sqrt{3}}{2}\right)$

  $\mathrm{sen}120° = \frac{\sqrt{3}}{2}$

- $\cos 240° = \cos(180° + 60°)$

  $\cos 240° = -\cos 60° = -\left(\frac{1}{2}\right)$

  $\cos 240° = -\frac{1}{2}$

- $\tan 300° = \tan(360° - 60°)$

  $\tan 300° = -\tan 60° = -(\sqrt{3})$

  $\tan 300° = -\sqrt{3}$

- $\sec 225° = \sec(270° - 45°)$

  $\sec 225° = -\csc 45° = -(\sqrt{2})$

  $\sec 225° = -\sqrt{2}$

Reemplazando en la expresión K:

$K = \frac{\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right)(-\sqrt{3})}{(-\sqrt{2})} = -\frac{3}{4\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$\therefore K = -\frac{3\sqrt{2}}{8}$

Clave D

10. $U = (\cos^2 135° - 3\tan 127°).\sec^2 240°$

Luego:

- $\cos 135° = \cos(180° - 45°)$

  $\cos 135° = -\cos 45° = -\left(\frac{\sqrt{2}}{2}\right)$

  $\cos 135° = -\frac{\sqrt{2}}{2}$

- $\tan 127° = \tan(180° - 53°)$

  $\tan 127° = -\tan 53° = -\left(\frac{4}{3}\right)$

  $\tan 127° = -\frac{4}{3}$

- $\sec 240° = \sec(180° + 60°)$

  $\sec 240° = -\sec 60° = -(2)$

  $\sec 240° = -2$

Reemplazando en la expresión U:

$U = \left(\left(-\frac{\sqrt{2}}{2}\right)^2 - 3\left(-\frac{4}{3}\right)\right)(-2)^2$

$U = \left(\frac{1}{2} + 4\right)(4) = \left(\frac{9}{2}\right)(4) \qquad \therefore U = 18$

Clave B

Nivel 2 (página 47) Unidad 2

Comunicación matemática

11.

12.

Razonamiento y demostración

13. $A = \frac{\mathrm{sen}(-x) + \cos(-x)}{\mathrm{sen}x - \cos x}$

$A = \frac{(-\mathrm{sen}x) + (\cos x)}{\mathrm{sen}x - \cos x}$

$A = \frac{-\mathrm{sen}x + \cos x}{\mathrm{sen}x - \cos x} = -\left(\frac{\mathrm{sen}x - \cos x}{\mathrm{sen}x - \cos x}\right)$

$\therefore A = -1$

Clave B

14.

$C = \frac{\mathrm{sen}(\pi + x).\tan\left(\frac{\pi}{2} + x\right).\mathrm{sen}\left(\frac{3\pi}{2} - x\right)}{\cot(\pi - x).\cos\left(\frac{\pi}{2} + x\right)}$

$C = \frac{(-\mathrm{sen}x)(-\cot x)(-\cos x)}{(-\cot x)(-\mathrm{sen}x)}$

$C = \frac{(-\mathrm{sen}x).\cot x.\cos x}{\cot x.\mathrm{sen}x} = -\cos x$

$\therefore C = -\cos x$

Clave D

15.

$I = \frac{\mathrm{sen}(x - \pi).\tan\left(x - \frac{\pi}{2}\right)}{\cos\left(x - \frac{3\pi}{2}\right)}$

$I = \frac{\mathrm{sen}(-(\pi - x)).\tan\left(-\left(\frac{\pi}{2} - x\right)\right)}{\cos\left(-\left(\frac{3\pi}{2} - x\right)\right)}$

$I = \frac{(-\mathrm{sen}(\pi - x))\left(-\tan\left(\frac{\pi}{2} - x\right)\right)}{\cos\left(\frac{3\pi}{2} - x\right)}$

$I = \frac{\mathrm{sen}(\pi - x).\tan\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{3\pi}{2} - x\right)}$

$I = \frac{(\mathrm{sen}x)(\cot x)}{(-\mathrm{sen}x)} = \qquad \therefore I = -\cot x$

Clave B

16.

$E = \frac{\mathrm{sen}(180° - \phi).\tan(360° - \phi)}{\cos(270° + \phi).\cot(90° + \phi)}$

$E = \frac{(\mathrm{sen}\phi)(-\tan\phi)}{(\mathrm{sen}\phi)(-\tan\phi)} = 1 \qquad \therefore E = 1$

Clave A

17. $L=\dfrac{\tan(\pi-x).\cot(2\pi-x).\sec(3\pi-x)}{\sec x.\tan(x-\pi).\cot(x-2\pi)}$

$L=\dfrac{(-\tan x)(-\cot x).\sec(2\pi+\pi-x)}{\sec x.\tan(-(\pi-x)).\cot(-(2\pi-x))}$

$L=\dfrac{\tan x.\cot x.\sec(\pi-x)}{\sec x[-\tan(\pi-x)][-\cot(2\pi-x)]}$

$L=\dfrac{\tan x.\cot x.(-\sec x)}{\sec x.\tan(\pi-x).\cot(2\pi-x)}$

$L=\dfrac{-\tan x.\cot x.\sec x}{\sec x.(-\tan x).(-\cot x)}$

$\therefore L=-1$

Clave A

18. Piden:
$P=\tan^3\dfrac{\pi}{12}+\tan^3\dfrac{5\pi}{12}+\tan^3\dfrac{7\pi}{12}+\tan^3\dfrac{11\pi}{12}$

Sabemos:
$\alpha+\beta=180°=\pi\text{ rad}\Rightarrow\tan\alpha=-\tan\beta$

Luego:

- $\left(\dfrac{\pi}{12}\right)+\left(\dfrac{11\pi}{12}\right)=\dfrac{12\pi}{12}=\pi$
  $\Rightarrow\tan\dfrac{\pi}{12}=-\tan\dfrac{11\pi}{12}$
- $\left(\dfrac{5\pi}{12}\right)+\left(\dfrac{7\pi}{12}\right)=\dfrac{12\pi}{12}=\pi$
  $\Rightarrow\tan\dfrac{5\pi}{12}=-\tan\dfrac{7\pi}{12}$

Reemplazando en la expresión P:

$P=\left(-\tan\dfrac{11\pi}{12}\right)^3+\left(-\tan\dfrac{7\pi}{12}\right)^3+\tan^3\dfrac{7\pi}{12}+\tan^3\dfrac{11\pi}{12}$

$P=-\tan^3\dfrac{11\pi}{12}-\tan^3\dfrac{7\pi}{12}+\tan^3\dfrac{7\pi}{12}+\tan^3\dfrac{11\pi}{12}$

$\therefore P=0$

Clave C

19. Por dato: **x** e **y** son ángulos complementarios.
Entonces: $x+y=90°$
$2x+2y=180°$

Piden:

$M=\dfrac{\text{sen}(2x+3y).\cos(x+2y)}{\text{sen}(y+2x).\cos(2y+3x)}$

$M=\dfrac{\text{sen}(2x+2y+y).\cos(x+y+y)}{\text{sen}(x+y+x).\cos(2x+2y+x)}$

$M=\dfrac{\text{sen}(180°+y).\cos(90°+y)}{\text{sen}(90°+x).\cos(180°+x)}$

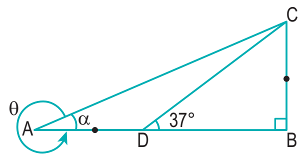
$M=\dfrac{(-\text{sen}y)(-\text{sen}y)}{(\cos x)(-\cos x)}=\dfrac{\text{sen}^2y}{-\cos^2x}$

$M=-\left(\dfrac{\text{sen}y}{\cos x}\right)^2=-\left[\dfrac{\text{sen}(90°-x)}{\cos x}\right]^2$

$M=-\left(\dfrac{\cos x}{\cos x}\right)^2=-(1)^2\qquad\therefore M=-1$

Clave A

20.



Del gráfico: $\theta+\alpha=360°$

En el $\triangle$DBC notable de 37° y 53°:
$DB=4k\ \wedge\ BC=3k$
En el $\triangle$ABC: $\tan\alpha=\dfrac{3k}{7k}$

$\Rightarrow\tan\alpha=\dfrac{3}{7}$

Piden:
$\tan\theta=\tan(360°-\alpha)$

$\tan\theta=-\tan\alpha=-\left(\dfrac{3}{7}\right)\qquad\therefore\tan\theta=-\dfrac{3}{7}$

Clave D

Nivel 3 (página 48) Unidad 2

Comunicación matemática

21.

22.

Razonamiento y demostración

23. Por dato: $\text{sen}40°=n$

Piden:

$K=\dfrac{\text{sen}140°.\cos130°}{\sec410°}$

$K=\dfrac{\text{sen}(180°-40°)\cos(90°+40°)}{\sec(360°+50°)}$

$K=\dfrac{(\text{sen}40°)(-\text{sen}40°)}{(\sec50°)}$

$K=-\dfrac{\text{sen}^240°}{\sec(90°-40°)}=-\dfrac{\text{sen}^240°}{\csc40°}$

$K=-\dfrac{\text{sen}^240°}{\left(\dfrac{1}{\text{sen}40°}\right)}=-\text{sen}^340°$

$\Rightarrow K=-(\text{sen}40°)^3=-(n)^3$

$\therefore K=-n^3$

Clave E

24. Por dato: A, B y C son los ángulos internos de un triángulo.
$\Rightarrow A+B+C=180°$

Piden:

$K=\dfrac{\sec(A+2B+C)}{\csc\left[\dfrac{1}{2}(A+3B+C)\right]}$

$K=\dfrac{\sec(A+B+C+B)}{\csc\left[\dfrac{1}{2}(A+B+C+2B)\right]}$

$K=\dfrac{\sec(180°+B)}{\csc\left[\dfrac{1}{2}(180°+2B)\right]}=\dfrac{(-\sec B)}{\csc(90°+B)}$

$K=-\dfrac{\sec B}{(\sec B)}\qquad\therefore K=-1$

Clave D

25. Por dato: $\sec\alpha=-2$
$\cos\alpha=-\dfrac{1}{2}$

Piden:

$A=\dfrac{1+\text{sen}\left(\alpha-\dfrac{7\pi}{2}\right).\cos(\alpha-3\pi)}{1-\cos\left(\dfrac{3\pi}{2}-\alpha\right).\cot(2\pi-\alpha)}$

Luego:

- $\text{sen}\left(\alpha-\dfrac{7\pi}{2}\right)=\text{sen}\left(-\left(\dfrac{7\pi}{2}-\alpha\right)\right)$
  $\text{sen}\left(\alpha-\dfrac{7\pi}{2}\right)=-\text{sen}\left(2\pi+\dfrac{3\pi}{2}-\alpha\right)$
  $\text{sen}\left(\alpha-\dfrac{7\pi}{2}\right)=-\text{sen}\left(\dfrac{3\pi}{2}-\alpha\right)=-(-\cos\alpha)$
  $\text{sen}\left(\alpha-\dfrac{7\pi}{2}\right)=\cos\alpha$
- $\cos(\alpha-3\pi)=\cos(-(3\pi-\alpha))$
  $\cos(\alpha-3\pi)=\cos(3\pi-\alpha)$
  $\cos(\alpha-3\pi)=\cos(2\pi+\pi-\alpha)$
  $\cos(\alpha-3\pi)=\cos(\pi-\alpha)=-\cos\alpha$
  $\cos(\alpha-3\pi)=-\cos\alpha$

Reemplazando en la expresión A:

$A=\dfrac{1+(\cos\alpha)(-\cos\alpha)}{1-(-\text{sen}\alpha)(-\cot\alpha)}$

$A=\dfrac{1-\cos^2\alpha}{1-\text{sen}\alpha\cot\alpha}=\dfrac{1-\cos^2\alpha}{1-\text{sen}\alpha\left(\dfrac{\cos\alpha}{\text{sen}\alpha}\right)}$

$A=\dfrac{1-\cos^2\alpha}{1-\cos\alpha}=\dfrac{(1-\cos\alpha)(1+\cos\alpha)}{1-\cos\alpha}$

$A=1+\cos\alpha=1+\left(-\dfrac{1}{2}\right)\qquad\therefore A=\dfrac{1}{2}$

Clave A

26. $L=\cos10°+\cos20°+\cos30°+...+\cos180°$

Sabemos:
Si $\alpha+\beta=180°\Rightarrow\cos\alpha=-\cos\beta$
$\Rightarrow\cos\alpha+\cos\beta=0$

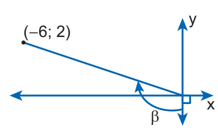
Luego:

$L=\cos10°+\cos20°+...+\cos160°+\cos170°+\cos180°$

Notamos que las parejas de los cosenos señalados van a sumar cero (ya que los ángulos suman 180°), quedando solo el término medio que es cos90°.
$\Rightarrow L=\cos90°+\cos180°=(0)+(-1)=-1$
$\therefore L=-1$

Clave D

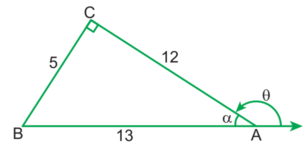
27.



Del gráfico:
$\cot(90°+\beta)=\dfrac{x}{y}=\dfrac{-6}{2}$
$\cot(90°+\beta)=-3$
$\Rightarrow(-\tan\beta)=-3$
$\therefore\tan\beta=3$

Clave B

28.



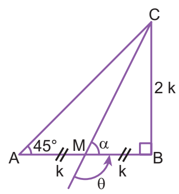
En el ◺BCA por el teorema de Pitágoras:
AC = 12
Del gráfico: $\theta + \alpha = 180°$
Piden:
$\tan\theta + \sec\theta = \tan(180° - \alpha) + \sec(180° - \alpha)$
$\tan\theta + \sec\theta = (-\tan\alpha) + (-\sec\alpha)$
$\tan\theta + \sec\theta = -(\tan\alpha + \sec\alpha)$

$$\tan\theta + \sec\theta = -\left(\frac{5}{12} + \frac{13}{12}\right) = -\frac{18}{12}$$

$$\therefore \tan\theta + \sec\theta = -\frac{3}{2}$$

Clave D

29.



Del gráfico: $\theta + \alpha = 180°$
En el ◺ABC notable de 45°:
AB = BC = 2k

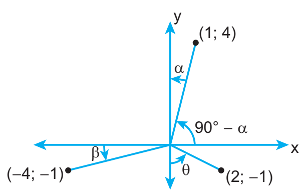
En el ◺MBC: $\tan\alpha = \dfrac{2k}{k}$

$\Rightarrow \tan\alpha = 2$

Piden:
$\tan\theta = \tan(180° - \alpha)$
$\tan\theta = -\tan\alpha = -(2)$ $\quad\therefore \tan\theta = -2$

Clave D

30.



Del gráfico:

- $\tan(90° - \alpha) = \dfrac{y}{x} = \dfrac{4}{1}$

  $\tan(90° - \alpha) = 4 \Rightarrow \cot\alpha = 4$

  $\tan\alpha = \dfrac{1}{4}$

- $\tan(180° + \beta) = \dfrac{y}{x} = \dfrac{-1}{-4}$

  $\tan(180° + \beta) = \dfrac{1}{4} \Rightarrow \tan\beta = \dfrac{1}{4}$

- $\tan(270° + \theta) = \dfrac{y}{x} = \dfrac{-1}{2}$

  $\tan(270° + \theta) = -\dfrac{1}{2} \Rightarrow -\cot\theta = -\dfrac{1}{2}$

  $\cot\theta = \dfrac{1}{2} \Rightarrow \tan\theta = 2$

Piden:
$P = (\tan\alpha + \tan\beta + \tan\theta)^2$

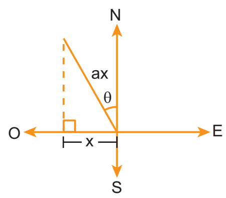
$$P = \left(\frac{1}{4} + \frac{1}{4} + 2\right)^2 = \left(\frac{5}{2}\right)^2$$

$$\therefore P = \frac{25}{4}$$

Clave A

MARATÓN MATEMÁTICA (página 50)

1. En el plano:



"θ" es el rumbo:

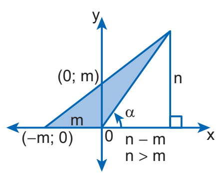
$$\cos\theta = \frac{\sqrt{(ax)^2 - x^2}}{ax}$$

$$\cos\theta = \sqrt{\frac{x^2(a^2 - 1)}{x^2(a^2)}}$$

$$\therefore \cos\theta = \sqrt{1 - \frac{1}{a^2}}$$

Clave A

2. Del gráfico tenemos:



$A_S = \dfrac{mn}{2} = 4{,}5\ u^2$
$mn = 9\ u^2$
1×9
3×3
9×1
$\Rightarrow n = 9\ u \ \wedge \ m = 1\ u$

$\therefore \cot\alpha = \dfrac{n - m}{n} = \dfrac{8}{9}$

Clave D

3. Si $\operatorname{sen}\theta < 0 \ \wedge \ \cos\theta > 0 \ \Rightarrow \ \theta \in$ IVC
Luego tenemos:
$\tan\theta = -3/4$
$\cot\theta = -4/3$
$k = \tan\theta + \cot\theta$

$k = \dfrac{-3}{4} + \dfrac{-4}{3} \qquad \therefore\ k = \dfrac{-25}{12}$

Clave B

4. $180° < \theta < 270°$
$P = \cos(\theta/4) \times \tan(\theta/2) \times \operatorname{sen}(2\theta)$
IC IIC IC ó IIC
$P = (+)\ (-)\ (+) \Rightarrow P = (-)$
$Q = \operatorname{sen}(\theta/4) \times \cot(\theta) \times \cos(\theta/3)$
IC IIIC IC
$Q = (+)\ (+)\ (+) \Rightarrow Q = (+)$
$\therefore (-); (+)$

Clave C

5. Si $\theta \in$ IIIC $\Rightarrow \cos\theta < 0$

$-1 < \dfrac{P}{a - q} < 0$

$1 > \dfrac{P}{q - a} > 0 \qquad p < q - a$
$a < q - p$
$\therefore q - p > a$

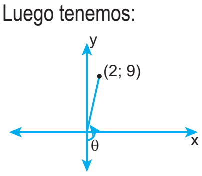
Clave B

6. $\dfrac{x}{2} + 8 = 11 - x$

$\dfrac{3x}{2} = 3$

$x = 2 \wedge y = 9 \Rightarrow (2; 9)$ punto de intersección

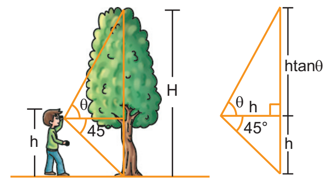
Luego tenemos:



$\therefore \tan\theta = -2/9$

Clave C

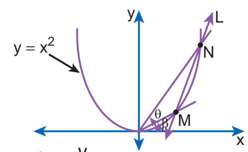
7.



$\Rightarrow H = h\tan\theta + h \qquad \therefore H = h(\tan\theta + 1)$

Clave C

8.



- $\tan\beta = \dfrac{1}{2} = \dfrac{y_\beta}{x_\beta}$

  $x_\beta = 2y_\beta$
  $x_\beta = 2x_\beta^2$
  $x_\beta = 1/2 \ \wedge \ y_\beta = 1/4$
  $M = \left(\dfrac{1}{2}; \dfrac{1}{4}\right)$

- $\tan\theta = 4 = \dfrac{y_\theta}{x_\theta}$

  $4x_\theta = y_\theta$
  $4x_\theta = x_\theta^2$
  $x_\theta = 4 \ \wedge \ y_\theta = 16$
  $N = (4; 16)$

- L: $\dfrac{x - x_M}{y - y_M} = \dfrac{x_N - x_M}{y_N - y_M}$

  $\dfrac{x - 1/2}{y - 1/4} = \dfrac{4 - 1/2}{16 - 1/4} \Rightarrow \dfrac{2(2x - 1)}{4y - 1} = \dfrac{7/2}{63/4}$

  $\dfrac{4x - 2}{4y - 1} = \dfrac{2}{9}$

  $36x - 18 = 8y - 2$
  $18x - 9 = 4y - 1$

  $y = \dfrac{18x - 8}{4} \qquad \therefore\ y = \dfrac{9x}{2} - 2$

Clave A

9. De la condición:
$\tan(3k\pi + \pi/2 + \theta) = -1/2 \Rightarrow \tan(\pi/2 + \theta) = -1/2$
$-\cot\theta = -1/2$
$\tan\theta = 2$

Nos piden:

$$P = \sqrt{\frac{-\csc^2\left(\frac{37\pi}{2} + \theta\right)}{\cos\left(\frac{-7\pi}{2} + \theta\right).\operatorname{sen}(-7\pi - \theta)}}$$

$$P = \sqrt{\frac{-\csc^2\left(18\pi + \frac{\pi}{2} + \theta\right)}{\cos\left(-4\pi + \frac{\pi}{2} + \theta\right).\operatorname{sen}(-8\pi + \pi - \theta)}}$$

$$P = \sqrt{\frac{-\csc^2\left(\frac{\pi}{2} + \theta\right)}{\cos\left(\frac{\pi}{2} + \theta\right).\operatorname{sen}(\pi - \theta)}}$$

$$P = \sqrt{\frac{-\sec^2\theta}{(-\operatorname{sen}\theta)(+\operatorname{sen}\theta)}} = \sqrt{\frac{-\sec^2\theta}{-\operatorname{sen}^2\theta}}$$

$P = \sqrt{(\sec\theta\,.\,\csc\theta)^2} = \sec\theta\,.\,\csc\theta$

$P = \tan\theta + \cot\theta = 2 + 1/2 \qquad \therefore\ P = 5/2$

Clave D

APLICAMOS LO APRENDIDO (página 53) Unidad 3

1. Operamos la expresión:

$\cos\beta = \frac{m-1}{3} + \frac{3-m}{2}$

$\cos\beta = \frac{2(m-1)+3(3-m)}{(3)(2)} = \frac{2m-2+9-3m}{6}$

$6\cos\beta = 7 - m \Rightarrow m = 7 - 6\cos\beta$

Sabemos:

$-1 \leq \cos\beta \leq 1$

$-6 \leq 6\cos\beta \leq 6$

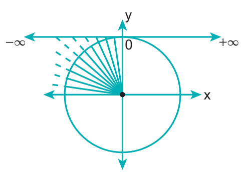
$6 \geq -6\cos\beta \geq -6$

$13 \geq 7 - 6\cos\beta \geq 1 \Rightarrow 13 \geq m \geq 1$

$\therefore\ m \in [1; 13]$

Clave C

2. Graficamos la RT cotangente en el IIC.



$\Rightarrow -\infty < \cot\alpha < 0$

Operamos para hallar el valor de P:

$-\infty < \cot\alpha < 0$

$-\infty < 8\cot\alpha < 0$

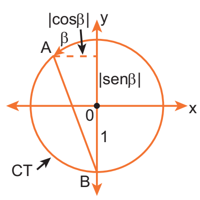
$-\infty < 8\cot\alpha + 7 < 7$

$-\infty < P < 7$

$\therefore\ P \in \langle -\infty; 7\rangle$

Clave B

3. Hallamos el valor de $\overline{AB}$:



$AB^2 = (|\cos\beta|)^2 + (|\text{sen}\beta| + 1)^2$

$AB^2 = \underbrace{\cos\beta^2 + \text{sen}\beta^2}_{1} + 2\text{sen}\beta + 1$

$(AB)^2 = 2 + 2\text{sen}\beta$ ... (1)

Hallamos BC en el ⊿ABC:

$(AB)^2 + (BC)^2 = (AC)^2$

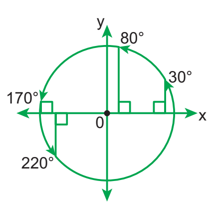
$2 + 2\text{sen}\beta + x^2 = (2)^2$

$x^2 = 4 - 2 - 2\text{sen}\beta$

$x^2 = 2 - 2\text{sen}\beta \Rightarrow x = \sqrt{2 - 2\text{sen}\beta}$

Clave D

4. Graficamos las razones trigonométricas en la CT:



Luego, tenemos:

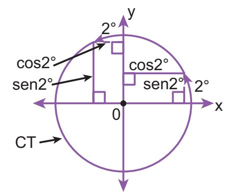
sen80° > sen30° > sen170° > sen220°

II > I > IV > III

Clave E

5. Debemos tener en cuenta:

2 = 2 rad ≅ 114° 35' 30"

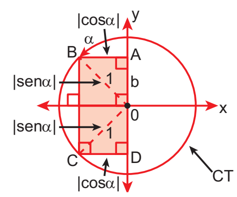


sen2° > sen2 F

cos2° > cos2 V

Clave C

6.



Para calcular el área de la región sombreada, utilizaremos distancias.

La figura es un rectángulo, sea su área: A

A = (base) . (altura)

A = (|cosα|) . (|senα| + |senα|)

$A = \underbrace{(|\cos\alpha|)}_{(-)}\underbrace{(2|\text{sen}\alpha|)}_{(+)}$

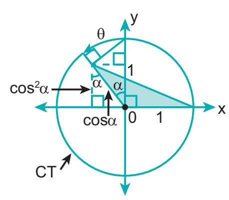
Entonces:

$A = (-\cos\alpha)(2\text{sen}\alpha)$

$\therefore A = -2\text{sen}\alpha\cos\alpha$

Clave C

7.



Del gráfico:

$A_{somb.} = \frac{(\text{base})(\text{altura})}{2}$

$A_{somb.} = \frac{(1).(\cos^2\alpha)}{2} = 0{,}5\cos^2\alpha$

Además: $\theta = 90° + \alpha$

$\Rightarrow \text{sen}\theta = \cos\alpha$

$\text{sen}^2\theta = \cos^2\alpha$

Luego:

$A_{somb.} = 0{,}5(\text{sen}^2\theta)$

$\therefore A_{somb.} = 0{,}5\text{sen}^2\theta$

Clave B

8. El ángulo doble no influye en la variación; entonces:

$-1 \leq \text{sen}2\alpha \leq 1$

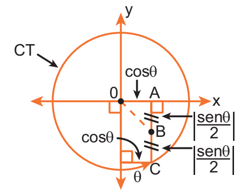
$-2 \leq 2\text{sen}2\alpha \leq 2$

$-3 \leq 2\text{sen}2\alpha - 1 \leq 1 \Rightarrow -3 \leq k \leq 1$

$\therefore\ k \in [-3; 1]$

Clave E

9. Del gráfico tenemos:



En el ⊿OAB:

$OB^2 = OA^2 + AB^2$

$x^2 = (\cos\theta)^2 + \left(\frac{\text{sen}\theta}{2}\right)^2$

$x^2 = \cos^2\theta + \frac{\text{sen}^2\theta}{4} = \frac{4\cos^2\theta + \text{sen}^2\theta}{4}$

$x^2 = \frac{3\cos^2\theta + 1}{4}$

$\therefore\ x = \frac{\sqrt{3\cos^2\theta + 1}}{2}$

Clave C

10. Reducimos la expresión:

$2\text{sen}\beta = \frac{P+2}{3} - \frac{5+P}{4} = \frac{4(P+2) - 3(5+P)}{(3)(4)}$

$2\text{sen}\beta = \frac{4P + 8 - 15 - 3P}{12} \Rightarrow 24\text{sen}\beta = P - 7$

$P = 24\text{sen}\beta + 7$

Sabemos:

$-1 \leq \text{sen}\beta \leq 1$

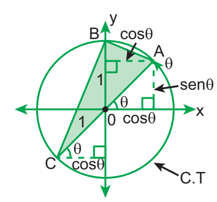
$-24 \leq 24\text{sen}\beta \leq 24$

$-17 \leq 24\text{sen}\beta + 7 \leq 31 \Rightarrow -17 \leq P \leq 31$

$P \in [-17; 31]$

Clave B

11.



Como θ ∈ IC, sus razones trigonométricas son positivas.

Sea A: el área de la región sombreada.

Del gráfico:

$A = A_{\triangle ABO} + A_{\triangle BOC}$

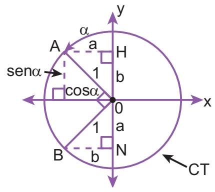
$A = \frac{1.\cos\theta}{2} + \frac{1.\cos\theta}{2}$

$A = \frac{2\cos\theta}{2}$

$\therefore A = \cos\theta$

Clave D

12.



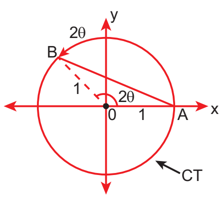
Las coordenadas del punto A serían:
A(–a; b) = A(cosα; senα) ... (I)

Del gráfico:
$\triangle$AHO ≅ $\triangle$ONB
Las coordenadas del punto B serán:
B(–b; –a) ... (II)

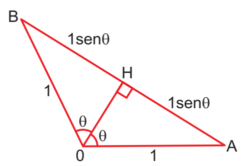
De (I): a = –cosα
b = senα
Reemplazando en (II):
B(–senα; –(–cosα))
∴ B(–senα; cosα)

Clave D

13.



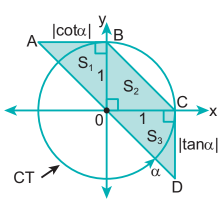
Entonces:



Del gráfico: AB = AH + HB
AB = senθ + senθ
∴ AB = 2senθ

Clave C

14.



Por definición:
AB = cotα ∧ CD = tanα

Del gráfico:
$A_{somb.} = S_1 + S_2 + S_3$

$$A_{somb.} = \frac{1.|\cot\alpha|}{2} + \frac{1.1}{2} + \frac{1.|\tan\alpha|}{2}$$

$$A_{somb.} = \frac{1}{2}(1 + \underbrace{|\tan\alpha|}_{(-)} + \underbrace{|\cot\alpha|}_{(-)})$$

$\therefore A_{somb.} = 0,5(1 - \tan\alpha - \cot\alpha)$

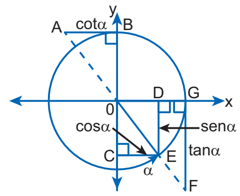
Clave E

PRACTIQUEMOS
Nivel 1 (página 55) Unidad 3

Comunicación matemática

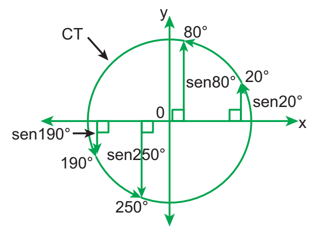
1.

2. Por teoría, tenemos:



Razonamiento y demostración

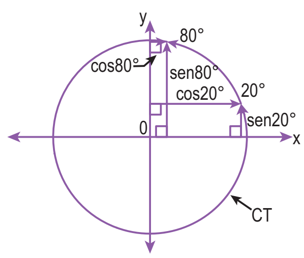
3.



Del gráfico:
sen80° > sen20° > 0
sen250° < sen190° < 0
Entonces:
I. sen20° > sen80° (F)
II. sen190° < sen250° (F)

Clave C

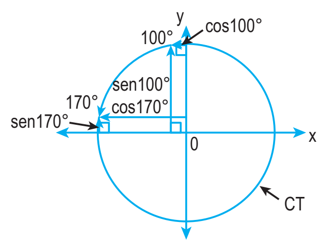
4.



Del gráfico:
cos20° > sen20° > 0
sen80° > cos80° > 0
Entonces:
I. sen20° < cos20° (V)
II. cos80° > sen80° (F)

Clave C

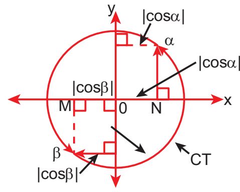
5.



Del gráfico:
sen100° > 0; cos100° < 0 ∧
|sen100°| > |cos100°|
⇒ sen100° > –(cos100°)
⇒ sen100° + cos100° > 0
sen170° > 0; cos170° < 0 ∧
|sen170°| < |cos170°|
⇒ sen170° < –(cos170°)
⇒ sen170° + cos170° < 0
Entonces:
I. sen100° + cos100° < 0 (F)
II. sen170° + cos170° > 0 (F)

Clave D

6.



Del gráfico:
MN = MO + ON
MN = |cosβ| + |cosα|
- Como α ∈ IC ⇒ cosα > 0
⇒ |cosα| = cosα
- Como β ∈ IIIC ⇒ cosβ < 0
⇒ |cosβ| = –cosβ

Entonces:
MN = (–cosβ) + (cosα) = cosα – cosβ
∴ MN = cosα – cosβ

Clave D

7.

Por dato: $\operatorname{sen}\theta = \dfrac{2x-5}{3}$

Sabemos: $-1 \le \operatorname{sen}\theta \le 1$

$$-1 \le \frac{2x-5}{3} \le 1$$
$$-3 \le 2x - 5 \le 3$$
$$2 \le 2x \le 8$$
$$1 \le x \le 4$$
$\therefore x \in [1; 4]$

Clave B

8.

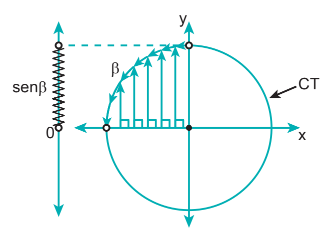
Piden la variación de:
P = tanx + 2

Sabemos que $\forall\, x \in \mathbb{R} - \{(2k+1)\frac{\pi}{2};\, k \in \mathbb{Z}\}$

$$-\infty < \tan x < +\infty$$
$$\Rightarrow -\infty < \tan x + 2 < +\infty$$
$$-\infty < P < +\infty$$
$\therefore P \in \langle -\infty; +\infty \rangle$ que es equivalente a $P \in \mathbb{R}$.

Clave E

9.



Luego:

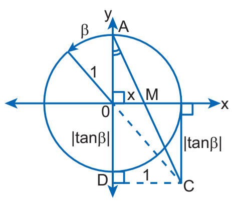
$$0 < \operatorname{sen}\beta < 1$$
$$-3 < -3\operatorname{sen}\beta < 0$$
$$4 - 3 < \underbrace{4 - 3\operatorname{sen}\beta}_{1 < E < 4} < 0 + 4$$

$\therefore E \in \langle 1; 4 \rangle$

Clave E

Resolución de problemas

10. La intersección es en M:



En el ◺ADC y el ◺AOM tenemos:

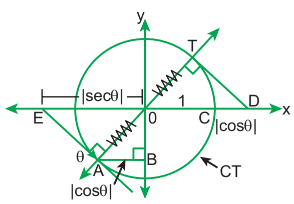
$$\frac{1}{x} = \frac{1 + |\tan\beta|}{1}$$

$$\frac{1}{x} = \frac{1 - \tan\beta}{1} \Rightarrow \frac{1}{1 - \tan\beta} = x$$

$$\therefore x = \frac{1}{(1 - \tan\beta)}$$

Clave D

11. Trazamos AE ⊥ AT:



Del gráfico tenemos: ($\overline{TD} \,//\, \overline{EA}$)

$\Rightarrow |\sec\theta| = 1 + |\cos\theta|$

$-\sec\theta = 1 - \cos\theta$

$\dfrac{-1}{\cos\theta} = 1 - \cos\theta \Rightarrow -1 = \cos\theta - \cos^2\theta$

$\cos^2\theta - \cos\theta - 1 = 0$

$x^2 - x - 1 = 0$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1+4}}{2}$$

$$x = \frac{1 \pm \sqrt{5}}{2};\ \cos\theta \text{ es negativo}$$

$$\Rightarrow \cos\theta = \frac{1 - \sqrt{5}}{2}$$

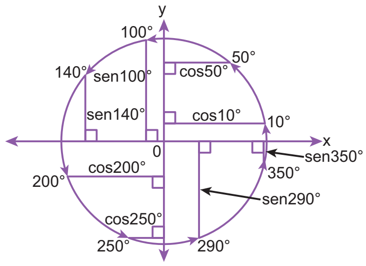
$$\therefore \cos = \frac{1 - \sqrt{5}}{2}$$

Clave C

Nivel 2 (página 55) Unidad 3

Comunicación matemática

12.

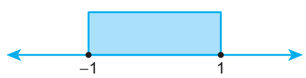


Del gráfico tenemos:

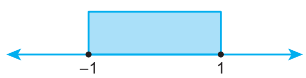
$\operatorname{sen}100° > \operatorname{sen}140°$	(V)
$\operatorname{esn}350° < \operatorname{sen}290°$	(F)
$\cos 10° < \cos 50°$	(F)
$\cos 200° > \cos 250°$	(F)

Clave D

13. I. senx



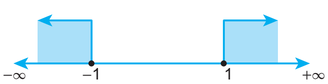
II. cosx



III. tanx:

IV. secx:

V. cscx:



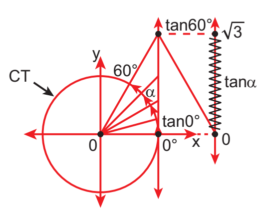
Razonamiento y demostración

14. Piden el máximo valor de:

$P = \sqrt{3}\tan\alpha + 1$

Por dato: $\alpha \in [0°; 60°]$

Analizando en la CT:



Entonces: $0 \leq \tan\alpha \leq \sqrt{3}$

$$0 \leq \sqrt{3}\tan\alpha \leq 3$$
$$1 \leq \underbrace{\sqrt{3}\tan\alpha + 1}_{1 \leq P \leq 4} \leq 4$$

$\Rightarrow P \in [1; 4]$

$\therefore P_{máx.} = 4$

Clave E

15.

Por dato: $\theta \in$ IIIC

Además:

$2 + \sqrt{\operatorname{sen}x - 1} = \sqrt{8 + 5\cos\theta}$ ...(α)

De la función raíz cuadrada, se debe cumplir:

$\operatorname{sen}x - 1 \geq 0 \Rightarrow \operatorname{sen}x \geq 1$ ...(I)

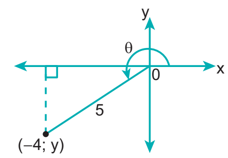
Sabemos que: $-1 \leq \operatorname{sen}x \leq 1$ ...(II)

De (I) y (II) deducimos: senx = 1

Reemplazando en (a):

$$2 + \sqrt{(1) - 1} = \sqrt{8 + 5\cos\theta}$$
$$2 = \sqrt{8 + 5\cos\theta}$$
$$4 = 8 + 5\cos\theta$$
$$-4 = 5\cos\theta \Rightarrow \cos\theta = -\frac{4}{5}$$

Luego:



Por radio vector:

$5^2 = (-4)^2 + y^2$

$9 = y^2$

$\Rightarrow y = 3 \vee y = -3$

Del gráfico: $y < 0$

$\Rightarrow y = -3$

Considerar que se debe calcular:

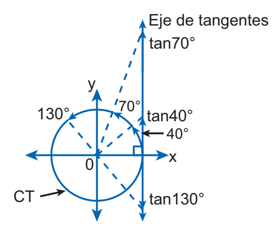
$$3\cot\theta + 2\csc x = 3\left(\frac{x}{y}\right) + 2\left(\frac{1}{\operatorname{sen}x}\right)$$

$$3\cot\theta + 2\csc x = 3\left(\frac{-4}{-3}\right) + 2\left(\frac{1}{1}\right) = 4 + 2$$

$$\therefore 3\cot\theta + 2\csc x = 6$$

Clave E

16.



Del gráfico:

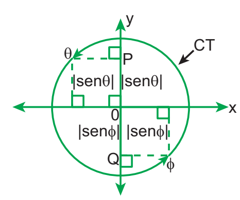
$\tan 70° > \tan 40° > 0 \wedge \tan 130° < 0$

Ordenando de menor a mayor tenemos:

$\tan 130° < \tan 40° < \tan 70°$

Clave C

17.



Del gráfico:

$PQ = PO + OQ$

$PQ = |\text{sen}\theta| + |\text{sen}\phi|$

Como $\theta \in \text{IIC} \Rightarrow \text{sen}\theta > 0$

$\Rightarrow |\text{sen}\theta| = \text{sen}\theta$

Como $\phi \in \text{IVC} \Rightarrow \text{sen}\phi < 0$

$\Rightarrow |\text{sen}\phi| = -\text{sen}\phi$

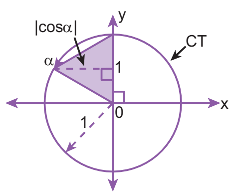
Entonces:

$PQ = (\text{sen}\theta) + (-\text{sen}\phi) = \text{sen}\theta - \text{sen}\phi$

$\therefore PQ = \text{sen}\theta - \text{sen}\phi$

Clave A

18.



Piden el área del triángulo sombreado (S).

$$\Rightarrow S = \frac{(\text{base}).(\text{altura})}{2} = \frac{(1).(|\cos\alpha|)}{2}$$

$$S = \frac{|\cos\alpha|}{2}$$

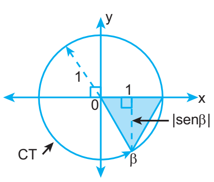
Como $\alpha \in \text{IIC} \Rightarrow \cos\alpha < 0$

$$S = \frac{-(\cos\alpha)}{2} = -\frac{\cos\alpha}{2}$$

$$\therefore S = -\frac{\cos\alpha}{2}$$

Clave D

19.



Piden el área del triángulo sombreado (S).

$$S = \frac{(\text{base}).(\text{altura})}{2} = \frac{(1).(|\text{sen}\beta|)}{2}$$

$$\Rightarrow S = \frac{|\text{sen}\beta|}{2}$$

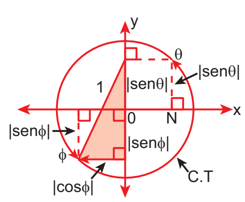
Como $\beta \in \text{IVC} \Rightarrow \text{sen}\beta < 0$

$$\Rightarrow S = \frac{-(\text{sen}\beta)}{2}$$

$$\therefore S = -\frac{\text{sen}\beta}{2}$$

Clave B

20.



Piden: el área del triángulo sombreado (S).

$$S = \frac{(\text{base}).(\text{altura})}{2}$$

$$S = \frac{(|\cos\phi|).(|\text{sen}\theta| + |\text{sen}\phi|)}{2}$$

Como $\theta \in \text{IC} \Rightarrow \text{sen}\theta > 0$

$\Rightarrow |\text{sen}\theta| = \text{sen}\theta$

Como $\phi \in \text{IIIC} \Rightarrow \text{sen}\phi < 0 \wedge \cos\phi < 0$

$\Rightarrow |\text{sen}\phi| = -\text{sen}\phi \wedge |\cos\phi| = -\cos\phi$

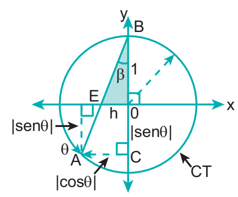
Entonces:

$$S = \frac{(-\cos\phi).(\text{sen}\theta + (-\text{sen}\phi))}{2}$$

$\therefore S = -0{,}5\cos\phi(\text{sen}\theta - \text{sen}\phi)$

Clave A

21.



Del gráfico:

$$\tan\beta = \frac{EO}{OB} = \frac{AC}{CB}$$

Como $\theta \in \text{IIIC} \Rightarrow \cos\theta < 0 \wedge \text{sen}\theta < 0$

$\Rightarrow |\cos\theta| = -\cos\theta \wedge |\text{sen}\theta| = -\text{sen}\theta$

Entonces:

$$h = \frac{|\cos\theta|}{1 + |\text{sen}\theta|}$$

$$\Rightarrow h = \frac{-\cos\theta}{1 + (-\text{sen}\theta)}$$

Piden el área del triángulo sombreado (S).

$$\Rightarrow S = \frac{(OB).(EO)}{2} = \frac{(1).(h)}{2}$$

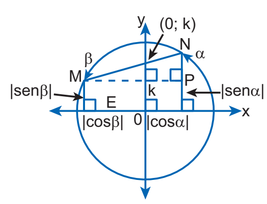
$$S = \frac{h}{2} = \frac{\cos\theta}{2(\text{sen}\theta - 1)}$$

$$\therefore S = \frac{\cos\theta}{2(\text{sen}\theta - 1)}$$

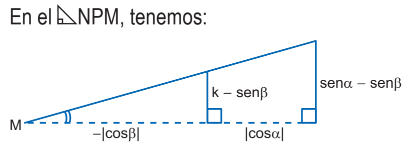
Clave B

Resolución de problemas

22. Graficamos α y β, $\alpha \in \text{IC}$ y $\beta \in \text{IIC}$



En el $\triangle$NPM, tenemos:



$$\frac{k - \text{sen}\beta}{-\cos\beta} = \frac{\text{sen}\alpha - \text{sen}\beta}{\cos\alpha - \cos\beta}$$

$$k = \text{sen}\beta = -\frac{\text{sen}\alpha\cos\beta + \text{sen}\beta\cos\beta}{\cos\alpha - \cos\beta}$$

$$k = \text{sen}\beta + \frac{\text{sen}\beta\cos\beta - \text{sen}\alpha\cos\beta}{\cos\alpha - \cos\beta}$$

$$k = \frac{(\text{sen}\beta\cos\alpha - \text{sen}\beta\cos\beta - \text{sen}\beta\cos\beta)}{(\cos\alpha - \cos\beta)}$$

$$k = \frac{\text{sen}\beta\cos\alpha - \text{sen}\alpha\cos\beta}{\cos\alpha - \cos\beta}$$

Clave C

23. $R = \dfrac{\text{sen}^2\theta + 1}{\text{sen}^2\theta + 4} = \dfrac{\text{sen}^2\theta + 1 + 3 - 3}{\text{sen}^2\theta + 4}$

$$R = \frac{\text{sen}^2\theta + 4}{\text{sen}^2\theta + 4} - \frac{3}{\text{sen}^2\theta + 4}$$

$$R = 1 - \frac{3}{\text{sen}^2\theta + 4}$$

Para $\theta \in \text{IIIC}$:

$-1 < \text{sen}\theta < 0$

$0 < \text{sen}^2\theta < 1$

$4 < \text{sen}^2\theta + 4 < 5$

$$\frac{1}{4} > \frac{1}{\text{sen}^2\theta + 4} > \frac{1}{5}$$

$$\frac{3}{4} > \frac{3}{\text{sen}^2\theta + 4} > \frac{3}{5}$$

$$\frac{3}{4} < \frac{-3}{\text{sen}^2\theta + 4} < \frac{-3}{5}$$

$$\frac{1}{4} < 1 - \frac{3}{\text{sen}^2\theta + 4} < \frac{2}{5}$$

$$\Rightarrow \frac{1}{4} < R < \frac{2}{5}$$

$$\therefore R \in \left\langle \frac{1}{4}; \frac{2}{5} \right\rangle$$

Clave A

Nivel 3 (página 56) Unidad 3

Comunicación matemática

24. M:

$-1 \le \cos x \le 1$

$-1 \le \dfrac{3a - 5}{4} \le 1 \Rightarrow -4 \le 3a - 5 \le 4$

$1 \le 3a \le 9 \Rightarrow \dfrac{1}{3} \le a \le 3$

$a = \{0; 1; 2; 3\}$

$M = 0 + 1 + 2 + 3 = 6$

N:

$-1 \le \text{sen}x \le 1$

$-1 \le \dfrac{5b - 4}{6} \le 1 ; -6 \le 5b - 4 \le 6$

$-2 \le 5b \le 10; \dfrac{-2}{5} \le b \le 2$

$b = \{0; 1; 2\}$

$N = 0 + 1 + 2 = 3$

$\therefore M = 2N$

Clave B

25.

Razonamiento y demostración

26. Por dato: $20° < \theta < \alpha \leq 90°$

Además: $\cos 2\alpha + \csc 3\alpha = 0$

$\Rightarrow \cos 2\alpha = -\csc 3\theta$

$\Rightarrow \cos 2\alpha = -\dfrac{1}{\text{sen}3\theta}$

$\Rightarrow \underbrace{\text{sen}3\theta}_{\substack{1\\-1}} \,.\, \underbrace{\cos 2\alpha}_{\substack{-1\\1}} = -1$

Luego:

Si $\text{sen}3\theta = 1 \wedge \cos 2\alpha = -1$, entonces:

$3\theta = 90° \wedge 2\alpha = 180°$

$\Rightarrow \theta = 30° \wedge \alpha = 90°$

(cumple con el dato inicial)

Si $\text{sen}3\theta = -1 \wedge \cos 2\alpha = 1$, entonces:

$3\theta = 270° \wedge 2\alpha = 0°$

$\Rightarrow \theta = 90° \wedge \alpha = 0°$

(no cumple con el dato inicial)

Por lo tanto: $\theta = 30° \wedge \alpha = 90°$

Piden:

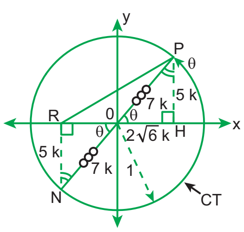
$\alpha + \theta = 90° + 30° = 120°$

$\Rightarrow \alpha + \theta = 120°\left(\dfrac{\pi \text{ rad}}{180°}\right) = \dfrac{2\pi}{3}$ rad

$\therefore \alpha + \theta = \dfrac{2\pi}{3}$ rad

Clave A

27.



Por dato: $\text{sen}\theta = \dfrac{5}{7}$

En el ◺OHP por el teorema de Pitágoras:

$OH = 2\sqrt{6}\,k$

Luego el ◺PHO ≅ ◺NRO (A–L–A)

$\Rightarrow OH = OR = 2\sqrt{6}\,k$

Además: $OP = ON = 1 \Rightarrow 7k = 1 \Rightarrow k = \dfrac{1}{7}$

En el ◺RHP por el teorema de Pitágoras:

$PR^2 = (4\sqrt{6}\,k)^2 + (5k)^2$

$PR^2 = 96k^2 + 25k^2 = 121k^2$

$\Rightarrow PR = 11k = 11\left(\dfrac{1}{7}\right) \quad \therefore PR = \dfrac{11}{7}$

Clave D

28. Por dato: $\cos 2\theta = \dfrac{x-3}{2}$

Sabemos: $-1 \leq \cos\theta \leq 1$

$0 \leq \cos 2\theta \leq 1$

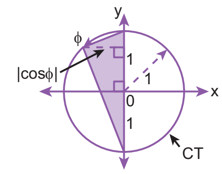
$0 \leq \dfrac{x-3}{2} \leq 1$

$0 \leq x - 3 \leq 2$

$3 \leq x \leq 5 \qquad \therefore x \in [3; 5]$

Clave A

29.



Piden el área del triángulo sombreado (S).

$S = \dfrac{(\text{base}).(\text{altura})}{2} = \dfrac{(2).(|\cos\phi|)}{2}$

$\Rightarrow S = |\cos\phi|$

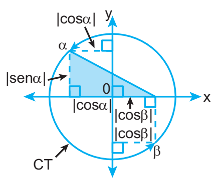
Como $\phi \in$ IIC $\Rightarrow \cos\phi < 0$

$\Rightarrow S = -(\cos\phi) = -\cos\phi$

$\therefore S = -\cos\phi$

Clave D

30.



Piden el área del triángulo sombreado (S).

$S = \dfrac{(\text{base}).(\text{altura})}{2}$

$S = \dfrac{(|\cos\alpha| + |\cos\beta|).(|\text{sen}\alpha|)}{2}$

Como $\alpha \in$ IIC $\Rightarrow \text{sen}\alpha > 0 \wedge \cos\alpha < 0$

$\Rightarrow |\text{sen}\alpha| = \text{sen}\alpha \wedge |\cos\alpha| = -\cos\alpha$

Como $\beta \in$ IVC $\Rightarrow \cos\beta > 0$

$\Rightarrow |\cos\beta| = \cos\beta$

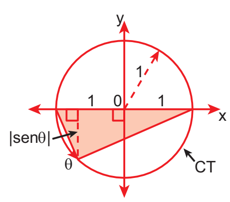
Entonces:

$S = \dfrac{(-\cos\alpha + \cos\beta).(\text{sen}\alpha)}{2}$

$\therefore S = 0{,}5\,\text{sen}\alpha(\cos\beta - \cos\alpha)$

Clave A

31.



Piden el área del triángulo sombreado (S).

$S = \dfrac{(\text{base}).(\text{altura})}{2} = \dfrac{(2).(|\text{sen}\theta|)}{2}$

$\Rightarrow S = |\text{sen}\theta|$

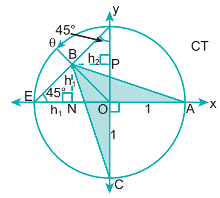
Como $\theta \in$ IIIC $\Rightarrow \text{sen}\theta < 0$

$\Rightarrow S = -(\text{sen}\theta) = -\text{sen}\theta$

$\therefore S = -\text{sen}\theta$

Clave D

32.



Del gráfico:

$BP = NO = h_2$

Además:

$EO = EN + NO$

$\Rightarrow EO = h_1 + h_2$

Pero EO es el radio de la CT:

$\Rightarrow EO = 1 \quad \Rightarrow h1 + h2 = 1$

Piden el área del cuadrilátero sombreado (S).

$S = S_{\triangle ABO} + S_{\triangle CBO}$

$S = \dfrac{(OA).h_1}{2} + \dfrac{(OC).h_2}{2}$

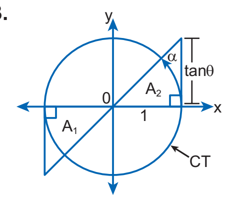
$S = \dfrac{(1).h_1}{2} + \dfrac{(1).h_2}{2}$

$\Rightarrow S = \dfrac{1}{2}(h1 + h2) = \dfrac{1}{2}(1)$ ` $S = \dfrac{1}{2}$

Clave E

Resolución de problemas

33.



$A_1 = A_2 = \dfrac{1.\tan\theta}{2} = \dfrac{\tan\theta}{2}$

$45° \leq \alpha \leq 60°$

$1 \leq \tan\alpha \leq \sqrt{3}$

$\dfrac{1}{2} \leq \tan\alpha \leq \dfrac{\sqrt{3}}{2} \qquad \therefore A \in \left[\dfrac{1}{2}; \dfrac{\sqrt{3}}{2}\right]$

Clave D

34. $\tan^3\alpha < 4\tan\alpha$

$0 < 4\tan\alpha - \tan\alpha^3$

$0 < \tan\alpha(4 - \tan^2\alpha)$

$\qquad (+) \qquad (+)$

$\qquad (-) \qquad (-)$

Valores negativos

Entonces:

$\tan\alpha < 0 \;\wedge\; 4 - \tan^2\alpha < 0$

$4 < \tan^2\alpha$

Se cumple:

$\tan\alpha \in \langle -\infty; -2\rangle \;\wedge\; \langle 2; +\infty\rangle$

$\Rightarrow \tan\alpha \in \langle -\infty; -2\rangle$

El máximo valor entero negativo:

$\tan\alpha = -3 = \dfrac{-3}{1} = \dfrac{x}{y}$

$x^2 + y^2 = r^2$

$(-3)^2 + (1)^2 = r^2 = 10 \Rightarrow r = \sqrt{10}$

$\sec\alpha \,.\, \csc\alpha = \dfrac{r}{y}.\dfrac{r}{x} = \dfrac{r^2}{xy}$

$= \dfrac{(\sqrt{10})^2}{(-3)(1)} = \dfrac{-10}{3}$

$\therefore \sec\alpha \,.\, \csc\alpha = \dfrac{-10}{3}$

Clave B

APLICAMOS LO APRENDIDO (página 58) Unidad 3

1. $L = (\csc\alpha + 1)(\sec\alpha - \tan\alpha)$

$$L = \left(\frac{1}{\text{sen}\alpha} + 1\right)\left(\frac{1}{\cos\alpha} - \frac{\text{sen}\alpha}{\cos\alpha}\right)$$

$$L = \left(\frac{1+\text{sen}\alpha}{\text{sen}\alpha}\right)\left(\frac{1-\text{sen}\alpha}{\cos\alpha}\right) = \frac{1-\text{sen}^2\alpha}{\text{sen}\alpha\,.\cos\alpha}$$

$$L = \frac{\cos^2\alpha}{\text{sen}\alpha\,.\cos\alpha} = \frac{\cos\alpha}{\text{sen}\alpha}$$

$\therefore\ L = \cot\alpha$

Clave C

2. Resolución:

$R = (\csc\alpha + \cot\alpha)(\sec\alpha - 1)$

$R = (\csc\alpha\,.\sec\alpha - \csc\alpha + \csc\alpha - \cot\alpha)$

$R = \tan\alpha + \cot\alpha - \cot\alpha$

$\therefore\ R = \tan\alpha$

Clave A

3.

$$(\sec\alpha - \cos\alpha)^2 = (3)^2$$

$$\sec^2\alpha - 2\,.\sec\alpha\,.\cos\alpha + \cos^2\alpha = 9$$

$$\sec^2\alpha - 2\,.\frac{1}{\cos\alpha}\,.\cos\alpha + \cos^2\alpha = 9$$

$$\sec^2\alpha + \cos^2\alpha - 2 = 9$$

$$M = \sqrt{\sec^2\alpha + \cos^2\alpha - 2} = \sqrt{9}$$

$M = 3$

Clave B

4. Factorizando:

$$\frac{\text{sen}\alpha(1-\text{sen}^2\alpha)}{\cos\alpha(1-\cos^2\alpha)} = \cot\alpha$$

$$\frac{\text{sen}\alpha\ .\ \cos^2\alpha}{\cos\alpha\ .\ \text{sen}^2\alpha} = \cot\alpha$$

Simplificando: $\dfrac{\cos\alpha}{\text{sen}\alpha} = \cot\alpha$

5. $$A = \frac{(\text{sen}x + \cos x)^2 - 1}{2\text{sen}x}$$

$$A = \frac{\text{sen}^2x + 2\text{sen}x\cos x + \cos^2x - 1}{2\text{sen}x}$$

$$A = \frac{\overbrace{\text{sen}^2x + \cos^2x}^{1} + 2\text{sen}x\cos x - 1}{2\text{sen}x}$$

$$A = \frac{2\text{sen}x\cos x}{2\text{sen}x}$$

$\therefore\ A = \cos x$

Clave A

6. Piden:

$R = \csc x - \text{sen}x$

Del dato:

$\text{sen}x + \text{sen}^2x = 1$

Multiplicamos por $(\csc x)$:

$(\csc x)\text{sen}x + (\csc x)\text{sen}^2x = (\csc x)\,.\,1$

$1 + \text{sen}x = \csc x$

$\Rightarrow \csc x - \text{sen}x = 1$

$\therefore\ R = 1$

Clave A

7. $$H = \frac{(1+\text{sen}x+\cos x)(1-\text{sen}x-\cos x)}{\text{sen}x\cos x}$$

$$H = \frac{[1+(\text{sen}x+\cos x)][1-(\text{sen}x+\cos x)]}{\text{sen}x\cos x}$$

$$H = \frac{1^2 - (\text{sen}x+\cos x)^2}{\text{sen}x\cos x}$$

$$H = \frac{1 - (\overbrace{\text{sen}^2x + \cos^2x}^{1} + 2\text{sen}x\cos x)}{\text{sen}x\cos x}$$

$$H = \frac{1-1-2\text{sen}x\cos x}{\text{sen}x\cos x} = \frac{-2\text{sen}x\cos x}{\text{sen}x\cos x}$$

$\therefore\ H = -2$

Clave C

8. $$N = \frac{\text{sen}x}{1+\cos x} + \frac{1+\cos x}{\text{sen}x}$$

$$N = \frac{\text{sen}^2x + (1+\cos x)^2}{(1+\cos x)\text{sen}x}$$

$$N = \frac{(\text{sen}^2x+\cos^2x) + 2\cos x + 1}{(1+\cos x)\text{sen}x}$$

$$N = \frac{1+1+2\cos x}{(1+\cos x)\text{sen}x} = \frac{2(1+\cos x)}{(1+\cos x)\text{sen}x}$$

$$N = \frac{2}{\text{sen}x} = 2\csc x$$

$\therefore\ N = 2\csc x$

Clave D

9. $$A = \frac{(\tan\theta + \cot\theta)^3}{\csc^3\theta}$$

Por identidad auxiliar:

$\tan\theta + \cot\theta = \sec\theta\csc\theta$

Entonces:

$$A = \frac{(\sec\theta\csc\theta)^3}{\csc^3\theta} = \frac{\sec^3\theta\csc^3\theta}{\csc^3\theta}$$

$\therefore\ A = \sec^3\theta$

Clave A

10. Por dato:

$\tan x - \cot x = \dfrac{3}{2}$

Elevando al cuadrado:

$$\tan^2x - \underbrace{2\tan x\cot x}_{1} + \cot^2x = \frac{9}{4}$$

$$\tan^2x + \cot^2x = \frac{17}{4}$$

$$\tan^2x + \cot^2x + 2 = \frac{17}{4} + 2$$

$$\tan^2x + 2\tan x\cot x + \cot^2x = \frac{25}{4}$$

$$(\tan x + \cot x)^2 = \frac{25}{4}$$

$\Rightarrow \tan x + \cot x = \dfrac{5}{2} \vee \tan x + \cot x = -\dfrac{5}{2}$

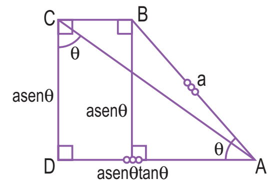
Piden:

$M = \tan x + \cot x + 0{,}5$

$M = \dfrac{5}{2} + 0{,}5 = 3 \vee M = -\dfrac{5}{2} + 0{,}5 = -2$

Clave C

11.



Por dato:

$AB = AD$

$a = a\text{sen}\theta\tan\theta$

$\Rightarrow \text{sen}\theta\,.\tan\theta = 1$

Piden:

$$P = \sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta$$

$$P = \frac{1-\cos^2\theta}{\cos\theta} = \frac{\text{sen}^2\theta}{\cos\theta} = \text{sen}\theta\frac{\text{sen}\theta}{\cos\theta}$$

$P = \text{sen}\theta\,.\tan\theta = 1$ (del dato)

$\therefore\ P = 1$

Clave C

12. $$E = \frac{1+\tan x+\sec x}{1+\cot x+\csc x}$$

$$E = \frac{\frac{\cos x}{\cos x} + \frac{\text{sen}x}{\cos x} + \frac{1}{\cos x}}{\frac{\text{sen}x}{\text{sen}x} + \frac{\cos x}{\text{sen}x} + \frac{1}{\text{sen}x}}$$

$$E = \frac{\frac{\cos x + \text{sen}x + 1}{\cos x}}{\frac{\text{sen}x + \cos x + 1}{\text{sen}x}}$$

$$E = \frac{\text{sen}x(1+\text{sen}x+\cos x)}{\cos x(1+\text{sen}x+\cos x)}$$

$$E = \frac{\text{sen}x}{\cos x} = \tan x$$

$\therefore\ E = \tan x$

Clave D

13. $$B = \frac{(1+\sec x)(1+\csc x)(1-\cos x)(1-\text{sen}x)}{1+\text{sen}x(1-\csc x)}$$

$$B = \frac{\left(1+\frac{1}{\cos x}\right)\left(1+\frac{1}{\text{sen}x}\right)(1-\cos x)(1-\text{sen}x)}{1+\text{sen}x-\text{sen}x\csc x}$$

$$B = \frac{\left(\frac{1+\cos x}{\cos x}\right)\left(\frac{1+\text{sen}x}{\text{sen}x}\right)(1-\cos x)(1-\text{sen}x)}{1+\text{sen}x-1}$$

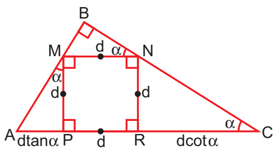
$$B = \frac{(1+\cos x)(1-\cos x)(1+\text{sen}x)(1-\text{sen}x)}{\cos x\,\text{sen}^2x}$$

$$B = \frac{(1-\cos^2x)(1-\text{sen}^2x)}{\cos x\,\text{sen}^2x} = \frac{(\text{sen}^2x)(\cos^2x)}{\cos x\,\text{sen}^2x}$$

$\therefore\ B = \cos x$

Clave B

14.



Por dato:
$AC = 4MN$
$d\tan\alpha + d + d\cot\alpha = 4 \,.\, d$
$d(\tan\alpha + \cot\alpha + 1) = 4d$
$\tan\alpha + \cot\alpha = 3$

Piden:
$K = \sec^2\alpha + \csc^2\alpha + 1$
$K = \sec^2\alpha \,.\, \csc^2\alpha + 1$
$K = (\sec\alpha\csc\alpha)^2 + 1$
$K = (\tan\alpha + \cot\alpha)^2 + 1$
$K = (3)^2 + 1 = 9 + 1 = 10$
$\therefore\ K = 10$

Clave C

PRACTIQUEMOS

Nivel 1 (página 60) Unidad 3

Comunicación matemática

1. Por teoría tenemos:

- Por teoría tenemos:
- $\cos x \,.\, \sec x = 1$ $\Rightarrow$ I. recíprocas
- $\cot x \,.\, \text{sen}\,x = \cos x$ $\Rightarrow$ I. por división
- $\text{sen}^2x = 1 - \cos^2x$ $\Rightarrow$ I. pitagórica
- $\sec^2x + \csc^2x = \sec^2x \,.\, \csc^2x$ $\Rightarrow$ I. auxiliares
- $\cot^2x = \csc^2x - 1$ $\Rightarrow$ I. pitagóricas

$\therefore$ Dos son pitagóricas

Clave B

2.

Razonamiento y demostración

3. $M = \dfrac{\text{sen}^2x + \cos^2x}{\sec^2x - \tan^2x}$

Por identidades pitagóricas:
$\text{sen}^2x + \cos^2x = 1 \ \wedge\ \sec^2x - \tan^2x = 1$
$\Rightarrow M = \dfrac{1}{1} = 1$
$\therefore\ M = 1$

Clave E

4. $S = \dfrac{\tan\alpha + \cot\alpha}{\sec\alpha\csc\alpha}$

$$S = \frac{\dfrac{\text{sen}\alpha}{\cos\alpha} + \dfrac{\cos\alpha}{\text{sen}\alpha}}{\sec\alpha\csc\alpha} = \frac{\dfrac{\text{sen}^2\alpha + \cos^2\alpha}{\cos\alpha\,\text{sen}\alpha}}{\sec\alpha\csc\alpha}$$

$$S = \frac{\left(\dfrac{1}{\cos\alpha\,\text{sen}\alpha}\right)}{\sec\alpha\csc\alpha} = \frac{(\sec\alpha\csc\alpha)}{\sec\alpha\csc\alpha} = 1$$

$\therefore\ S = 1$

Clave A

5. $S = \cot\alpha \,.\, \dfrac{\text{sen}\alpha}{\cos\alpha} + \tan\alpha \,.\, \dfrac{\cos\alpha}{\text{sen}\alpha}$

$$S = \left(\frac{\cos\alpha}{\text{sen}\alpha}\right).\frac{\text{sen}\alpha}{\cos\alpha} + \left(\frac{\text{sen}\alpha}{\cos\alpha}\right).\frac{\cos\alpha}{\text{sen}\alpha}$$

$\Rightarrow S = 1 + 1 = 2$
$\therefore\ S = 2$

Clave A

6. Por dato:
$\tan\alpha + \cot\alpha = \sqrt{6}$

Elevando al cuadrado:
$$\tan^2\alpha + \underbrace{2\tan\alpha\cot\alpha}_{1} + \cot^2\alpha = \left(\sqrt{6}\right)^2$$

$\Rightarrow \tan^2\alpha + 2 + \cot^2\alpha = 6$
$\Rightarrow \tan^2\alpha + \cot^2\alpha = 4$

Piden:
$R = \sqrt{\tan^2\alpha + \cot^2\alpha + 5}$
$\Rightarrow\ R = \sqrt{(4) + 5} = \sqrt{9}$
$\therefore\ R = 3$

Clave A

7. Piden:
$A = \csc^2\theta + \text{sen}^2\theta$

Por dato:
$\csc\theta - \text{sen}\theta = 2$

Elevando al cuadrado:
$$\csc^2\theta - \underbrace{2\csc\theta\,\text{sen}\theta}_{1} + \text{sen}^2\theta = 2^2$$

$\Rightarrow \csc^2\theta - 2 + \text{sen}^2\theta = 4$
$\Rightarrow \csc^2\theta + \text{sen}^2\theta = 6$
$\therefore\ A = 6$

Clave C

8. $E = \dfrac{(1 + \cos\alpha)(1 - \cos\alpha)}{\text{sen}^2\alpha}$

$E = \dfrac{1 - \cos^2\alpha}{\text{sen}^2\alpha} = \dfrac{\text{sen}^2\alpha}{\text{sen}^2\alpha} = 1$

$\therefore\ E = 1$

Clave C

9. Por dato:
$\cot\theta - \csc\theta = \sqrt{9}$
$\Rightarrow \cot\theta - \csc\theta = 3$
$\Rightarrow \csc\theta - \cot\theta = -3$

Piden:
$S = 9(\cot\theta + \csc\theta)$

Por identidad pitagórica:
$\csc^2\theta - \cot^2\theta = 1$
$(\csc\theta - \cot\theta)(\csc\theta + \cot\theta) = 1$
$(-3)(\csc\theta + \cot\theta) = 1$
$\Rightarrow \csc\theta + \cot\theta = -\dfrac{1}{3}$

Reemplazando en la expresión S:
$S = 9\left(-\dfrac{1}{3}\right) = -3$
$\therefore\ S = -3$

Clave B

10.

$S = \dfrac{1 + \tan^3x}{1 + \tan x} + \tan x$

Sea: $A = \dfrac{1 + \tan^3x}{1 + \tan x}$

$A = \dfrac{(1 + \tan x)(1^2 - 1.\tan x + \tan^2x)}{1 + \tan x}$

$A = 1 - \tan x + \tan^2x$
$\Rightarrow A = (1 + \tan^2x) - \tan x = (\sec^2x) - \tan x$
$\Rightarrow A = \sec^2x - \tan x$

Luego:
$S = A + \tan x$
$\Rightarrow S = (\sec^2x - \tan x) + \tan x$
$\therefore S = \sec^2x$

Clave A

Resolución de problemas

11. Se tiene:

$$\text{sen}\theta\ ;\ \tan\theta\ ;\ \sec\theta$$
$\times r$ $\quad \times r$

$\text{sen}\theta \times r = \tan\theta$
$\sec\theta \times r = \dfrac{\text{sen}\theta}{\cos\theta}$
$r = \sec\theta$ ... (I)
$\tan\theta \times r = \sec\theta$
$\tan\theta \times \sec\theta = \sec\theta$
$\tan\theta = 1$
$\therefore\ \theta = 45° = \pi/4$

Clave C

12. • Simplificamos:

$$K = \sqrt{\frac{1+\text{sen}x}{1-\text{sen}x}} + \sqrt{\frac{1-\text{sen}x}{1+\text{sen}x}} + \sqrt{\frac{1-\cos x}{1+\cos x}} + \sqrt{\frac{1+\cos x}{1-\cos x}}$$

$$\sqrt{\frac{1+\text{sen}x}{1-\text{sen}x}} = \sqrt{\frac{1+\text{sen}x}{1-\text{sen}x} \times \frac{1+\text{sen}x}{1+\text{sen}x}} = \sqrt{\frac{(1+\text{sen}x)^2}{1-\text{sen}^2x}}$$

$$= \sqrt{\frac{(1+\text{sen}x)^2}{\cos^2x}} = \frac{1+\text{sen}x}{|\cos x|}$$

$$\sqrt{\frac{1-\text{sen}x}{1+\text{sen}x} \times \frac{1-\text{sen}x}{1-\text{sen}x}} = \sqrt{\frac{(1-\text{sen}x)^2}{1-\text{sen}^2x}} = \frac{1-\text{sen}x}{|\cos x|}$$

$$\sqrt{\frac{1-\cos x}{1+\cos x} \times \frac{1-\cos x}{1-\cos x}} = \sqrt{\frac{(1-\cos x)^2}{1-\cos^2x}} = \frac{1-\cos x}{|\text{sen}x|}$$

$$\sqrt{\frac{1+\cos x}{1-\cos x} \times \frac{1+\cos x}{1+\cos x}} = \sqrt{\frac{(1+\cos x)^2}{1-\cos^2x}} = \frac{1+\cos x}{|\text{sen}x|}$$

• Luego tenemos:

$$K = \frac{1+\text{sen}x}{|\cos x|} + \frac{1-\text{sen}x}{|\cos x|} + \frac{1-\cos x}{|\text{sen}x|} + \frac{1+\cos x}{|\text{sen}x|}$$

$$K = \frac{2}{|\cos x|} + \frac{2}{|\text{sen}x|} = 2(|\csc x| + |\sec x|)$$

• Evaluamos el resultado del alumno y el nuestro:

$-2(\csc x + \sec x) = 2(|\csc x| + |\sec x|)$
$|\csc x| = -\csc x$
$|\sec x| = -\sec x$ $\Rightarrow\ x \in \text{IIIC}$

$\therefore\ \left\langle 10\dfrac{\pi}{9}; 4\dfrac{\pi}{3}\right\rangle \in C$

Clave D

Nivel 2 (página 60) Unidad 3

Comunicación matemática

13. I. $\text{sen}^4x + \cos^4x = \quad = 1 - 2\text{sen}^2x \,.\, \cos^2x$ (V)

II. $(\csc x - \cot x)(\csc x + \cot x) = \csc^2x - \cot^2x = 1$ (V)

III. $\dfrac{\cot x.\text{sen}x}{\cos x} = 1 \Rightarrow \cot x = \dfrac{\cos x}{\text{sen}x}$ (V)

IV. $(1 - \text{sen}x - \cos x)^2 = 2(1 - \text{sen}x)(1 - \cos x)$
$\neq 2(1 + \text{sen}x) \,.\, (1 - \cos x)$ (F)

$\therefore$ VVVF

Clave B

14. Por identidades auxiliares, tenemos:

$\text{asenx} + \text{bcosx} = c \ \wedge \ c = \sqrt{a^2+b^2}; \ \ x \in \mathbb{R}$

$\Rightarrow \ \text{senx} = \frac{a}{c} \ \wedge \ \text{cosx} = b/c$

$\therefore$ I y II son necesarios.

Clave E

Razonamiento y demostración

15.

$$E = \frac{\csc^2\theta - \cot^2\theta}{\text{sen}^2\theta + \cos^2\theta}$$

Por identidades pitagóricas:

$\csc^2\theta - \cot^2\theta = 1 \wedge \text{sen}^2\theta + \cos^2\theta = 1$

$\Rightarrow E = \frac{1}{1} = 1$

$\therefore E = 1$

Clave B

16. $A = \left[\dfrac{\sec^2\theta + \csc^2\theta}{\sec^2\theta.\csc^2\theta}\right]^3$

Luego:

$$\sec^2\theta + \csc^2\theta = \frac{1}{\cos^2\theta} + \frac{1}{\text{sen}^2\theta}$$

$$\sec^2\theta + \csc^2\theta = \frac{\text{sen}^2\theta + \cos^2\theta}{\cos^2\theta\text{sen}^2\theta}$$

$$\sec^2\theta + \csc^2\theta = \frac{1}{\cos^2\theta\text{sen}^2\theta}$$

$$\sec^2\theta + \csc^2\theta = \sec^2\theta \ . \ \csc^2\theta$$

$$\Rightarrow \ A = \left[\frac{\sec^2\theta.\csc^2\theta}{\sec^2\theta.\csc^2\theta}\right]^3 = (1)^3$$

$\therefore \ A = 1$

Clave D

17. $N = \dfrac{\sec^2x - 1}{\tan^2x} + \dfrac{\csc^2x - 1}{\cot^2x}$

Por identidades pitagóricas:

$\tan^2x = \sec^2x - 1 \ \wedge \ \cot^2x = \csc^2x - 1$

$$\Rightarrow \ N = \frac{(\tan^2x)}{\tan^2x} + \frac{(\cot^2x)}{\cot^2x}$$

$\Rightarrow N = 1 + 1 = 2$

$\therefore N = 2$

Clave D

18. $M = \dfrac{\text{sen}^8x - \cos^8x}{\text{sen}^2x - \cos^2x} - \text{sen}^4x$

Sea:

$$H = \frac{\text{sen}^8x - \cos^8x}{\text{sen}^2x - \cos^2x}$$

$$H = \frac{(\text{sen}^4x + \cos^4x)(\text{sen}^4x - \cos^4x)}{\text{sen}^2x - \cos^2x}$$

$$H = \frac{(\text{sen}^4x+\cos^4x)(\text{sen}^2x+\cos^2x)(\text{sen}^2x-\cos^2x)}{\text{sen}^2x-\cos^2x}$$

$H = (\text{sen}^4x + \cos^4x)(1)$

$\Rightarrow H = \text{sen}^4x + \cos^4x$

Luego:

$M = H - \text{sen}^4x$

$\Rightarrow M = (\text{sen}^4x + \cos^4x) - \text{sen}^4x$

$\therefore \ M = \cos^4x$

Clave B

19.

Por dato:

$\text{sen}\theta + \csc\theta = 4$

Elevando al cuadrado:

$\text{sen}^2\theta + \underbrace{2\text{sen}\theta\csc\theta}_{1} + \csc^2\theta = (4)^2$

$\Rightarrow \text{sen}^2\theta + 2 + \csc^2\theta = 16$

$\Rightarrow \text{sen}^2\theta + \csc^2\theta = 14$

Piden:

$S = \sqrt[3]{\text{sen}^2\theta + \csc^2\theta + 13}$

$\Rightarrow \ S = \sqrt[3]{(14) + 13} = \sqrt[3]{27}$

$\therefore \ S = 3$

Clave C

20. $L = \dfrac{\text{senx}}{1 - \cos x} - \csc x$

$$L = \frac{\text{senx}}{1 - \cos x} - \frac{1}{\text{senx}} = \frac{\text{sen}^2x - (1 - \cos x)}{(1 - \cos x)\text{senx}}$$

$$L = \frac{(1 - \cos^2x) - 1 + \cos x}{(1 - \cos x)\text{senx}} = \frac{\cos x - \cos^2x}{(1 - \cos x)\text{senx}}$$

$$L = \frac{\cos x(1 - \cos x)}{(1 - \cos x)\text{senx}} = \frac{\cos x}{\text{senx}} = \cot x$$

$\therefore \ L = \cot x$

Clave A

21. Piden:

$S = \tan^4\theta + \cot^4\theta$

Por dato:

$\tan\theta - \cot\theta = 5$

Elevando al cuadrado:

$\tan^2\theta - \underbrace{2\tan\theta\cot\theta}_{1} + \cot^2\theta = (5)^2$

$\Rightarrow \tan^2\theta - 2 + \cot^2\theta = 25$

$\Rightarrow \tan^2\theta + \cot^2\theta = 27$

Elevando nuevamente al cuadrado:

$(\tan^2\theta)^2 + 2\tan^2\theta\cot^2\theta + (\cot^2\theta)^2 = (27)^2$

$\tan^4\theta + 2(\tan\theta\cot\theta)^2 + \cot^4\theta = 729$

$\Rightarrow \tan^4\theta + 2(1)^2 + \cot^4\theta = 729$

$\Rightarrow \tan^4\theta + \cot^4\theta = 727$

$\therefore \ S = 727$

Clave D

22. $M = \dfrac{\text{sen}^3x + \cos^3x}{1 - \text{senx}\cos x} - \cos x$

Sea: $B = \dfrac{\text{sen}^3x + \cos^3x}{1 - \text{senx}\cos x}$

$$B = \frac{(\text{senx} + \cos x)(\text{sen}^2x - \text{senx}\cos x + \cos^2x)}{1 - \text{senx}\cos x}$$

$$B = \frac{(\text{senx} + \cos x)(\text{sen}^2x + \cos^2x - \text{senx}\cos x)}{1 - \text{senx}\cos x}$$

$$B = \frac{(\text{senx} + \cos x)(1 - \text{senx}\cos x)}{1 - \text{senx}\cos x}$$

$\Rightarrow B = \text{senx} + \text{cosx}$

Luego:

$M = B - \text{cosx}$

$\Rightarrow M = (\text{senx} + \text{cosx}) - \text{cosx}$

$\therefore \ M = \text{senx}$

Clave B

23. Por dato: $(\sec\theta\tan\theta)^{-1} = \dfrac{1}{5}$

$\Rightarrow \ \dfrac{1}{\sec\theta\tan\theta} = \dfrac{1}{5} \Rightarrow \sec\theta\tan\theta = 5$

Piden:

$M = \sqrt{\sec^4\theta + \tan^4\theta - 2}$

Por identidad pitagórica:

$\sec^2\theta - \tan^2\theta = 1$

Elevando al cuadrado:

$(\sec^2\theta)^2 - 2\sec^2\theta\tan^2\theta + (\tan^2\theta)^2 = 12$

$\sec^4\theta - 2(\sec\theta\tan\theta)^2 + \tan^4\theta = 1$

$\Rightarrow \sec^4\theta - 2(5)^2 + \tan^4\theta = 1$

$\Rightarrow \sec^4\theta + \tan^4\theta = 51$

Reemplazando en la expresión M:

$M = \sqrt{(51) - 2} = \sqrt{49}$

$\therefore \ M = 7$

Clave A

Resolución de problemas

24. Simplificamos:

$$K = \frac{(1 + \csc^2\theta)\text{sen}\theta}{\sec\theta \ . \ \csc\theta - \text{sen}^2\theta \ . \ \tan\theta}$$

$$K = \frac{\dfrac{\text{sen}^2\theta + 1}{\text{sen}^2\theta} . (\text{sen}\theta)}{\dfrac{1}{\cos\theta} . \dfrac{1}{\text{sen}\theta} - \text{sen}^2\theta \times \dfrac{\text{sen}\theta}{\cos\theta}}$$

$$K = \frac{\dfrac{\text{sen}^2\theta + 1}{\cancel{\text{sen}\theta}}}{\dfrac{1 - \text{sen}^4\theta}{\cancel{\text{sen}\theta} \ . \ \cos\theta}} = \frac{(\text{sen}^2\theta + 1)\cos\theta}{(1 - \text{sen}^4\theta)}$$

$$K = \frac{\cancel{(\text{sen}^2\theta + 1)}(\cos\theta)}{\cancel{(1 + \text{sen}^2\theta)}(1 - \text{sen}^2\theta)}$$

$$K = \frac{\cos\theta}{1 - \text{sen}^2\theta} = \frac{\cos\theta}{\cos^2\theta} = \frac{1}{\cos\theta}$$

Sabemos:

$\frac{\pi}{6} \leq \ \theta \ \leq \frac{\pi}{3}$

$\frac{1}{2} \leq \cos\theta \ \leq \frac{\sqrt{3}}{2}$

$\frac{2}{\sqrt{3}} \leq \frac{1}{\cos\theta} \leq 2$

$\therefore \ k_{máx.} = 2$

Clave D

25. Simplificamos:

$$P=\frac{\csc^2\beta-\cos^4\beta\csc^2\beta}{\cot\beta.\sec\beta+\cos\beta\cot\beta}$$

$$P=\frac{\csc^2\beta(1-\cos^4\beta)}{\cot\beta(\sec\beta+\cos\beta)}$$

$$P=\frac{\cancel{(1+\cos^2\beta)}(1-\cos^2\beta)}{\frac{\cos\beta}{\cancel{\text{sen}\beta}}\times\text{sen}^2\beta\left(\frac{\cancel{1+\cos^2\beta}}{\cos\beta}\right)}$$

$$P=\frac{(1-\cos^2\beta)}{\cos\beta\times\text{sen}\beta\left(\frac{1}{\cos\beta}\right)}$$

$$P=\frac{(1-\cos^2\beta)}{\cancel{\cos\beta}\times\text{sen}\beta\left(\frac{1}{\cancel{\cos\beta}}\right)}$$

$$P=\frac{\text{sen}^2\beta}{\text{sen}\beta}=\text{sen}\beta$$

- Sabemos

$$\frac{\pi}{6}\le\beta\le 2\pi/3$$

$$\frac{1}{2}\le\text{sen}\beta\le 1\Rightarrow\frac{1}{2}\le P\le 1$$

$\therefore P_{mín}=1/2$

Clave C

Nivel 3 (página 61) Unidad 3

Comunicación matemática

26. En M:

$\text{sen}^2x\,.\,\cos x+\cos^3x=k\cos x$

$\cos x\,.\,(\text{sen}^2x+\cos^2x)=k\cos x$

$\dfrac{\cancel{\cos x}(1)}{\cancel{\cos x}}=k\Rightarrow k=1$

$\therefore M=k=1$

- Notamos que "k" no depende del ángulo.

$\Rightarrow M=N=k=1$

$\therefore \sqrt{2(M+N)}=2$

Clave E

27. En la sucesión:

$\text{sen}^2\theta\ ;\ 1\ ;\ 2\ ;\ 3+\text{sen}^2\theta;\ 4+3\text{sen}^2\theta$

$\cos^2\theta\quad 1\quad 1+\text{sen}^2\theta\quad 1+2\text{sen}^2\theta$

$\text{sen}^2\theta\quad \text{sen}^2\theta\quad \text{sen}^2\theta$

$\Rightarrow t_5=4+3\text{sen}^2\theta$

$A=t_5+\cos^2\theta-2\text{sen}^2\theta$

$A=4+3\text{sen}^2\theta+\cos^2\theta-2\text{sen}^2\theta$

$A=5$

Clave E

28. $E=\dfrac{1-\cos^2\alpha}{\text{sen}^2\alpha}+\dfrac{1-\text{sen}^2\alpha}{\cos^2\alpha}$

Por identidades pitagóricas:

$\text{sen}^2\alpha=1-\cos^2\alpha\wedge\cos^2\alpha=1-\text{sen}^2\alpha$

$\Rightarrow E=\dfrac{(\text{sen}^2\alpha)}{\text{sen}^2\alpha}+\dfrac{(\cos^2\alpha)}{\cos^2\alpha}$

$\Rightarrow E=1+1=2$

$\therefore E=2$

Clave E

29. $M=\left[\dfrac{\text{sen}^4x-\cos^4x}{\text{sen}^2x-\cos^2x}\right]^4$

$M=\left[\dfrac{(\text{sen}^2x+\cos^2x)(\text{sen}^2x-\cos^2x)}{\text{sen}^2x-\cos^2x}\right]^4$

$M=(\text{sen}^2x+\cos^2x)^4=(1)^4$

$\therefore M=1$

Clave B

30. $L=\dfrac{\text{sen}^6x+\cos^6x}{1-3\text{sen}^2x\cos^2x}+\dfrac{1-2\text{sen}^2x\cos^2x}{\text{sen}^4x+\cos^4x}$

Por identidad pitagórica:

$\text{sen}^2x+\cos^2x=1\quad ...(I)$

Elevando (I) al cuadrado:

$\text{sen}^4x+2\text{sen}^2x\cos^2x+\cos^4x=1^2$

$\Rightarrow\text{sen}^4x+\cos^4x=1-2\text{sen}^2x\cos^2x$

Elevando (I) al cubo:

$(\text{sen}^2x)^3+(\cos^2x)^3+3\text{sen}^2x\cos^2x(\text{sen}^2x+\cos^2x)=1^3$

$\text{sen}^6x+\cos^6x+3\text{sen}^2x\cos^2x(1)=1$

$\Rightarrow\text{sen}^6x+\cos^6x=1-3\text{sen}^2x\cos^2x$

Reemplazando en la expresión L:

$L=\dfrac{(1-3\text{sen}^2x\cos^2x)}{1-3\text{sen}^2x\cos^2x}+\dfrac{1-2\text{sen}^2x\cos^2x}{(1-2\text{sen}^2x\cos^2x)}$

$\Rightarrow L=1+1=2$

$\therefore L=2$

Clave B

31. $A=\dfrac{\text{sen}^3\alpha-\cos^3\alpha}{1+\text{sen}\alpha\cos\alpha}-\text{sen}\alpha$

Sea: $E=\dfrac{\text{sen}^3\alpha-\cos^3\alpha}{1+\text{sen}\alpha\cos\alpha}$

$E=\dfrac{(\text{sen}\alpha-\cos\alpha)(\text{sen}^2\alpha+\text{sen}\alpha\cos\alpha+\cos^2\alpha)}{1+\text{sen}\alpha\cos\alpha}$

$E=\dfrac{(\text{sen}\alpha-\cos\alpha)(\text{sen}^2\alpha+\cos^2\alpha+\text{sen}\alpha\cos\alpha)}{1+\text{sen}\alpha\cos\alpha}$

$E=\dfrac{(\text{sen}\alpha-\cos\alpha)(1+\text{sen}\alpha\cos\alpha)}{1+\text{sen}\alpha\cos\alpha}$

$\Rightarrow E=\text{sen}\alpha-\cos\alpha$

Luego:

$A=E-\text{sen}\alpha$

$\Rightarrow A=(\text{sen}\alpha-\cos\alpha)-\text{sen}\alpha$

$\therefore A=-\cos\alpha$

Clave D

32. $A=\dfrac{(\csc\alpha+1)(\csc\alpha-1)}{\cot^2\alpha}+\dfrac{(\sec\alpha+1)(\sec\alpha-1)}{\tan^2\alpha}$

$A=\dfrac{\csc^2\alpha-1}{\cot^2}+\dfrac{\sec^2\alpha-1}{\tan^2\alpha}$

$A=\dfrac{(\cot^2\alpha)}{\cot^2\alpha}+\dfrac{(\tan^2\alpha)}{\tan^2\alpha}$

$\Rightarrow A=1+1=2$

$\therefore A=2$

Clave C

33. Por dato: $\text{sen}\alpha+\cos\alpha=\dfrac{2}{3}$

Elevando al cuadrado:

$\text{sen}^2\alpha+2\text{sen}\alpha\cos\alpha+\cos^2\alpha=\left(\dfrac{2}{3}\right)^2$

$(\text{sen}^2\alpha+\cos^2\alpha)+2\text{sen}\alpha\cos\alpha=\dfrac{4}{9}$

$(1)+2\text{sen}\alpha\cos\alpha=\dfrac{4}{9}$

$2\text{sen}\alpha\cos\alpha=-\dfrac{5}{9}$

$\Rightarrow\text{sen}\alpha\cos\alpha=-\dfrac{5}{18}$

Piden:

$M=\sqrt{162(\text{sen}^4\alpha+\cos^4\alpha)+7}$

Por identidad auxiliar:

$\text{sen}^4\alpha+\cos^4\alpha=1-2\text{sen}^2\alpha\cos^2\alpha$

$\text{sen}^4\alpha+\cos^4\alpha=1-2(\text{sen}\alpha\cos\alpha)^2$

$\text{sen}^4\alpha+\cos^4\alpha=1-2\left(-\dfrac{5}{18}\right)^2$

$\Rightarrow\text{sen}^4\alpha+\cos^4\alpha=\dfrac{137}{162}$

Reemplazando en la expresión M:

$M=\sqrt{162\left(\dfrac{137}{162}\right)+7}=\sqrt{144}$

$\therefore M=12$

Clave E

34. $T=\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}}-\csc\theta$

* Considerar: $\theta\in IC$

$T=\sqrt{\dfrac{(1+\cos\theta).(1+\cos\theta)}{(1-\cos\theta).(1+\cos\theta)}}-\csc\theta$

$T=\sqrt{\dfrac{(1+\cos\theta)^2}{1-\cos^2\theta}}-\csc\theta$

$T=\dfrac{|1+\cos\theta|}{\sqrt{\text{sen}^2\theta}}-\csc\theta=\dfrac{|1+\cos\theta|}{|\text{sen}\theta|}-\csc\theta$

Como $\theta\in IC\Rightarrow\text{sen}\theta>0\ \wedge\ \cos\theta>0$

$\Rightarrow T=\dfrac{1+\cos\theta}{\text{sen}\theta}-\dfrac{1}{\text{sen}\theta}=\dfrac{1+\cos\theta-1}{\text{sen}\theta}$

$\Rightarrow T=\dfrac{\cos\theta}{\text{sen}\theta}=\cot\theta$

$\therefore T=\cot\theta$

Clave E

35. Por dato: $\cos x+\sec x=3$

Elevando al cuadrado:

$\cos^2x+\underbrace{2\cos x\sec x}_{1}+\sec^2x=3^2$

$\Rightarrow\cos^2x+2+\sec^2x=9$

$\Rightarrow\cos^2x+\sec^2x=7$

Piden:

$T=\sqrt[4]{\cos^3x+\sec^3x-2}$

Luego:

$\cos^3x+\sec^3x=(\cos x+\sec x)(\cos^2x-\underbrace{\cos x\sec x}_{1}+\sec^2x)$

$\cos^3x + \sec^3x = (\cos x + \sec x)(\cos^2x + \sec^2x - 1)$

$\cos^3x + \sec^3x = (3)(7 - 1) = 18$

$\Rightarrow \cos^3x + \sec^3x = 18$

Reemplazando en la expresión T:

$$T = \sqrt[4]{(18) - 2} = \sqrt[4]{16} = 2$$

$\therefore\ T = 2$

Clave D

Resolución de problemas

36. En A:

$$A = \sqrt{\tan\alpha + \cot\alpha}$$

$$A = \sqrt{\frac{\operatorname{sen}\alpha}{\cos\alpha} + \frac{\cos\alpha}{\operatorname{sen}\alpha}}$$

$$A = \sqrt{\frac{\operatorname{sen}^2\alpha + \cos^2\alpha}{\operatorname{sen}\alpha \cdot \cos\alpha}} = \sqrt{\frac{1}{\operatorname{sen}\alpha \cdot \cos\alpha}}$$

$$A = \sqrt{\sec\alpha \cdot \csc\alpha}$$

$(+)\quad(+)$

$\vee$

$(-)\quad(-)$

De B, tenemos:

$B = \sqrt{\operatorname{sen}\alpha} \Rightarrow \operatorname{sen}\alpha > 0$

$\Rightarrow$ $0 < \alpha < \pi \qquad \therefore\ \alpha \in \text{IC}$

De A: $0 < \alpha < \frac{\pi}{2}$

Clave A

37. En (1):

$$\cos\beta(\csc\beta - \operatorname{sen}\beta) = M$$

$$\cos\beta\left(\frac{1}{\operatorname{sen}\beta} - \operatorname{sen}\beta\right) = M$$

$$\cos\beta\left(\frac{1 - \operatorname{sen}^2\beta}{\operatorname{sen}\beta}\right) = M$$

$$\frac{\cos\beta\,(\cos^2\beta)}{\operatorname{sen}\beta} = \frac{\cos^3\beta}{\operatorname{sen}\beta} = M \quad \ldots (3)$$

En (2):

$$\operatorname{sen}\beta\left(\frac{1}{\cos\beta} - \cos\beta\right) = N$$

$$\operatorname{sen}\beta\left(\frac{1 - \cos^2\beta}{\cos\beta}\right) = N$$

$$\operatorname{sen}\beta\left(\frac{\operatorname{sen}^2\beta}{\cos\beta}\right) = \frac{\operatorname{sen}^3\beta}{\cos\beta} = N \quad \ldots (4)$$

Dividimos (3) ÷ (4):

$$\frac{\dfrac{\cos^3\beta}{\operatorname{sen}\beta}}{\dfrac{\operatorname{sen}^3\beta}{\cos\beta}} = \frac{\cos^4\beta}{\operatorname{sen}^4\beta} = \frac{M}{N} = K$$

$\Rightarrow \cos^4\beta = MK \ \wedge\ \operatorname{sen}^4\beta = NK$

$(3) \times (4)$:

$$\frac{\cos^3\beta}{\operatorname{sen}\beta} \times \frac{\operatorname{sen}^3\beta}{\cos\beta} = M.N = \cos^2\beta\operatorname{sen}^2\beta$$

Sabemos:

$\operatorname{sen}^4\beta + \cos^4\beta = 1 - 2\operatorname{sen}^2\beta\cos^2\beta$

$NK + MK = 1 - 2\,MN$

$\therefore\ (M + N)\,K + 2\,MN = 1$

Clave C

APLICAMOS LO APRENDIDO (página 62) Unidad 3

1. $E = \text{sen}10° + 2\cos20°\cos80°$

$E = \text{sen}10° + 2\text{sen}70°\cos80°$

$E = \text{sen}(80° - 70°) + 2\text{sen}70°\cos80°$

$E = \text{sen}80°\cos70° - \cos80°\text{sen}70° + 2\text{sen}70°\cos80°$

$E = \text{sen}80°\cos70° + \cos80°\text{sen}70°$

$E = \text{sen}(80° + 70°) = \text{sen}150°$

$\Rightarrow E = \text{sen}(180° - 30°) = \text{sen}30° = \dfrac{1}{2}$

$\therefore E = \dfrac{1}{2}$

Clave A

2. $P = \cos80° + 2\text{sen}70°\text{sen}10°$

$P = \cos(70° + 10°) + 2\text{sen}70°\text{sen}10°$

$P = \cos70°\cos10° - \text{sen}70°\text{sen}10° + 2\text{sen}70°\text{sen}10°$

$P = \cos70°\cos10° + \text{sen}70°\text{sen}10°$

$P = \cos(70° - 10°) = \cos60° = \dfrac{1}{2}$

$\therefore P = \dfrac{1}{2}$

Clave B

3. $E = \sqrt{3}\cot10°(\tan50° - \tan40°)$

$E = \dfrac{\sqrt{3}(\tan50° - \tan40°)}{\tan10°}$

$E = \dfrac{\sqrt{3}(\tan50° - \tan40°)}{\tan(50° - 40°)} = \dfrac{\sqrt{3}(\tan50° - \tan40°)}{\dfrac{(\tan50° - \tan40°)}{(1 + \tan50°\tan40°)}}$

$E = \sqrt{3}(1 + \tan50°\tan40°)$

$E = \sqrt{3}(1 + \underbrace{\tan50°\cot50°}_{1}) \Rightarrow E = \sqrt{3}(1 + 1)$

$\therefore E = 2\sqrt{3}$

Clave D

4. $\tan(\alpha + \beta) = 5 \quad \ldots \text{(I)}$

$\tan\alpha = 7 \quad \ldots \text{(II)}$

De (I):

$\dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 5$

$\dfrac{7 + \tan\beta}{1 - 7.\tan\beta} = 5$

$7 + \tan\beta = 5 - 35\tan\beta$

$36\tan\beta = -2$

$\therefore \tan\beta = -\dfrac{1}{18}$

Clave C

5. $E = \dfrac{\text{sen}(x + y) - \cos x\,\text{sen}y}{\text{sen}(x - y) + \cos x\,\text{sen}y}$

$E = \dfrac{\text{sen}x\cos y + \cos x\,\text{sen}y - \cos x\,\text{sen}y}{\text{sen}x\cos y - \cos x\,\text{sen}y + \cos x\,\text{sen}y}$

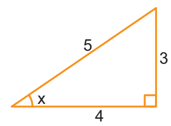
$E = \dfrac{\text{sen}x\cos y}{\text{sen}x\cos y} = 1$

$\therefore E = 1$

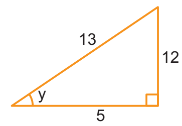
Clave A

6.

$\tan x = \dfrac{3}{4} \quad \Rightarrow$



$\sec y = \dfrac{13}{5} \quad \Rightarrow$



Piden: $\text{sen}(x + y)$

$\text{sen}(x + y) = \text{sen}x\cos y + \cos x\,\text{sen}y$

$\text{sen}(x + y) = \left(\dfrac{3}{5}\right)\left(\dfrac{5}{13}\right) + \left(\dfrac{4}{5}\right)\left(\dfrac{12}{13}\right)$

$\text{sen}(x + y) = \dfrac{15}{65} + \dfrac{48}{65} = \dfrac{63}{65}$

$\therefore \text{sen}(x + y) = \dfrac{63}{65}$

Clave C

7. $E = \tan21° + \tan24° + \tan21°.\tan24°$

De las relaciones auxiliares:

$\tan(\alpha + \beta) = \tan\alpha + \tan\beta + \tan\alpha\tan\beta\tan(\alpha + \beta)$

Para: $\alpha = 21 \;\wedge\; \beta = 24°$

$\tan(21° + 24°) = \tan21° + \tan24° + \tan21°\tan24°\tan(21° + 24°)$

$\tan45° = \tan21° + \tan24° + \tan21°\tan24°.\tan45°$

$\Rightarrow 1 = \underbrace{\tan21° + \tan24° + \tan21°\tan24°(1)}_{E}$

$\therefore E = 1$

Clave D

8. $Q = \tan34° + \tan19° + \dfrac{4}{3}\tan34°\tan19°$

$Q = \tan34° + \tan19° + \tan53°.\tan34°.\tan19°$

$Q = \tan34° + \tan19° + \tan34°.\tan19°.\tan(34° + 19°)$

Empleando relaciones auxiliares:

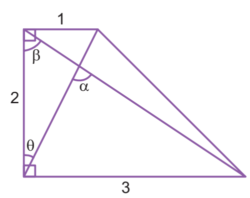
$Q = \tan(34° + 19°)$

$Q = \tan53° = \dfrac{4}{3}$

$\therefore Q = \dfrac{4}{3}$

Clave B

9.



Del gráfico: $\tan\theta = \dfrac{1}{2} \;\wedge\; \tan\beta = \dfrac{3}{2}$

Además:

$\alpha = \theta + \beta$

$\tan\alpha = \tan(\theta + \beta)$

Sabemos:

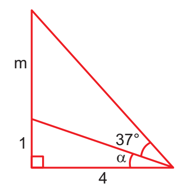
$\tan\alpha = \dfrac{\tan\theta + \tan\beta}{1 - \tan\theta\tan\beta}$

$\tan\alpha = \dfrac{\left(\frac{1}{2}\right) + \left(\frac{3}{2}\right)}{1 - \left(\frac{1}{2}\right)\left(\frac{3}{2}\right)} = \dfrac{2}{\frac{1}{4}} = 8$

$\therefore \tan\alpha = 8$

Clave C

10.



Del gráfico: $\tan\alpha = \dfrac{1}{4}$

Además:

$\tan(\alpha + 37°) = \dfrac{m + 1}{4}$

Resolviendo:

$\dfrac{\tan\alpha + \tan37°}{1 - \tan\alpha\tan37°} = \dfrac{m + 1}{4}$

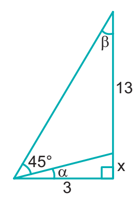
$\dfrac{\frac{1}{4} + \frac{3}{4}}{1 - \frac{1}{4}.\frac{3}{4}} = \dfrac{m + 1}{4}$

$\dfrac{16}{13} = \dfrac{m + 1}{4}$

$\therefore m = \dfrac{51}{13}$

Clave A

11.



$\alpha + \beta + 45° = 90 \Rightarrow \alpha + \beta = 45°$

$\tan(\alpha + \beta) = \tan45°$

$\dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta} = 1$

$\dfrac{\frac{x}{3} + \frac{3}{x + 13}}{1 - \frac{x}{\cancel{3}}.\frac{\cancel{3}}{x + 13}} = 1$

$\dfrac{x^2 + 13x + 9}{3\cancel{(x + 13)}} = \dfrac{13}{\cancel{x + 13}}$

$x^2 + 13x + 9 = 39$

$x^2 + 13x - 30 = 0$

$x \qquad +15$

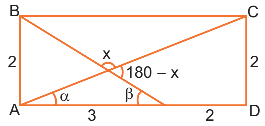
$x \qquad -2$

$(x + 15)(x - 2) = 0$

$x = 2$

Clave A

12.



Del gráfico:

$180 - x = \alpha + \beta$

$\tan(180° - x) = \tan(\alpha + \beta)$

$$-\tan x = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta}$$

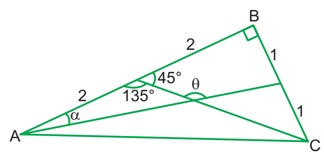
$$-\tan x = \frac{\frac{2}{5}+\frac{2}{3}}{1-\frac{2}{5}.\frac{2}{3}}$$

$$-\tan x = \frac{16}{11}$$

$$\tan x = -\frac{16}{11}$$

Clave C

13.



$\theta = \alpha + 135°$

$\tan\theta = \tan(\alpha + 135°)$

$$\tan\theta = \frac{\tan\alpha + \tan 135}{1 - \tan\alpha.\tan 135}$$

$$\tan\theta = \frac{\tan\alpha - \tan 45}{1 - \tan\alpha(-\tan 45)}$$

$$= \frac{\frac{1}{4}-1}{1-\frac{1}{4}(-1)}$$

$$\tan\theta = -\frac{3}{5}$$

Clave D

14. $$\tan(\theta - \alpha) = \frac{\tan\theta - \tan\alpha}{1 + \tan\theta.\tan\alpha}$$

$$\frac{1}{5} = \frac{\tan\theta - \frac{2}{3}}{1 + \frac{2}{3}\tan\theta}$$

$$\frac{1}{5} + \frac{2}{15}\tan\theta = \tan\theta - \frac{2}{3}$$

$$\frac{13}{15} = \frac{13}{15}\tan\theta$$

$\tan\theta = 1$

$$\tan(\theta + \alpha) = \frac{\tan\theta + \tan\alpha}{1 + \tan\theta.\tan\alpha}$$

$$\frac{x+2}{3} = \frac{1+\frac{2}{3}}{1-1.\frac{2}{3}}$$

$$\frac{x+2}{3} = 5 \qquad \therefore\ x = 13$$

Clave E

Nivel 1 (página 64) Unidad 3

Comunicación matemática

1.

2.

Razonamiento y demostración

3. Piden:

$C = \text{sen}17°\cos 43° + \text{sen}43°\cos 17°$

$C = \text{sen}17°\cos 43° + \cos 17°\text{sen}43°$

$C = \text{sen}(17° + 43°)$

$$\Rightarrow C = \text{sen}60° = \frac{\sqrt{3}}{2}$$

$$\therefore C = \frac{\sqrt{3}}{2}$$

Clave D

4. Piden:

$L = \cos 42°\cos 18° - \text{sen}42°\text{sen}18°$

$L = \cos(42° + 18°)$

$$\Rightarrow L = \cos 60° = \frac{1}{2}$$

$\therefore L = 0{,}5$

Clave A

5. $C = \text{sen}4x\cos x - \text{sen}x\cos 4x$

$C = \text{sen}4x\cos x - \cos 4x\,\text{sen}x$

$C = \text{sen}(4x - x) = \text{sen}3x$

$\therefore C = \text{sen}3x$

Clave A

6. $C = \cos 3x\cos 2x + \text{sen}3x\,\text{sen}2x$

$C = \cos(3x - 2x) = \cos x$

$\therefore C = \cos x$

Clave B

7. $$C = \frac{\text{sen}(45° + x) + \text{sen}(45° - x)}{\cos x}$$

Sabemos:

$\text{sen}(45° + x) = \text{sen}45°\cos x + \cos 45°\text{sen}x$

$\text{sen}(45° - x) = \text{sen}45°\cos x - \cos 45°\text{sen}x$

Sumando estas dos expresiones:

$\text{sen}(45° + x) + \text{sen}(45° - x) = 2\text{sen}45°\cos x$

Luego, al reemplazar en la expresión inicial tenemos:

$$C = \frac{2\text{sen}45°\cos x}{\cos x} = 2\text{sen}45°$$

$$\Rightarrow C = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$$

$\therefore C = \sqrt{2}$

Clave B

8. $$C = \frac{\text{sen}(x + \theta)}{\text{sen}x\,\text{sen}\theta} - \cot\theta$$

$$C = \frac{\text{sen}x\cos\theta + \cos x\,\text{sen}\theta}{\text{sen}x\,\text{sen}\theta} - \cot\theta$$

$$C = \frac{\text{sen}x\cos\theta}{\text{sen}x\,\text{sen}\theta} + \frac{\cos x\,\text{sen}\theta}{\text{sen}x\,\text{sen}\theta} - \cot\theta$$

$$C = \frac{\cos\theta}{\text{sen}\theta} + \frac{\cos x}{\text{sen}x} - \cot\theta$$

$\Rightarrow C = \cot\theta + \cot x - \cot\theta = \cot x$

$\therefore C = \cot x$

Clave C

9. $$C = \frac{\cos(x - \beta) - \text{sen}x\,\text{sen}\beta}{\cos x\cos\beta}$$

$$C = \frac{(\cos x\cos\beta + \text{sen}x\,\text{sen}\beta) - \text{sen}x\,\text{sen}\beta}{\cos x\cos\beta}$$

$$\Rightarrow C = \frac{\cos x\cos\beta}{\cos x\cos\beta} = 1$$

$\therefore C = 1$

Clave A

10. $$L = \frac{\text{sen}(\alpha + \theta) - \text{sen}\alpha\cos\theta}{\cos(\alpha + \theta) + \text{sen}\alpha\,\text{sen}\theta}$$

$$L = \frac{(\text{sen}\alpha\cos\theta + \cos\alpha\,\text{sen}\theta) - \text{sen}\alpha\cos\theta}{(\cos\alpha\cos\theta - \text{sen}\alpha\,\text{sen}\theta) + \text{sen}\alpha\,\text{sen}\theta}$$

$$\Rightarrow L = \frac{\cos\alpha\,\text{sen}\theta}{\cos\alpha\cos\theta} = \frac{\text{sen}\theta}{\cos\theta} = \tan\theta$$

$\therefore L = \tan\theta$

Clave B

Nivel 2 (página 64) Unidad 3

Comunicación matemática

11. Por teoría:

I. F

II. F

III. V

12. Por teoría:

I. V

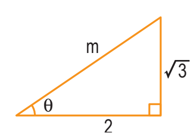
II. V

III. V

Razonamiento y demostración

13. Por dato: $\tan\theta = \frac{\sqrt{3}}{2}$

Además: θ es un ángulo agudo.



Por el teorema de Pitágoras:

$m^2 = (\sqrt{3})^2 + 2^2 = 7$

$\Rightarrow m = \sqrt{7}$

Luego:

$$\text{sen}\theta = \frac{\sqrt{3}}{m} = \frac{\sqrt{3}}{\sqrt{7}}$$

$$\cos\theta = \frac{2}{m} = \frac{2}{\sqrt{7}}$$

Piden:

$\text{sen}(60° + \theta) = \text{sen}60°\cos\theta + \cos 60°\text{sen}\theta$

$$\text{sen}(60° + \theta) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{2}{\sqrt{7}}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{\sqrt{7}}\right)$$

$\Rightarrow\ \text{sen}(60° + \theta) = \dfrac{3\sqrt{3}}{2\sqrt{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}}$

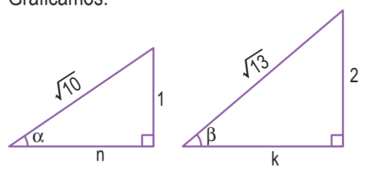
$\therefore\ \text{sen}(60° + \theta) = \dfrac{3\sqrt{21}}{14}$

Clave C

14. Por dato: α y β son ángulos agudos.

Además: $\text{sen}\alpha = \dfrac{1}{\sqrt{10}} \ \wedge\ \text{sen}\beta = \dfrac{2}{\sqrt{13}}$

Graficamos:



Por el teorema de Pitágoras: n = k = 3

Luego:

$\tan\alpha = \dfrac{1}{n} = \dfrac{1}{3}$

$\tan\beta = \dfrac{2}{k} = \dfrac{2}{3}$

Piden:

$\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\tan(\alpha + \beta) = \dfrac{\left(\frac{1}{3}\right) + \left(\frac{2}{3}\right)}{1 - \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)} = \dfrac{1}{\left(\frac{7}{9}\right)}$

$\therefore\ \tan(\alpha + \beta) = \dfrac{9}{7}$

Clave D

15. Piden:

$C = (\text{sen}\alpha + \text{sen}\beta)^2 + (\cos\alpha + \cos\beta)^2$

Luego:

$$\begin{array}{r} (\text{sen}\alpha + \text{sen}\beta)^2 = \text{sen}^2\alpha + 2\text{sen}\alpha\text{sen}\beta + \text{sen}^2\beta \\ (\cos\alpha + \cos\beta)^2 = \cos^2\alpha + 2\cos\alpha\cos\beta + \cos^2\beta \\ \hline C = 1 + 2(\text{sen}\alpha\text{sen}\beta + \cos\alpha\cos\beta) + 1 \end{array} \ (+)$$

$\Rightarrow C = 2 + 2(\cos\alpha\cos\beta + \text{sen}\alpha\text{sen}\beta)$

$\Rightarrow C = 2 + 2\cos(\alpha - \beta)$

Por dato: $\alpha - \beta = \dfrac{\pi}{6}$

$\Rightarrow C = 2 + 2\cos\dfrac{\pi}{6} = 2 + 2\left(\dfrac{\sqrt{3}}{2}\right)$

$\therefore\ C = 2 + \sqrt{3}$

Clave E

16. Piden:

$L = (\cos\alpha + \cos\theta)^2 + (\text{sen}\alpha - \text{sen}\theta)^2$

Luego:

$$\begin{array}{r} (\cos\alpha + \cos\theta)^2 = \cos^2\alpha + 2\cos\alpha\cos\theta + \cos^2\theta \\ (\text{sen}\alpha - \text{sen}\theta)^2 = \text{sen}^2\alpha - 2\text{sen}\alpha\text{sen}\theta + \text{sen}^2\theta \\ \hline L = 1 + 2(\cos\alpha\cos\theta - \text{sen}\alpha\text{sen}\theta) + 1 \end{array} \ (+)$$

$\Rightarrow L = 2 + 2(\cos\alpha\cos\theta - \text{sen}\alpha\text{sen}\theta)$

$\Rightarrow L = 2 + 2\cos(\alpha + \theta)$

Por dato: $\alpha + \theta = 37°$

$\Rightarrow\ L = 2 + 2\cos 37° = 2 + 2\left(\dfrac{4}{5}\right)$

$\therefore\ L = 3{,}6$

Clave E

17. Piden el valor máximo de:

$C = 3\text{sen}x - \sqrt{2}\cos x$

Por propiedad:

Si $y = a\text{sen}x \pm b\cos x$, entonces:

$-\sqrt{a^2 + b^2} \leq\ y\ \leq \sqrt{a^2 + b^2}$

Entonces:

$-\sqrt{(3)^2 + (\sqrt{2})^2} \leq\ C\ \leq \sqrt{(3)^2 + (\sqrt{2})^2}$

$-\sqrt{11} \leq\ C\ \leq \sqrt{11}$

$\Rightarrow C \in \left[-\sqrt{11};\ \sqrt{11}\right]$

$\therefore\ C_{\text{máx.}} = \sqrt{11}$

Clave D

18. Piden el máximo valor de:

$3\text{sen}x + 4\cos x + 5$

Sea: $H = 3\text{sen}x + 4\cos x + 5$

$H - 5 = 3\text{sen}x + 4\cos x$

Por propiedad:

Si $y = a\text{sen}x + b\cos x$, entonces:

$-\sqrt{a^2 + b^2} \leq y \leq \sqrt{a^2 + b^2}$

Luego:

$-\sqrt{3^2 + 4^2} \leq H - 5 \leq \sqrt{3^2 + 4^2}$

$-5 \leq H - 5 \leq 5$

$0 \leq H \leq 10$

$\Rightarrow\ H \in [0;\ 10]$

$\therefore\ H_{\text{máx.}} = 10$

Clave C

19. Piden el máximo valor de:

$L = 5(\text{sen}x - 1) + 12(\cos x + 1)$

$L = 5\text{sen}x - 5 + 12\cos x + 12$

$L = 5\text{sen}x + 12\cos x + 7$

Sea: $y = 5\text{sen}x + 12\cos x$, entonces:

$L = y + 7$

Por propiedad:

$-\sqrt{5^2 + 12^2} \leq\ y\ \leq \sqrt{5^2 + 12^2}$

$-13 \leq\ y\ \leq 13$

$-13 + 7 \leq \underbrace{y + 7} \leq 13 + 7$

$-6 \leq\ L \leq 20$

$\Rightarrow L \in [-6;\ 20]$

$\therefore\ L_{\text{máx.}} = 20$

Clave D

20. Sea:

$M = \tan 20° + \tan 25° + \tan 20°\tan 25°$

Empleando la relación auxiliar:

$\tan(\alpha + \beta) = \tan\alpha + \tan\beta + \tan\alpha\tan\beta\tan(\alpha + \beta)$

Luego tenemos:

$M = \tan 20° + \tan 25° + \tan 20°\tan 25°\ .\ (1)$

$M = \tan 20° + \tan 25° + \tan 20°\tan 25°(\tan 45°)$

$M = \tan 20° + \tan 25° + \tan 20°\tan 25°\tan(20° + 25°)$

Finalmente:

$M = \tan(20° + 25°) = \tan 45°$

$\Rightarrow M = 1$

$\therefore\ \tan 20° + \tan 25° + \tan 20°\tan 25° = 1$

Clave A

Nivel 3 (página 65) Unidad 3

Comunicación matemática

21.

22.

Razonamiento y demostración

23. Piden:

$\dfrac{\tan 65° - \tan 25°}{\tan 40°} = \dfrac{\tan 65° - \tan 25°}{\tan(65° - 25°)}$

$\dfrac{\tan 65° - \tan 25°}{\tan 40°} = \dfrac{(\tan 65° - \tan 25°)}{\left(\dfrac{\tan 65° - \tan 25°}{1 + \tan 65°\tan 25°}\right)}$

$\dfrac{\tan 65° - \tan 25°}{\tan 40°} = 1 + \tan 65°\tan 25°\ \ ...(1)$

Luego:

$\tan 25° = \tan(90° - 65°) = \cot 65°$

$\Rightarrow \tan 25° = \cot 65°$

Reemplazando:

$\dfrac{\tan 65° - \tan 25°}{\tan 40°} = 1 + \dfrac{\tan 65°\cot 65°}{1}$

$\therefore\ \dfrac{\tan 65° - \tan 25°}{\tan 40°} = 2$

Clave E

24. Piden: $\dfrac{\text{sen}20°}{\text{sen}25° - \cos 25°}$

Por propiedad:

$a\text{sen}x \pm b\cos x = \sqrt{a^2 + b^2}\,\text{sen}(x \pm \alpha)$

Donde: $\tan\alpha = \dfrac{b}{a}$

De: $\text{sen}25° - \cos 25°$; $a = 1\ \wedge\ b = 1$

Además: $\tan\alpha = \dfrac{b}{a} = \dfrac{1}{1} = 1$

$\Rightarrow \tan\alpha = \tan 45° \Rightarrow \alpha = 45°$

Entonces:

$\text{sen}25° - \cos 25° = \sqrt{1^2 + 1^2}\,\text{sen}(25° - 45°)$

$\text{sen}25° - \cos 25° = \sqrt{2}\,\text{sen}(-20°)$

Luego:

$\dfrac{\text{sen}20°}{\text{sen}25° - \cos 25°} = \dfrac{\text{sen}20°}{\sqrt{2}\,\text{sen}(-20°)}$

$\dfrac{\text{sen}20°}{\text{sen}25° - \cos 25°} = \dfrac{\text{sen}20°}{\sqrt{2}(-\text{sen}20°)}$

$\dfrac{\text{sen}20°}{\text{sen}25° - \cos 25°} = -\dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$\therefore\ \dfrac{\text{sen}20°}{\text{sen}25° - \cos 25°} = -\dfrac{\sqrt{2}}{2}$

Clave B

25. Sea: $A = \dfrac{\tan^2 5\alpha - \tan^2 3\alpha}{1 - \tan^2 5\alpha\tan^2 3\alpha}$

Luego:

$A = \dfrac{(\tan 5\alpha + \tan 3\alpha)\ .\ (\tan 5\alpha - \tan 3\alpha)}{(1 + \tan 5\alpha\tan 3\alpha)\ .\ (1 - \tan 5\alpha\tan 3\alpha)}$

$A = \left[\dfrac{\tan 5\alpha + \tan 3\alpha}{1 - \tan 5\alpha\tan 3\alpha}\right] \cdot \left[\dfrac{\tan 5\alpha - \tan 3\alpha}{1 + \tan 5\alpha\tan 3\alpha}\right]$

$A = [\tan(5\alpha + 3\alpha)]\,[\tan(5\alpha - 3\alpha)]$

$\Rightarrow A = \tan 8\alpha \,.\, \tan 2\alpha$

$\therefore \dfrac{\tan^2 5\alpha - \tan^2 3\alpha}{1 - \tan^2 5\alpha \tan^2 3\alpha} = \tan 8\alpha \tan 2\alpha$

Clave E

26. Sea:

$P = \tan 20° + \tan 40° + \sqrt{3}\tan 20° \tan 40°$

Empleando la relación auxiliar:

$\tan(\alpha + \beta) = \tan\alpha + \tan\beta + \tan\alpha \tan\beta \tan(\alpha + \beta)$

Luego tenemos:

$P = \tan 20° + \tan 40° + \tan 20° \tan 40°.(\sqrt{3})$

$P = \tan 20° + \tan 40° + \tan 20° \tan 40°(\tan 60°)$

$P = \tan 20° + \tan 40° + \tan 20° \tan 40° \tan(20° + 40°)$

Finalmente:

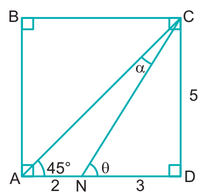
$P = \tan(20° + 40°) = \tan 60°$

$\Rightarrow P = \sqrt{3}$

$\therefore\ \tan 20° + \tan 40° + \sqrt{3}\tan 20° \tan 40° = \sqrt{3}$

Clave D

27. Por dato: ABCD es un cuadrado.



Del gráfico: $\tan\theta = \dfrac{5}{3}$

Además: $45° + \alpha = \theta$

$\Rightarrow \alpha = \theta - 45° \Rightarrow \tan\alpha = \tan(\theta - 45°)$

Luego:

$\tan\alpha = \dfrac{\tan\theta - \tan 45°}{1 + \tan\theta \tan 45°}$

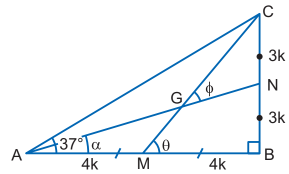
$\tan\alpha = \dfrac{\left(\frac{5}{3}\right) - (1)}{1 + \left(\frac{5}{3}\right)(1)} = \dfrac{\left(\frac{2}{3}\right)}{\left(\frac{8}{3}\right)}$

$\Rightarrow \tan\alpha = \dfrac{2}{8} = \dfrac{1}{4}$

$\therefore\ \tan\alpha = \dfrac{1}{4}$

Clave D

28.



Por dato: G es el baricentro del ⊿ABC.

Del ⊿ABC notable de 37° y 53°:

$BC = 6k \ \wedge\ AB = 8k$

Del gráfico: $\alpha + \phi = \theta$

$\Rightarrow \phi = \theta - \alpha \Rightarrow \tan\phi = \tan(\theta - \alpha)$

Entonces:

$\tan\phi = \dfrac{\tan\theta - \tan\alpha}{1 - \tan\theta \tan\alpha}$

$\tan\phi = \dfrac{\left(\frac{6k}{4k}\right) - \left(\frac{3k}{8k}\right)}{1 + \left(\frac{6k}{4k}\right)\left(\frac{3k}{8k}\right)}$

$\tan\phi = \dfrac{\left(\frac{3}{2}\right) - \left(\frac{3}{8}\right)}{1 + \left(\frac{3}{2}\right)\left(\frac{3}{8}\right)} = \dfrac{\left(\frac{9}{8}\right)}{\left(\frac{25}{16}\right)}$

$\Rightarrow \tan\phi = \dfrac{9.16}{8.25} = \dfrac{18}{25}$

$\therefore\ \tan\phi = \dfrac{18}{25}$

Clave B

29. Por dato:

$\tan A + \tan B = 7\tan C$

Además: A, B y C son los ángulos internos de un triángulo.

$\Rightarrow A + B + C = 180°$

Por relaciones angulares, entonces:

$\underbrace{\tan A + \tan B}_{7\tan C} + \tan C = \tan A \tan B \tan C$

$\Rightarrow 8\tan C = \tan A \tan B \tan C$

$\Rightarrow \tan A \tan B = 8$

Piden:

$L = \tan A \tan B = 8$

$\therefore\ L = 8$

Clave C

30. Por dato:

$\dfrac{\tan A}{2} = \dfrac{\tan B}{3} = \dfrac{\tan C}{4} = k$

$\Rightarrow \tan A = 2k;\ \tan B = 3k;\ \tan C = 4k$

Además: A; B y C son los ángulos internos de un triángulo.

$\Rightarrow A + B + C = 180°$

Por relaciones angulares:

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$(2k) + (3k) + (4k) = (2k)(3k)(4k)$

$9k = 24k^3$

$\dfrac{3}{8} = k^2$

$\Rightarrow k = \dfrac{\sqrt{6}}{4}$

Piden:

$L = \sqrt{6}\tan A + 3 = \sqrt{6}(2k) + 3$

$\Rightarrow L = 2\sqrt{6}\left(\dfrac{\sqrt{6}}{4}\right) + 3 = 3 + 3$

$\therefore\ L = 6$

Clave B

ÁNGULOS MÚLTIPLES

APLICAMOS LO APRENDIDO (página 66) Unidad 3

1. $N = \dfrac{(\cos 35° + \text{sen}35°)(\cos 35° - \text{sen}35°)}{4\cos 10°\text{sen}10°}$

$N = \dfrac{\cos^2 35° - \text{sen}^2 35°}{2\text{sen}20°} = \dfrac{(\cos 70°)}{2\text{sen}20°}$

$N = \dfrac{(\text{sen}20°)}{2\text{sen}20°} = \dfrac{1}{2}$

$\therefore N = \dfrac{1}{2} = 0{,}5$

Clave E

2. $\tan x = 3$

Entonces:

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

$\tan 2x = \dfrac{2(3)}{1-(3)^2}$

$\tan 2x = \dfrac{6}{-8} = -\dfrac{3}{4}$

Luego:

$\tan^2 2x = \left(-\dfrac{3}{4}\right)^2 = \dfrac{9}{16}$

$\therefore \tan^2 2x = \dfrac{9}{16}$

Clave B

3. $E = 2\sqrt{2 - \sqrt{2 + 2\cos 24°}}$

$E = 2\sqrt{2 - \sqrt{2(1 + \cos 24°)}}$

$E = 2\sqrt{2 - \sqrt{2(2\cos^2 12°)}}$

$E = 2\sqrt{2 - 2\cos 12°}$

$E = 2\sqrt{2(1 - \cos 12°)} = 2\sqrt{2(2\text{sen}^2 6°)}$

$E = 2\sqrt{4\text{sen}^2 6°} = 2 \,.\, 2\text{sen}6° = 4\text{sen}6°$

$\therefore E = 4\text{sen}6°$

Clave A

4. $K = \dfrac{\text{sen}\theta - 2\text{sen}^3\theta}{\sec\theta}$

$K = \dfrac{\text{sen}\theta(1 - 2\text{sen}^2\theta)}{\sec\theta} = \dfrac{\text{sen}\theta(\cos 2\theta)}{\sec\theta}$

$K = \text{sen}\theta\cos\theta\cos 2\theta$

$K = \dfrac{(2\text{sen}\theta\cos\theta)\cos 2\theta}{2} = \dfrac{(\text{sen}2\theta)\cos 2\theta}{2}$

$K = \dfrac{2(\text{sen}2\theta\cos 2\theta)}{2.2} = \dfrac{\text{sen}4\theta}{4}$

Por dato: $\theta = \dfrac{\pi}{8} \Rightarrow 4\theta = \dfrac{\pi}{2}$

Entonces:

$K = \dfrac{\text{sen}\frac{\pi}{2}}{4} = \dfrac{1}{4}$

$\therefore K = \dfrac{1}{4}$

Clave B

5. Por dato:

$\text{sen}\dfrac{x}{2} = \dfrac{4}{7}$

Luego:

$\text{sen}^2\dfrac{x}{2} = \dfrac{1 - \cos x}{2}$

$\left(\dfrac{4}{7}\right)^2 = \dfrac{1 - \cos x}{2}$

$\dfrac{32}{49} = 1 - \cos x$

$\Rightarrow \cos x = 1 - \dfrac{32}{49}$

$\therefore \cos x = \dfrac{17}{49}$

Clave C

6. $\tan\dfrac{x}{2} = \dfrac{2}{3}$

Luego:

$\tan^2\dfrac{x}{2} = \dfrac{1 - \cos x}{1 + \cos x}$

$\left(\dfrac{2}{3}\right)^2 = \dfrac{1 - \cos x}{1 + \cos x}$

Entonces:

$4 + 4\cos x = 9 - 9\cos x$

$13\cos x = 5$

$\therefore \cos x = \dfrac{5}{13}$

Clave B

7. $\cos x = -\dfrac{23}{25} \quad \wedge \quad 90° < x < 180°$

$\Rightarrow 45° < \dfrac{x}{2} < 90°$

$\Rightarrow \left(\dfrac{x}{2}\right) \in \text{IC}$

Luego:

$\cos\dfrac{x}{2} = +\sqrt{\dfrac{1 + \cos x}{2}}$

$\cos\dfrac{x}{2} = \sqrt{\dfrac{1 + \left(-\frac{23}{25}\right)}{2}} = \sqrt{\dfrac{\frac{2}{25}}{2}}$

$\cos\dfrac{x}{2} = \sqrt{\dfrac{1}{25}} = \dfrac{1}{5}$

$\therefore \cos\dfrac{x}{2} = \dfrac{1}{5}$

Clave A

8. Piden: $\tan 22{,}5°$

$\tan 22{,}5° = \tan\dfrac{45°}{2}$

Por identidad auxiliar de ángulo mitad:

$\tan\dfrac{x}{2} = \csc x - \cot x$

$\tan\dfrac{45°}{2} = \csc 45° - \cot 45°$

$\tan 22{,}5° = (\sqrt{2}) - (1)$

$\therefore \tan 22{,}5° = \sqrt{2} - 1$

Clave C

9. Piden: $y = \tan 159°$

Luego:

$\tan(3 \,.\, 53°) = \dfrac{3\tan 53° - \tan^3 53°}{1 - 3\tan^2 53°}$

$\tan 159° = \dfrac{3\left(\frac{4}{3}\right) - \left(\frac{4}{3}\right)^3}{1 - 3\left(\frac{4}{3}\right)^2}$

$\tan 159° = \dfrac{4 - \frac{64}{27}}{1 - \frac{16}{3}} = \dfrac{\frac{44}{27}}{\frac{-13}{3}}$

$\therefore \tan 159° = -\dfrac{44}{117}$

Clave A

10. Por dato: $\tan x = -\dfrac{1}{2}$

Luego:

$\tan 3x = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

$\tan 3x = \dfrac{3\left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right)^3}{1 - 3\left(-\frac{1}{2}\right)^2} = \dfrac{-\frac{3}{2} + \frac{1}{8}}{1 - \frac{3}{4}}$

$\tan 3x = \dfrac{-\frac{11}{8}}{\frac{1}{4}} = -\dfrac{11}{2}$

Por identidad de ángulo doble:

$\tan(2 \,.\, 3x) = \dfrac{2\tan 3x}{1 - \tan^2 3x}$

$\tan 6x = \dfrac{2\left(-\frac{11}{2}\right)}{1 - \left(-\frac{11}{2}\right)^2} = \dfrac{-11}{-\frac{117}{4}}$

$\tan 6x = \dfrac{-44}{-117} = \dfrac{44}{117} \qquad \therefore \tan 6x = \dfrac{44}{117}$

Clave B

11. Por dato:

$(1 + \cos x)^2 + (1 - \cos x)^2 = 2$

$2(1^2 + \cos^2 x) = 2$

$2 + 2\cos^2 x = 2$

$\cos^2 x = 0$

$\Rightarrow \cos x = 0$

Piden: $\cos 6x$

$\cos 6x = 4\cos^3 2x - 3\cos 2x \qquad ...(I)$

Por ángulo doble:

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 2(0)^2 - 1$

$\cos 2x = -1$

Reemplazando en (I):

$\cos 6x = 4(-1)^3 - 3(-1)$

$\cos 6x = 4(-1) + 3 = -4 + 3 = -1$

$\therefore \cos 6x = -1$

Clave C

12. $C=\dfrac{\text{sen}8°\ .\ \text{sen}52°\ .\ \text{sen}68°}{\cos 66°}$

$C=\dfrac{4\text{sen}8°\ .\ \text{sen}(60°-8°)\ .\ \text{sen}(60°+8°)}{4\cos 66°}$

$C=\dfrac{\text{sen}3(8°)}{4\cos 66°}=\dfrac{\text{sen}24°}{4\cos 66°}=\dfrac{\text{sen}24°}{4(\text{sen}24°)}$

$\therefore C=\dfrac{1}{4}$

Clave E

13. $\tan\alpha=\dfrac{4}{12}\ \Rightarrow\ \tan\alpha=\dfrac{1}{3}$ ...(I)

$\tan2\alpha=\dfrac{x+4}{12}$

$\dfrac{2\tan\alpha}{1-\tan^2\alpha}=\dfrac{x+4}{12}$ ...(II)

(I) en (II):

$$\dfrac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2}=\dfrac{x+4}{12}$$

Resolviendo: $x=5$

Clave A

14. Se sabe que:

$\text{sen}\theta(2\cos2\theta+1)=\text{sen}3\theta$

$\cos\theta(2\cos2\theta-1)=\cos3\theta$

De la condición:

$\dfrac{2\cos\theta+1}{2\cos\theta-1}=\dfrac{\cos\frac{\theta}{2}}{\text{sen}\frac{\theta}{2}}$

$\dfrac{\text{sen}\frac{\theta}{2}(2\cos\theta+1)}{\cos\frac{\theta}{2}(2\cos\theta-1)}=1$

$\dfrac{\text{sen}\frac{3\theta}{2}}{\cos\frac{3\theta}{2}}=1\ \Rightarrow\ \tan\dfrac{3\theta}{2}=1$

$\dfrac{3\theta}{2}=\dfrac{\pi}{4}\Rightarrow\theta=\dfrac{\pi}{6}$

Clave B

PRACTIQUEMOS

Nivel 1 (página 68) Unidad 3

Comunicación matemática

1.

2.

Razonamiento y demostración

3. $A=\text{sen}2x\tan x+2\cos^2x$

$A=(2\text{sen}x\cos x)\left(\dfrac{\text{sen}x}{\cos x}\right)+2\cos^2x$

$A=2\text{sen}^2x+2\cos^2x$

$\Rightarrow A=2(\text{sen}^2x+\cos^2x)=2(1)$

$\therefore\ A=2$

Clave C

4. $R=\text{sen}\theta\cos\theta\cos2\theta$

Sabemos que: $2\text{sen}\theta\cos\theta=\text{sen}2\theta$

$\Rightarrow \text{sen}\theta\cos\theta=\dfrac{\text{sen}2\theta}{2}$

Entonces:

$R=\left(\dfrac{\text{sen}2\theta}{2}\right)\cos2\theta=\dfrac{\text{sen}2\theta\cos2\theta}{2}$

$\Rightarrow R=\dfrac{2\text{sen}2\theta\cos2\theta}{2.2}=\dfrac{\text{sen}4\theta}{4}$

$\therefore R=\dfrac{\text{sen}4\theta}{4}$

Clave B

5. Por dato: $\text{sen}\theta=\dfrac{2}{\sqrt5}$

Piden:

$\cos2\theta=1-2\text{sen}^2\theta$

$\Rightarrow \cos2\theta=1-2\left(\dfrac{2}{\sqrt5}\right)^2=1-\dfrac{8}{5}$

$\therefore \cos2\theta=-\dfrac{3}{5}$

Clave D

6. Por dato: $\cos\theta=\dfrac{1}{\sqrt3}$

Piden:

$\cos2\theta=2\cos^2\theta-1$

$\Rightarrow \cos2\theta=2\left(\dfrac{1}{\sqrt3}\right)^2-1=\dfrac{2}{3}-1$

$\therefore \cos2\theta=-\dfrac{1}{3}$

Clave A

7. Por dato: $\tan\theta=\dfrac{1}{2}$

Piden:

$\tan2\theta=\dfrac{2\tan\theta}{1-\tan^2\theta}$

$\Rightarrow \tan2\theta=\dfrac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2}=\dfrac{1}{\frac{3}{4}}$

$\therefore \tan2\theta=\dfrac{4}{3}$

Clave C

8. $M=(\sec x-\cos x)(\csc x-\text{sen}x)$

$M=\left(\dfrac{1}{\cos x}-\cos x\right)\left(\dfrac{1}{\text{sen}x}-\text{sen}x\right)$

$M=\left(\dfrac{1-\cos^2x}{\cos x}\right)\left(\dfrac{1-\text{sen}^2x}{\text{sen}x}\right)$

$M=\dfrac{(\text{sen}^2x)}{\cos x}.\dfrac{(\cos^2x)}{\text{sen}x}$

$\Rightarrow M=\text{sen}x\cos x=\dfrac{2\text{sen}x\cos x}{2}$

$\therefore M=\dfrac{\text{sen}2x}{2}$

Clave D

9. Por dato: $\cos\dfrac{x}{2}=-\dfrac{1}{5}$

Piden: $\cos x$

Sabemos: $\cos\dfrac{x}{2}=\pm\sqrt{\dfrac{1+\cos x}{2}}$

Entonces deducimos que:

$-\dfrac{1}{5}=-\sqrt{\dfrac{1+\cos x}{2}}$

$\left(-\dfrac{1}{5}\right)^2=\left(-\sqrt{\dfrac{1+\cos x}{2}}\right)^2$

$\dfrac{1}{25}=\dfrac{1+\cos x}{2}$

$2=25+25\cos x$

$\Rightarrow 25\cos x=-23$

$\therefore \cos x=-\dfrac{23}{25}$

Clave B

10. Por dato: $\tan\dfrac{x}{2}=\dfrac{1}{3}$

Piden: $\cos x$

Sabemos: $\tan\dfrac{x}{2}=\pm\sqrt{\dfrac{1-\cos x}{1+\cos x}}$

Entonces deducimos que:

$\dfrac{1}{3}=\sqrt{\dfrac{1-\cos x}{1+\cos x}}$

$\left(\dfrac{1}{3}\right)^2=\left(\sqrt{\dfrac{1-\cos x}{1+\cos x}}\right)^2$

$\dfrac{1}{9}=\dfrac{1-\cos x}{1+\cos x}\Rightarrow 1+\cos x=9-9\cos x$

$\Rightarrow 10\cos x=8\Rightarrow \cos x=\dfrac{8}{10}$

$\therefore \cos x=\dfrac{4}{5}$

Clave A

11. Por dato: $\tan\dfrac{x}{2}=-2$

Piden: $\cos x$

Sabemos: $\tan\dfrac{x}{2}=\pm\sqrt{\dfrac{1-\cos x}{1+\cos x}}$

Entonces deducimos que:

$-2=-\sqrt{\dfrac{1-\cos x}{1+\cos x}}$

$(-2)^2=\left(-\sqrt{\dfrac{1-\cos x}{1+\cos x}}\right)^2$

$4=\dfrac{1-\cos x}{1+\cos x}$

Luego:

$4+4\cos x=1-\cos x$

$5\cos x=-3$

$\therefore \cos x=-\dfrac{3}{5}$

Clave B

12. $E=\sec40°-\tan40°$

$E=\sec(90°-50°)-\tan(90°-50°)$

$E=\csc50°-\cot50°=\tan\dfrac{50°}{2}$

$\therefore\ E=\tan25°$

Clave A

13. Piden:

$E = \tan 22°30' = \tan 22{,}5°$

$E = \tan\frac{45°}{2} = \csc 45° - \cot 45°$

$\Rightarrow E = (\sqrt{2}) - (1) = \sqrt{2} - 1$

$\therefore E = \sqrt{2} - 1$

Clave C

14. Piden:

$E = \tan\frac{\pi}{8} - \cot\frac{\pi}{8}$

Sea $\frac{\pi}{8} = \frac{\alpha}{2}$; entonces:

$E = \tan\frac{\alpha}{2} - \cot\frac{\alpha}{2}$

$E = (\csc\alpha - \cot\alpha) - (\csc\alpha + \cot\alpha)$

$E = \csc\alpha - \cot\alpha - \csc\alpha - \cot\alpha$

$\Rightarrow E = -2\cot\alpha$

Como: $\frac{\alpha}{2} = \frac{\pi}{8} \Rightarrow \alpha = \frac{\pi}{4}$

$\Rightarrow E = -2\cot\frac{\pi}{4} = -2\cot 45°$

$\therefore E = -2(1) = -2$

Clave A

15. Piden:

$E = \cos 80° \,.\, \cos 20° \,.\, \cos 40°$

$E = \cos 40° \,.\, \cos 20° \,.\, \cos 80°$

$E = \cos(60° - 20°)\cos 20°\cos(60° + 20°)$

$E = \frac{\cos 3(20°)}{4} = \frac{\cos 60°}{4}$

$\Rightarrow E = \frac{\left(\frac{1}{2}\right)}{4} = \frac{1}{8}$

$\therefore E = \frac{1}{8}$

Clave E

16. Piden:

$E = \text{sen}6° \,.\, \text{sen}54° \,.\, \text{sen}66°$

$E = \text{sen}54° \,.\, \text{sen}6° \,.\, \text{sen}66°$

$E = \text{sen}(60° - 6°)\text{sen}6°\text{sen}(60° + 6°)$

$E = \frac{\text{sen}3(6°)}{4} = \frac{\text{sen}18°}{4}$

$\therefore E = \frac{\text{sen}18°}{4}$

Clave E

17. $C = (\cos 3x + 2\cos x)\tan x$

$C = \cos 3x \,.\, \tan x + 2\cos x \,.\, \tan x$

$C = \cos x(2\cos 2x - 1) . \frac{\text{sen}x}{\cos x} + 2\cos x \,.\, \frac{\text{sen}x}{\cos x}$

$C = (2\cos 2x - 1) \,.\, \text{sen}x + 2\text{sen}x$

$C = \text{sen}x[(2\cos 2x - 1) + 2]$

$C = \text{sen}x(2\cos 2x + 1) = \text{sen}3x$

$\therefore C = \text{sen}3x$

Clave C

18. Piden:

$\text{sen}111° = \text{sen}3(37°)$

$\text{sen}111° = 3\text{sen}37° - 4\text{sen}^3 37°$

$\text{sen}111° = 3\left(\frac{3}{5}\right) - 4\left(\frac{3}{5}\right)^3$

$\text{sen}111° = \frac{9}{5} - \frac{108}{125} = \frac{117}{125}$

$\therefore \text{sen}111° = \frac{117}{125}$

Clave C

19. $A = \frac{\text{sen}3x}{\text{sen}x} - \frac{\cos 3x}{\cos x}$

$A = \frac{\text{sen}x(2\cos 2x + 1)}{\text{sen}x} - \frac{\cos x(2\cos 2x - 1)}{\cos x}$

$A = (2\cos 2x + 1) - (2\cos 2x - 1)$

$A = 2\cos 2x + 1 - 2\cos 2x + 1 = 2$

$\therefore A = 2$

Clave E

20. Por dato: $\text{sen}\theta = \frac{1}{3}$

Piden:

$L = \frac{\cos 3\theta}{\cos\theta} = \frac{\cos\theta(2\cos 2\theta - 1)}{\cos\theta}$

$L = 2\cos 2\theta - 1 = 2(1 - 2\text{sen}^2\theta) - 1$

$L = 2 - 4\text{sen}^2\theta - 1 = 1 - 4\text{sen}^2\theta$

$\Rightarrow L = 1 - 4\left(\frac{1}{3}\right)^2 = 1 - \frac{4}{9}$

$\therefore L = \frac{5}{9}$

Clave E

Resolución de problemas

21. Dato:

$\tan\theta = \frac{2}{3} \;\Rightarrow\; \tan 2\theta = \frac{2\tan\theta}{1 - \text{tg}^2\theta}$

$\tan 2\theta = \frac{2(2/3)}{1 - (2/3)^2}$

$\therefore\; \tan 2\theta = 12/5$

Clave C

22. Dato:

$\cot\theta = 2$

$\tan\theta = \frac{1}{2} \;\Rightarrow\; \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

$\tan 2\theta = \frac{2(1/2)}{1 - (1/2)^2}$

$\therefore\; \tan 2\theta = 4/3$

Clave E

Nivel 2 (página 68) Unidad 3

Comunicación matemática

23.

24.

Razonamiento y demostración

25. Piden: sen8x

Por dato:

$\text{sen}x\cos x\cos 2x\cos 4x = m$

$\Rightarrow \cos x\cos 2x\cos 2^2x = \frac{m}{\text{sen}x}$ ...(I)

Por propiedad:

• $\cos x\cos 2x\cos 2^2x = \frac{\text{sen}2^{2+1}x}{2^{2+1}\text{sen}x}$

$\cos x\cos 2x\cos 2^2x = \frac{\text{sen}2^3x}{2^3\text{sen}x} = \frac{\text{sen}8x}{8\text{sen}x}$

Reemplazando en (I):

$\Rightarrow \frac{\text{sen}8x}{8\text{sen}x} = \frac{m}{\text{sen}x} \Rightarrow \frac{\text{sen}8x}{8} = m$

$\therefore \text{sen}8x = 8m$

Clave D

26. Piden:

$M = \sqrt{1 + \text{sen}2x} - \text{sen}x$

$M = \sqrt{(\text{sen}x + \cos x)^2} - \text{sen}x$

$M = |\text{sen}x + \cos x| - \text{sen}x$

Por dato: $0° < x < 90°$

$\Rightarrow x \in \text{IC} \Rightarrow \text{sen}x > 0 \;\wedge\; \cos x > 0$

$\Rightarrow \text{sen}x + \cos x > 0$

Entonces:

$M = (\text{sen}x + \cos x) - \text{sen}x = \cos x$

$\therefore M = \cos x$

Clave B

27. Piden:

$A = 2(\cos^4x - \text{sen}^4x)^2 - 1$

Luego:

$\cos^4x - \text{sen}^4x = (\cos^2x - \text{sen}^2x) \,.\, (\cos^2x + \text{sen}^2x)$

$\cos^4x - \text{sen}^4x = (\cos 2x)(1)$

$\Rightarrow \cos^4x - \text{sen}^4x = \cos 2x$

Reemplazando en la expresión A:

$A = 2(\cos 2x)^2 - 1$

$\Rightarrow A = 2\cos^2 2x - 1 = \cos 4x$

$\therefore A = \cos 4x$

Clave C

28. $M = \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8}$

$M = \cos^4\frac{\pi}{8} + \left[\cos\left(\frac{\pi}{2} - \frac{\pi}{8}\right)\right]^4$

$M = \cos^4\frac{\pi}{8} + \left(\text{sen}\frac{\pi}{8}\right)^4$

$M = \text{sen}^4\frac{\pi}{8} + \cos^4\frac{\pi}{8}$

Sea $\frac{\pi}{8} = \alpha$, entonces:

$M = \text{sen}^4\alpha + \cos^4\alpha$

$M = 1 - 2\text{sen}^2\alpha\cos^2\alpha$

$M = 1 - \frac{4\text{sen}^2\alpha\cos^2\alpha}{2}$

$M = 1 - \frac{(2\text{sen}\alpha\cos\alpha)^2}{2} = 1 - \frac{(\text{sen}2\alpha)^2}{2}$

Como: $a = \frac{\pi}{8} \Rightarrow 2a = \frac{\pi}{4}$

$$\Rightarrow M = 1 - \frac{\left(\text{sen}\frac{\pi}{4}\right)^2}{2} = 1 - \frac{\left(\frac{\sqrt{2}}{2}\right)^2}{2}$$

$$\Rightarrow M = 1 - \frac{1}{4} = \frac{3}{4} = 0{,}75$$

$\therefore\ M = 0{,}75$

Clave B

29. $A = \dfrac{\text{sen}^3x - \cos^3x}{\text{sen}x - \cos x} - 1$

$$A = \frac{(\text{sen}x - \cos x)(\text{sen}^2x + \text{sen}x\cos x + \cos^2x)}{\text{sen}x - \cos x} - 1$$

$$A = \text{sen}x\cos x + \underbrace{\text{sen}^2x + \cos^2x}_{1} - 1$$

$$\Rightarrow A = \text{sen}x\cos x = \frac{2\text{sen}x\cos x}{2}$$

$$\therefore A = \frac{\text{sen}2x}{2}$$

Clave D

30. Piden: sen2x

Por dato: $\tan x + \cot x = n$

Por propiedad:

$2\csc 2x = \tan x + \cot x$

$\Rightarrow 2\csc 2x = n$

$2\left(\dfrac{1}{\text{sen}2x}\right) = n \Rightarrow \dfrac{2}{n} = \text{sen}2x$

$\therefore\ \text{sen}2x = \dfrac{2}{n}$

Clave A

31. $E = \left(\cot\frac{x}{2} + \tan\frac{x}{2}\right)(\csc 2x - \cot 2x)$

Por identidad del ángulo doble:

$\tan\frac{x}{2} + \cot\frac{x}{2} = 2\csc x$

Por identidad del ángulo mitad:

$\csc 2x - \cot 2x = \tan x$

Reemplazando en la expresión E, tenemos:

$E = (2\csc x)(\tan x)$

$$E = 2\left(\frac{1}{\text{sen}x}\right)\left(\frac{\text{sen}x}{\cos x}\right)$$

$$\Rightarrow E = 2\left(\frac{1}{\cos x}\right) = 2(\sec x)$$

$\therefore E = 2\sec x$

Clave E

32.

$$E = \frac{\cot\frac{x}{2} - \tan\frac{x}{2}}{\csc 2x + \cot 2x}$$

Por identidad del ángulo doble:

$\cot\frac{x}{2} - \tan\frac{x}{2} = 2\cot x$

Por identidad del ángulo mitad:

$\csc 2x + \cot 2x = \cot x$

Reemplazando en la expresión E, tenemos:

$$E = \frac{2\cot x}{\cot x} = 2$$

$\therefore E = 2$

Clave D

33. Por dato: $\text{sen}\frac{x}{2} = \frac{3}{4}$

Piden: cosx

Sabemos: $\text{sen}\frac{x}{2} = \pm\sqrt{\dfrac{1-\cos x}{2}}$

Entonces deducimos que:

$$\frac{3}{4} = \sqrt{\frac{1-\cos x}{2}}$$

$$\left(\frac{3}{4}\right)^2 = \left(\sqrt{\frac{1-\cos x}{2}}\right)^2$$

$$\frac{9}{16} = \frac{1-\cos x}{2}$$

$18 = 16 - 16\cos x$

$\Rightarrow 16\cos x = -2$

$\therefore \cos x = -\dfrac{1}{8}$

Clave A

34. Piden:

$$E = \sqrt{\frac{1-\cos 100°}{2}}$$

Sabemos:

$$\text{sen}\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

Otra forma de expresar esta identidad es:

$$\left|\text{sen}\frac{x}{2}\right| = \sqrt{\frac{1-\cos x}{2}}$$

Luego:

$$E = \sqrt{\frac{1-\cos 100°}{2}} = \left|\text{sen}\frac{100°}{2}\right|$$

$\Rightarrow E = |\text{sen}50°|$

Como: $50° \in \text{IC} \Rightarrow \text{sen}50° > 0$

$\Rightarrow |\text{sen}50°| = \text{sen}50°$

$\therefore\ E = \text{sen}50°$

Clave C

35. Piden:

$$E = \sqrt{\frac{1+\cos 80°}{2}}$$

Sabemos:

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

Otra forma de expresar esta identidad es:

$$\left|\cos\frac{x}{2}\right| = \sqrt{\frac{1+\cos x}{2}}$$

Luego:

$$E = \sqrt{\frac{1+\cos 80°}{2}} = \left|\cos\frac{80°}{2}\right|$$

$\Rightarrow E = |\cos 40°|$

Como: $40° \in \text{IC} \Rightarrow \cos 40° > 0$

$\Rightarrow |\cos 40°| = \cos 40°$

$\therefore E = \cos 40°$

Clave B

36. Por dato: $\cos x = -\dfrac{3}{4}$

Además: $180° < x < 270°$

$90° < \dfrac{x}{2} < 135° \Rightarrow \dfrac{x}{2} \in \text{IIC}$

Piden:

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

Como: $\dfrac{x}{2} \in \text{IIC} \Rightarrow \tan\dfrac{x}{2}$ es (–)

Entonces:

$$\tan\frac{x}{2} = -\sqrt{\frac{1-\left(\frac{-3}{4}\right)}{1+\left(\frac{-3}{4}\right)}}$$

$$\Rightarrow \tan\frac{x}{2} = -\sqrt{\frac{\left(\frac{7}{4}\right)}{\left(\frac{1}{4}\right)}} = -\sqrt{7}$$

$\therefore \tan\dfrac{x}{2} = -\sqrt{7}$

Clave D

37. Sea:

$A = \tan 3\theta(2\cos 2\theta - 1) - (2\cos 2\theta + 1)\tan\theta$

Sabemos:

• $\cos 3\theta = \cos\theta(2\cos 2\theta - 1)$

$\Rightarrow \dfrac{\cos 3\theta}{\cos\theta} = 2\cos 2\theta - 1$

• $\text{sen}3\theta = \text{sen}\theta(2\cos 2\theta + 1)$

$\Rightarrow \dfrac{\text{sen}3\theta}{\text{sen}\theta} = 2\cos 2\theta + 1$

Entonces:

$$A = \tan 3\theta\left(\frac{\cos 3\theta}{\cos\theta}\right) - \left(\frac{\text{sen}3\theta}{\text{sen}\theta}\right)\tan\theta$$

$$A = \left(\frac{\text{sen}3\theta}{\cos 3\theta}\right)\frac{\cos 3\theta}{\cos\theta} - \frac{\text{sen}3\theta}{\text{sen}\theta}\left(\frac{\text{sen}\theta}{\cos\theta}\right)$$

$$A = \frac{\text{sen}3\theta}{\cos\theta} - \frac{\text{sen}3\theta}{\cos\theta} = 0$$

$\therefore \tan 3\theta(2\cos 2\theta - 1) - (2\cos 2\theta + 1)\tan\theta = 0$

Clave C

38. Sea:

$$H = \sec\frac{2\pi}{9} + 8\cos^2\frac{2\pi}{9}$$

Como $\dfrac{2\pi}{9}$ rad $= 40°$, entonces:

$H = \sec 40° + 8\cos^2 40°$

$$H = \frac{1}{\cos 40°} + 8\cos^2 40°$$

$$H = \frac{1 + 8\cos^3 40°}{\cos 40°}$$

$$H = \frac{1 + 2(4\cos^3 40°)}{\cos 40°}$$

$$H = \frac{1 + 2(3\cos 40° + \cos 3(40°))}{\cos 40°}$$

$$H = \frac{1 + 6\cos 40° + 2\cos 120°}{\cos 40°}$$

$$H=\frac{1+6\cos 40°+2\left(-\frac{1}{2}\right)}{\cos 40°}$$

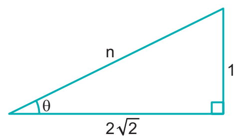
$$H=\frac{1+6\cos 40°-1}{\cos 40°}=\frac{6\cos 40°}{\cos 40°}$$

$\Rightarrow H = 6$

$\therefore\ \sec\frac{2\pi}{9}+8\cos^2\frac{2\pi}{9}=6$

Clave E

39. Por dato: $\cot\theta = 2\sqrt{2}$; θ agudo.



Por el teorema de Pitágoras:

$n^2 = 1^2 + (2\sqrt{2})^2 = 9 \Rightarrow n = 3$

Entonces: $\operatorname{sen}\theta = \frac{1}{n} = \frac{1}{3}$

Piden:

$\operatorname{sen}3\theta = 3\operatorname{sen}\theta - 4\operatorname{sen}^3\theta$

$\operatorname{sen}3\theta = 3\left(\frac{1}{3}\right)-4\left(\frac{1}{3}\right)^3$

$\Rightarrow \operatorname{sen}3\theta = 1-\frac{4}{27}=\frac{23}{27}$

$\therefore\ \operatorname{sen}3\theta = \frac{23}{27}$

Clave C

40. Por dato:

$\operatorname{sen}3x = 0{,}25\operatorname{sen}x$

$\operatorname{sen}x(2\cos 2x + 1) = 0{,}25\operatorname{sen}x$

Entonces:

$2\cos 2x + 1 = \frac{1}{4}$

$\underbrace{\cos 2x}_{\frac{1-\tan^2 x}{1+\tan^2 x}} = -\frac{3}{8}$

$\frac{1-\tan^2 x}{1+\tan^2 x} = -\frac{3}{8}$

$8\tan^2 x - 8 = 3 + 3\tan^2 x$

$\Rightarrow 5\tan^2 x = 11$

Piden:

$K = 5\tan^2 x + 1 = (11) + 1$

$\therefore K = 12$

Clave E

41. Por dato:

$$\tan 3x = 5\tan x$$

$$\frac{\operatorname{sen}3x}{\cos 3x} = 5\left(\frac{\operatorname{sen}x}{\cos x}\right)$$

$$\frac{\operatorname{sen}x(2\cos 2x+1)}{\cos x(2\cos 2x-1)} = \frac{5\operatorname{sen}x}{\cos x}$$

$$2\cos 2x + 1 = 5(2\cos 2x - 1)$$

$$6 = 8\cos 2x$$

$$\Rightarrow \cos 2x = \frac{3}{4}$$

Luego elevamos al cuadrado:

$\cos^2 2x = \frac{9}{16} \Rightarrow \sec^2 2x = \frac{16}{9}$

$\Rightarrow \tan^2 2x + 1 = \frac{16}{9} \Rightarrow \tan^2 2x = \frac{7}{9}$

$\therefore\ |\tan 2x| = \frac{\sqrt{7}}{3}$

Clave D

42. $4\cos 18° - \frac{3}{\cos 18°} = k\tan 18°$

$$\frac{4\cos^2 18° - 3}{\cos 18°} = k\ .\ \frac{\operatorname{sen}18°}{\cos 18°}$$

$$4\cos^2 18° - 3 = k\operatorname{sen}18°$$

$$\cos 18°(4\cos^2 18° - 3) = (k\operatorname{sen}18°)\ .\ \cos 18°$$

$$4\cos^3 18° - 3\cos 18° = k\operatorname{sen}18°\cos 18°$$

$$\cos 3(18°) = k\left(\frac{2\operatorname{sen}18°\cos 18°}{2}\right)$$

$$\cos 54° = k\left(\frac{\operatorname{sen}36°}{2}\right)$$

$$\cos(90° - 36°) = \frac{k\operatorname{sen}36°}{2}$$

$$\operatorname{sen}36° = \frac{k\operatorname{sen}36°}{2}$$

$$1 = \frac{k}{2}$$

$\therefore k = 2$

Clave A

Resolución de problemas

43. Dato: $\operatorname{sen}\theta = \frac{1}{2} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$$\operatorname{sen}\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\operatorname{sen}\frac{\theta}{2} = +\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$$

$$\therefore\ \operatorname{sen}\frac{\theta}{2} = \frac{1}{2}.\sqrt{2-\sqrt{3}}$$

Clave E

44. Dato:

$\sec\theta = 3 \Rightarrow \cos\theta = 1/3$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = +\sqrt{\frac{1+\frac{1}{3}}{2}}$$

$$\therefore\ \cos\frac{\theta}{2} = \frac{\sqrt{6}}{3}$$

Clave B

Nivel 3 (página 69) Unidad 3

Comunicación matemática

45.

46.

47. $E = \tan\theta + 2\tan 2\theta + 4\tan 4\theta + 8\cot 8\theta$

Por propiedad:

$2\cot 2x = \cot x - \tan x$

Luego:

$E = \tan\theta + 2\tan 2\theta + 4\tan 4\theta + 4(2\cot 8\theta)$

$E = \tan\theta + 2\tan 2\theta + 4\tan 4\theta + 4(\cot 4\theta - \tan 4\theta)$

$E = \tan\theta + 2\tan 2\theta + 2(2\cot 4\theta)$

$E = \tan\theta + 2\tan 2\theta + 2(\cot 2\theta - \tan 2\theta)$

$E = \tan\theta + 2\cot 2\theta$

$\Rightarrow E = \tan\theta + (\cot\theta - \tan\theta) = \cot\theta$

$\therefore E = \cot\theta$

Clave B

48. Piden:

$Q = (1+\sec 2\theta)(1+\sec 4\theta)(1+\sec 8\theta)(1+\sec 16\theta)$

Sabemos: $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$$\frac{\tan 2x}{\tan x} = \frac{2}{1-\left(\frac{\operatorname{sen}^2 x}{\cos^2 x}\right)} = \frac{2\cos^2 x}{\cos^2 x - \operatorname{sen}^2 x}$$

$$\frac{\tan 2x}{\tan x} = \frac{(1+\cos 2x)}{\cos 2x} = \frac{1}{\cos 2x} + \frac{\cos 2x}{\cos 2x}$$

$$\Rightarrow \frac{\tan 2x}{\tan x} = \sec 2x + 1$$

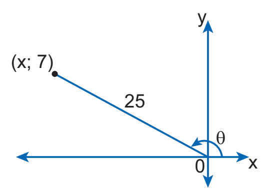
Aplicando esta equivalencia en Q, tenemos:

$$Q = \left(\frac{\tan 2\theta}{\tan\theta}\right)\left(\frac{\tan 4\theta}{\tan 2\theta}\right)\left(\frac{\tan 8\theta}{\tan 4\theta}\right)\left(\frac{\tan 16\theta}{\tan 8\theta}\right)$$

$$\therefore Q = \frac{\tan 16\theta}{\tan\theta}$$

Clave B

49. Por dato: $\operatorname{sen}\theta = \frac{7}{25}\ \wedge\ \theta \in \langle 90°; 180°\rangle$



Por radio vector: $x^2 + y^2 = r^2$

$\Rightarrow x^2 + 7^2 = 25^2 \Rightarrow x^2 = 576$

$\Rightarrow x = 24\ \vee\ x = -24$

Del gráfico: $x < 0 \Rightarrow x = -24$

Entonces: $\cos\theta = \frac{x}{r} = \frac{-24}{25}$

$\Rightarrow \cos\theta = -\frac{24}{25}$

Piden:

$\operatorname{sen}2\theta = 2\operatorname{sen}\theta\cos\theta$

$\Rightarrow \operatorname{sen}2\theta = 2\left(\frac{7}{25}\right)\left(-\frac{24}{25}\right) = -\frac{336}{625}$

$\therefore\ \operatorname{sen}2\theta = -\frac{336}{625}$

Clave D

50. $K = \tan\frac{x}{2} + 2\text{sen}^2\frac{x}{2}\cot x$

$\Rightarrow K = \tan\frac{x}{2} + 2\text{sen}^2\frac{x}{2}\left(\frac{\cos x}{\text{sen} x}\right)$

$K = \tan\frac{x}{2} + 2\text{sen}^2\frac{x}{2}\left(\frac{\cos x}{2\text{sen}\frac{x}{2}\cos\frac{x}{2}}\right)$

$K = \frac{\text{sen}\frac{x}{2}}{\cos\frac{x}{2}} + \frac{\text{sen}\frac{x}{2}\cos x}{\cos\frac{x}{2}}$

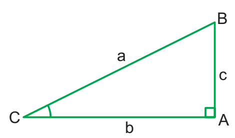
$K = \frac{\text{sen}\frac{x}{2}(1+\cos x)}{\cos\frac{x}{2}} = \frac{\text{sen}\frac{x}{2}\left(2\cos^2\frac{x}{2}\right)}{\cos\frac{x}{2}}$

$\Rightarrow K = 2\text{sen}\frac{x}{2}\cos\frac{x}{2} = \text{sen}x$

$\therefore K = \text{sen}x$

Clave D

51. Por dato:



Piden:

$\tan\frac{C}{2} = \frac{\text{sen}\frac{C}{2}}{\cos\frac{C}{2}} = \frac{\text{sen}\frac{C}{2}\times 2\left(\cos\frac{C}{2} - \text{sen}\frac{C}{2}\right)}{\cos\frac{C}{2}\times 2\left(\cos\frac{C}{2} - \text{sen}\frac{C}{2}\right)}$

$\tan\frac{C}{2} = \frac{2\text{sen}\frac{C}{2}\cos\frac{C}{2} - 2\text{sen}^2\frac{C}{2}}{2\cos^2\frac{C}{2} - 2\text{sen}\frac{C}{2}\cos\frac{C}{2}}$

$\tan\frac{C}{2} = \frac{(\text{sen}C) - (1-\cos C)}{(1+\cos C) - (\text{sen}C)}$

$\Rightarrow \tan\frac{C}{2} = \frac{\cos C + \text{sen}C - 1}{\cos C - \text{sen}C + 1}$

$\Rightarrow \tan\frac{C}{2} = \frac{\left(\frac{b}{a}\right) + \left(\frac{c}{a}\right) - 1}{\left(\frac{b}{a}\right) - \left(\frac{c}{a}\right) + 1} = \frac{b+c-a}{b-c+a}$

$\therefore \tan\frac{C}{2} = \frac{b+c-a}{b-c+a}$

Clave A

52. Sea:

$S = \tan x + \frac{1}{2}\tan\frac{x}{2} + \frac{1}{4}\tan\frac{x}{4} + ... + \frac{1}{2^n}\tan\frac{x}{2^n}$

Sabemos: $2\cot 2\theta = \cot\theta - \tan\theta$

$\Rightarrow \tan\theta = \cot\theta - 2\cot 2\theta$

$\Rightarrow \cot 2\theta = \frac{1}{2}\cot\theta - \frac{1}{2}\tan\theta$

Para 2 términos:

$S_2 = \tan x + \frac{1}{2}\tan\frac{x}{2} = \cot x - 2\cot 2x + \frac{1}{2}\tan\frac{x}{2}$

$S_2 = \left(\frac{1}{2}\cot\frac{x}{2} - \frac{1}{2}\tan\frac{x}{2}\right) + \frac{1}{2}\tan\frac{x}{2} - 2\cot 2x$

$\Rightarrow S_2 = \frac{1}{2}\cot\frac{x}{2} - 2\cot 2x$

Para 3 términos:

$S_3 = \underbrace{\tan x + \frac{1}{2}\tan\frac{x}{2}} + \frac{1}{4}\tan\frac{x}{4}$

$S_3 = \left(\frac{1}{2}\cot\frac{x}{2} - 2\cot 2x\right) + \frac{1}{4}\tan\frac{x}{4}$

$S_3 = \frac{1}{2}\left(\frac{1}{2}\cot\frac{x}{4} - \frac{1}{2}\tan\frac{x}{4}\right) + \frac{1}{4}\tan\frac{x}{4} - 2\cot 2x$

$S_3 = \frac{1}{4}\cot\frac{x}{4} - 2\cot 2x$

$\Rightarrow S_3 = \frac{1}{2^2}\cot\frac{x}{2^2} - 2\cot 2x$

Para 4 términos, se obtiene:

$\Rightarrow S4 = \frac{1}{2^3}\cot\frac{x}{2^3} - 2\cot 2x$

Como la serie original tiene (n + 1) términos:

$\therefore S = \frac{1}{2^n}\cot\frac{x}{2^n} - 2\cot 2x$

Clave D

53. $E = \csc 10° + \csc 20° + \csc 40° + \underbrace{\csc 80° + \cot 80°}_{\cot 40°}$

$E = \csc 10° + \csc 20° + \underbrace{\csc 40° + \cot 40°}_{\cot 20°}$

$E = \csc 10° + \underbrace{\csc 20° + \cot 20°}_{\cot 10°}$

$\Rightarrow E = \csc 10° + \cot 10° = \cot 5°$

$\therefore E = \cot 5°$

Clave A

54. $E = \sec x + \tan x$

$E = \csc(90° - x) + \cot(90° - x)$

Sabemos: $\cot\left(\frac{x}{2}\right) = \csc x + \cot x$

$\Rightarrow E = \cot\left(\frac{90° - x}{2}\right) = \cot\left(45° - \frac{x}{2}\right)$

$\therefore E = \cot\left(45° - \frac{x}{2}\right)$

Clave B

55. Piden:

$E = \sqrt{\frac{1-\cos 200°}{1+\cos 200°}}$

Sabemos:

$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

Otra forma de expresar esta identidad es:

$\left|\tan\frac{x}{2}\right| = \sqrt{\frac{1-\cos x}{1+\cos x}}$

Luego:

$E = \sqrt{\frac{1-\cos 200°}{1+\cos 200°}} = \left|\tan\frac{200°}{2}\right|$

$\Rightarrow E = |\tan 100°|$

Como: $100° \in$ IIC $\Rightarrow \tan 100° < 0$

$\Rightarrow |\tan 100°| = -\tan 100°$

$\therefore E = -\tan 100°$

Clave A

56. Piden:

$E = \sqrt{\frac{1-\cos 400°}{1+\cos 400°}}$

Sabemos:

$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

Otra forma de expresar esta identidad es:

$\left|\tan\frac{x}{2}\right| = \sqrt{\frac{1-\cos x}{1+\cos x}}$

Luego:

$E = \sqrt{\frac{1-\cos 400°}{1+\cos 400°}} = \left|\tan\frac{400°}{2}\right|$

$\Rightarrow E = |\tan 200°|$

Como: $200° \in$ IIIC $\Rightarrow \tan 200° > 0$

$\Rightarrow |\tan 200°| = \tan 200°$

$\therefore E = \tan 200°$

Clave B

57. Por dato: $\text{sen}\theta = \frac{a-b}{a+b}$

Piden:

$E = \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

Sea $\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = \frac{x}{2}$; entonces:

$E = \tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$ ...(I)

Como: $\frac{x}{2} = \frac{\pi}{4} - \frac{\theta}{2} \Rightarrow x = \frac{\pi}{2} - \theta$

$\Rightarrow \cos x = \cos\left(\frac{\pi}{2} - \theta\right) = \text{sen}\theta$

$\Rightarrow \cos x = \frac{a-b}{a+b}$

Reemplazando en (I):

$E = \pm\sqrt{\frac{1 - \left(\frac{a-b}{a+b}\right)}{1 + \left(\frac{a-b}{a+b}\right)}}$

$\Rightarrow E = \pm\sqrt{\frac{\left(\frac{2b}{a+b}\right)}{\left(\frac{2a}{a+b}\right)}} = \pm\sqrt{\frac{b}{a}}$

$\therefore E = \pm\sqrt{\frac{b}{a}}$

Clave B

58. Por dato: $\text{sen}\theta = \frac{m-n}{m+n}$

Piden: $\tan\left(45° + \frac{\theta}{2}\right)$

Sea $\left(45° + \frac{\theta}{2}\right) = \frac{x}{2}$; entonces:

$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$ ...(I)

Como: $\frac{x}{2} = 45° + \frac{\theta}{2} \Rightarrow x = 90° + \theta$

$\Rightarrow \cos x = \cos(90° + \theta) = -\text{sen}\theta$

$\Rightarrow \cos x = -\left(\frac{m-n}{m+n}\right) = \frac{n-m}{m+n}$

Reemplazando en (I):

$$\tan\frac{x}{2}=\pm\sqrt{\frac{1-\left(\frac{n-m}{m+n}\right)}{1+\left(\frac{n-m}{m+n}\right)}}$$

$$\Rightarrow \tan\frac{x}{2}=\pm\sqrt{\frac{\left(\frac{2m}{m+n}\right)}{\left(\frac{2n}{m+n}\right)}}=\pm\sqrt{\frac{m}{n}}$$

$$\therefore \tan\left(45°+\frac{\theta}{2}\right)=\pm\sqrt{\frac{m}{n}}$$

Clave A

59. Sea:

$$H=\frac{\text{sen}3\theta\cos^2\theta\,\text{sen}\theta-\cos3\theta\,\text{sen}^2\theta\cos\theta}{(\text{sen}\theta\cos\theta)^2}$$

$$H=\frac{\text{sen}3\theta\cos^2\theta\,\text{sen}\theta}{\text{sen}^2\theta\cos^2\theta}-\frac{\cos3\theta\,\text{sen}^2\theta\cos\theta}{\text{sen}^2\theta\cos^2\theta}$$

$$H=\frac{\text{sen}3\theta}{\text{sen}\theta}-\frac{\cos3\theta}{\cos\theta}$$

$$H=\frac{\text{sen}\theta(2\cos2\theta+1)}{\text{sen}\theta}-\frac{\cos\theta(2\cos2\theta-1)}{\cos\theta}$$

$H=(2\cos2\theta+1)-(2\cos2\theta-1)$

$H=2\cos2\theta+1-2\cos2\theta+1=2$

$$\therefore \frac{\text{sen}3\theta\cos^2\theta\,\text{sen}\theta-\cos3\theta\,\text{sen}^2\theta\cos\theta}{(\text{sen}\theta\cos\theta)^2}=2$$

Clave C

60. Piden: $\tan3\alpha$

Por dato: $2\tan^3\alpha=3\tan^2\alpha+6\tan\alpha-1$

$\Rightarrow 1-3\tan^2\alpha=6\tan\alpha-2\tan^3\alpha$

$1-3\tan^2\alpha=2(3\tan\alpha-\tan^3\alpha)$

$$\frac{1}{2}=\left(\frac{3\tan\alpha-\tan^3\alpha}{1-3\tan^2\alpha}\right)$$

$$\frac{1}{2}=(\tan3\alpha)$$

$$\therefore \tan3\alpha=\frac{1}{2}$$

Clave D

61. Por dato: $\tan\alpha=\frac{1}{3}$

Luego:

$$\tan3\alpha=\frac{3\tan\alpha-\tan^3\alpha}{1-3\tan^2\alpha}$$

$$\tan3\alpha=\frac{3\left(\frac{1}{3}\right)-\left(\frac{1}{3}\right)^3}{1-3\left(\frac{1}{3}\right)^2}$$

$$\tan3\alpha=\frac{1-\frac{1}{27}}{1-\frac{1}{3}}=\frac{\left(\frac{26}{27}\right)}{\left(\frac{2}{3}\right)}$$

$$\Rightarrow \tan3\alpha=\frac{13}{9}$$

Piden:

$$F=\frac{3\tan3\alpha-\tan\alpha}{3\tan\alpha-\tan3\alpha}$$

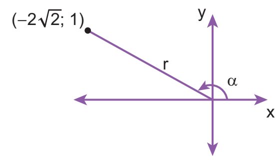
$$F=\frac{3\left(\frac{13}{9}\right)-\left(\frac{1}{3}\right)}{3\left(\frac{1}{3}\right)-\left(\frac{13}{9}\right)}=\frac{\frac{13}{3}-\frac{1}{3}}{1-\frac{13}{9}}$$

$$\Rightarrow F=\frac{\left(\frac{12}{3}\right)}{\left(-\frac{4}{9}\right)}=-9$$

$\therefore F=-9$

Clave B

62. Por dato: $\cot\alpha=-2\sqrt{2}$; $\alpha\in \text{IIC}$



Por radio vector: $r=3$

Entonces:

$$\text{sen}\alpha=\frac{y}{r}=\frac{1}{3}\Rightarrow \text{sen}\alpha=\frac{1}{3}$$

$$\cos\alpha=\frac{x}{r}=\frac{-2\sqrt{2}}{3}\Rightarrow \cos\alpha=-\frac{2\sqrt{2}}{3}$$

Piden:

$$C=\text{sen}3\alpha\,.\,\sec\alpha=\frac{\text{sen}3\alpha}{\cos\alpha}$$

$$C=\frac{3\text{sen}\alpha-4\text{sen}^3\alpha}{\cos\alpha}$$

$$C=\frac{3\left(\frac{1}{3}\right)-4\left(\frac{1}{3}\right)^3}{\left(-\frac{2\sqrt{2}}{3}\right)}=-\frac{\left(\frac{23}{27}\right)}{\left(\frac{2\sqrt{2}}{3}\right)}$$

$$\Rightarrow C=-\frac{23}{18\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=-\frac{23\sqrt{2}}{36}$$

$$\therefore C=-\frac{23\sqrt{2}}{36}$$

Clave D

63. Por razones trigonométricas de ángulos complementarios se cumple:

$\text{sen}36°=\cos54°$

$\Rightarrow \text{sen}2(18°)=\cos3(18°)$

Luego, utilizando las identidades de ángulo doble y triple, tenemos:

$2\text{sen}18°\cos18°=4\cos^3 18°-3\cos18°$

$2\text{sen}18°\cos18°=\cos18°(4\cos^2 18°-3)$

$2\text{sen}18°=4(1-\text{sen}^2 18°)-3$

$2\text{sen}18°=1-4\text{sen}^2 18°$

$\Rightarrow 4\text{sen}^2 18°+2\text{sen}18°-1=0$

$$\Rightarrow \text{sen}18°=\frac{-2\pm\sqrt{2^2-4(4)(-1)}}{2(4)}$$

$$\text{sen}18°=\frac{-2\pm\sqrt{20}}{8}=\frac{-2\pm2\sqrt{5}}{8}$$

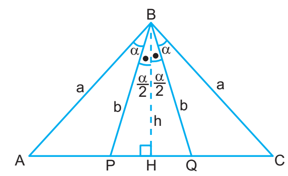
$$\Rightarrow \text{sen}18°=\frac{-1+\sqrt{5}}{4}\vee \text{sen}18°=\frac{-1-\sqrt{5}}{4}$$

Como: $18°\in \text{IC}\Rightarrow \text{sen}18°>0$

$$\therefore \text{sen}18°=\frac{-1+\sqrt{5}}{4}$$

Clave A

64.



Del ⊿BHQ: $h=b\cos\frac{\alpha}{2}$

Del ⊿BHC: $h=a\cos\frac{3\alpha}{2}$

Entonces: $b\cos\frac{\alpha}{2}=a\cos\frac{3\alpha}{2}$ ...(1)

Por dato: $a=2b$

Reemplazando en (1):

$$b\cos\frac{\alpha}{2}=(2b)\cos\frac{3\alpha}{2}$$

$$\cos\frac{\alpha}{2}=2\left[\cos\frac{\alpha}{2}(2\cos\alpha-1)\right]$$

$$\frac{1}{2}=2\cos\alpha-1$$

$$\frac{3}{2}=2\cos\alpha\Rightarrow\cos\alpha=\frac{3}{4}$$

$$\therefore \alpha=\arccos\frac{3}{4}$$

Clave E

65. Dato: $\cot\theta=1/2\Rightarrow\tan\theta=2$

$$\cos2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\cos2\theta=\frac{1-2^2}{1+2^2}$$

$\therefore \cos2\theta=-3/5$

Clave B

66. Dato: $\cot\theta=1/2\Rightarrow\tan\theta=2$

$$\tan3\theta=\frac{3\tan\theta-\text{tg}^3\theta}{1-3\tan^2\theta}$$

$$\tan3\theta=\frac{3.(2)-(2)^3}{1-3(2)^2}$$

$\therefore \tan3\theta=2/11$

Clave D

APLICAMOS LO APRENDIDO (página 71) Unidad 3

1. $A = 2\text{sen}3x \cdot \cos x - \text{sen}2x$

$A = \text{sen}(3x + x) + \text{sen}(3x - x) - \text{sen}2x$

$A = \text{sen}4x + \text{sen}2x - \text{sen}2x$

$\therefore A = \text{sen}4x$

Clave D

2. $B = \text{sen}15° \cdot \cos5° + \cos25° \cdot \text{sen}15°$

$B = \text{sen}15°(\cos25° + \cos5°)$

$B = \text{sen}15°(2\cos15° \cdot \cos10°)$

$B = \underbrace{2\text{sen}15°\cos15°}_{(\text{sen}30°)}\cos10°$

Luego:

$B = \text{sen}30° \cdot \cos10° = \frac{1}{2} \cdot \cos10°$

$\therefore B = \frac{\cos 10°}{2}$

Clave C

3. $R = \cos40° + \cos80° + \cos160°$

$R = \underbrace{2\cos\left(\frac{80° + 40°}{2}\right)\cos\left(\frac{80° - 40°}{2}\right)}\ -(\cos20°)$

$R = 2\cos60°\cos20° - \cos20°$

$R = 2\left(\frac{1}{2}\right)\cos20° - \cos20°$

$R = \cos20° - \cos20° = 0$

$\therefore R = 0$

Clave C

4. $A = 2\text{sen}7x \cdot \cos2x - \text{sen}5x$

$A = \text{sen}(7x + 2x) + \text{sen}(7x - 2x) - \text{sen}5x$

$A = \text{sen}9x + \text{sen}5x - \text{sen}5x$

$A = \text{sen}9x$

Por dato: $x = \frac{\pi}{18} \Rightarrow 9x = \frac{\pi}{2}$

$\Rightarrow A = \text{sen}\frac{\pi}{2} = 1$

$\therefore A = 1$

Clave B

5. $P = \text{sen}135° + \cos225° + \sec315°$

$P = (\text{sen}45°) + (-\cos45°) + (\sec45°)$

$P = \text{sen}45° - \cos45° + \sec45°$

$P = \left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right) + (\sqrt{2}) = \sqrt{2}$

$\therefore P = \sqrt{2}$

Clave E

6. $L = \frac{\text{sen}10° + \text{sen}30° + \text{sen}50°}{\cos 10° + \cos 30° + \cos 50°}$

$L = \frac{(\text{sen}50° + \text{sen}10°) + \text{sen}30°}{(\cos 50° + \cos 10°) + \cos 30°}$

$L = \frac{(2\text{sen}30° \cos 20°) + \text{sen}30°}{(2\cos 30° \cos 20°) + \cos 30°}$

$L = \frac{\text{sen}30°(2\cos 20° + 1)}{\cos 30°(2\cos 20° + 1)}$

$L = \frac{\text{sen}30°}{\cos 30°} = \tan 30° = \frac{\sqrt{3}}{3}$

$\therefore L = \frac{\sqrt{3}}{3}$

Clave E

7. $H = \frac{1 - \cos 2x + \cos 4x - \cos 6x}{\text{sen}2x - \text{sen}4x + \text{sen}6x}$

$H = \frac{(1 - \cos 2x) - (\cos 6x - \cos 4x)}{(\text{sen}6x + \text{sen}2x) - \text{sen}4x}$

$H = \frac{2\text{sen}^2 x - (-2\text{sen}5x \cdot \text{sen}x)}{2\text{sen}4x \cos 2x - 2\text{sen}2x \cos 2x}$

$H = \frac{2\text{sen}x(\text{sen}x + \text{sen}5x)}{2\cos 2x(\text{sen}4x - \text{sen}2x)}$

$H = \frac{\text{sen}x(2\text{sen}3x \cos 2x)}{\cos 2x(2\cos 3x\text{sen}x)} = \frac{\text{sen}3x}{\cos 3x}$

$H = \frac{\text{sen}3x}{\cos 3x} = \tan 3x$

$\therefore H = \tan3x$

Clave D

8. $R = \frac{\text{sen}5A \cdot \text{sen}A - (\cos 7A - \cos 3A)}{\cos 5A \cdot \text{sen}A + (\text{sen}7A - \text{sen}3A)}$

$R = \frac{\text{sen}5A \cdot \text{sen}A - (-2\text{sen}5A \cdot \text{sen}2A)}{\cos 5A \cdot \text{sen}A + (2\cos 5A \cdot \text{sen}2A)}$

$R = \frac{\text{sen}5A(\text{sen}A + 2\text{sen}2A)}{\cos 5A(\text{sen}A + 2\text{sen}2A)}$

$R = \frac{\text{sen}5A}{\cos 5A} = \tan 5A$

$\therefore R = \tan5A$

Clave A

9. $M = \frac{\text{sen}(2x + 30°) + \text{sen}(2y + 30°)}{\cos(2x + 45°) + \cos(2y + 45°)}$

$M = \frac{2\text{sen}\left(\frac{2x + 30° + 2y + 30°}{2}\right)\cos\left(\frac{2x + 30° - 2y - 30°}{2}\right)}{2\cos\left(\frac{2x + 45° + 2y + 45°}{2}\right)\cos\left(\frac{2x + 45° - 2y - 45°}{2}\right)}$

$M = \frac{\text{sen}(x + y + 30°) \cdot \cos(x - y)}{\cos(x + y + 45°) \cdot \cos(x - y)}$

$M = \frac{\text{sen}(x + y + 30°)}{\cos(x + y + 45°)}$

Por dato: $x + y = 15°$

Entonces:

$M = \frac{\text{sen}(15° + 30°)}{\cos(15° + 45°)} = \frac{\text{sen}45°}{\cos 60°} = \frac{\frac{\sqrt{2}}{2}}{\frac{1}{2}} = \sqrt{2}$

$\therefore M = \sqrt{2}$

Clave B

10. $R = \csc10° - 4\text{sen}70°$

$R = \frac{1}{\text{sen}10°} - 4\text{sen}70°$

$R = \frac{1 - 4\text{sen}10°\text{sen}70°}{\text{sen}10°}$

$R = \frac{1 - 2(2\text{sen}10°\text{sen}70°)}{\text{sen}10°}$

$R = \frac{1 - 2(\cos 60° - \cos 80°)}{\text{sen}10°}$

$R = \frac{1 - 2\cos 60° + 2\cos 80°}{\text{sen}10°}$

$R = \frac{1 - 2\left(\frac{1}{2}\right) + 2\cos 80°}{\text{sen}10°} = \frac{2\cos 80°}{\cos 80°} = 2$

$\therefore R = 2$

Clave C

11. Por dato: $x + y = 30°$

$C = \cos2x + \cos2y$

$C = 2\cos\left(\frac{2x + 2y}{2}\right)\cos\left(\frac{2x - 2y}{2}\right)$

$C = 2\cos(x + y)\cos(x - y)$

$C = 2\cos(30°)\cos(x - y)$

$C = 2\left(\frac{\sqrt{3}}{2}\right)\cos(x - y)$

$\Rightarrow C = \sqrt{3}\cos(x - y)$

Sabemos:

$-1 \le \cos(x - y) \le 1$

$-\sqrt{3} \le \sqrt{3}\cos(x - y) \le \sqrt{3}$

$\Rightarrow C \in [-\sqrt{3}; \sqrt{3}]$

$\therefore C_{máx.} = \sqrt{3}$

Clave E

12. Usamos la transformación a producto:

$2\text{sen}\frac{(A + B)}{2} \cdot \cos\frac{(A - B)}{2} = x$

$2\cos\frac{(A + B)}{2} \cdot \cos\frac{(A - B)}{2} = y$

Luego de dividir las dos igualdades obtenemos:

$\frac{2\text{sen}\frac{(A + B)}{2}}{2\cos\frac{(A + B)}{2}} = \tan\frac{(A + B)}{2} = \frac{x}{y}$

Sabemos que: $\text{sen}2\theta = \frac{2\tan\theta}{1 + \tan^2\theta}$

Reemplazando:

$\text{sen}(A + B) = \frac{2\tan\frac{(A + B)}{2}}{1 + \tan^2\frac{(A + B)}{2}}$

$\text{sen}(A + B) = \frac{2\frac{x}{y}}{1 + \frac{x^2}{y^2}} = \frac{2xy}{x^2 + y^2}$

Clave D

13. Por dato: A, B y C son los ángulos internos de un triángulo.

$\Rightarrow A + B + C = 180°$

Piden:

$$L = \frac{\text{senA} - \text{senC}}{\text{sen}\left(\frac{A - C}{2}\right)} \cdot \cos\frac{B}{2}$$

$$L = \frac{2\cos\left(\frac{A + C}{2}\right)\text{sen}\left(\frac{A - C}{2}\right)}{\text{sen}\left(\frac{A - C}{2}\right)} \cdot \cos\frac{B}{2}$$

$$L = 2\cos\left(\frac{A + C}{2}\right) \cdot \cos\frac{B}{2}$$

$$L = 2\cos\left(\frac{180° - B}{2}\right) \cdot \cos\frac{B}{2}$$

$$L = 2\cos\left(90° - \frac{B}{2}\right) \cdot \cos\frac{B}{2}$$

$$L = 2\left(\text{sen}\frac{B}{2}\right) \cdot \cos\frac{B}{2} = \text{sen}2\left(\frac{B}{2}\right)$$

$\therefore L = \text{senB}$

Clave A

14. Piden:

$$C = \frac{\cos 5x + \cos 3x}{\text{sen}5x + \text{sen}3x}$$

$$C = \frac{2\cos\left(\frac{5x + 3x}{2}\right)\cos\left(\frac{5x - 3x}{2}\right)}{2\text{sen}\left(\frac{5x + 3x}{2}\right)\cos\left(\frac{5x - 3x}{2}\right)}$$

$$C = \frac{2\cos 4x \cos x}{2\text{sen}4x \cos x} = \frac{\cos 4x}{\text{sen}4x}$$

$\therefore C = \cot 4x$

Clave C

PRACTIQUEMOS
Nivel 1 (página 73) Unidad 3

Comunicación matemática

1.
- $\cos 95° - \cos 15° = -2\text{sen}55°\ \text{sen}40°$
- $\cos 70° - \cos 80° = +2\text{sen}75°\ \text{sen}5°$
- $\text{sen}\frac{2\pi}{7} - \text{sen}\frac{\pi}{3} = -2\text{sen}\frac{\pi}{21}\ \cos\frac{13\pi}{21}$
- $\cos 50° + \cos 18° = 2\cos 34 \cdot \cos 16°$
- $\text{sen}20° + \cos 40° = \cos 70° + \cos 40° = 2\cos 55° \cdot \cos 15°$
- $\text{sen}50° + \cos 20° = \cos 40° + \cos 20° = 2\cos 30°\ \cos 10°$

2.
- $2\text{sen}8x \cdot \cos x = \text{sen}(8x + x) + \text{sen}(8x - x) = \text{sen}9x + \text{sen}7x$
- $2\text{sen}10\alpha \cdot \text{sen}2\alpha = \cos(10\alpha - 2\alpha) - \cos(10\alpha + 2\alpha) = \cos 8\alpha - \cos 12\alpha$
- $\text{sen}3\theta \cdot \cos 5\theta = \frac{1}{2}(\text{sen}8\theta - \text{sen}2\theta)$
- $\cos 8\beta \cdot \text{sen}5\beta = \frac{1}{2}(\text{sen}13\beta - \text{sen}3\beta)$
- $2\cos 70° \cdot \cos 25° = \cos 95° + \cos 45°$
- $\text{sen}\frac{\pi}{8} \cdot \cos\frac{\pi}{12} = \frac{1}{2}\left(\text{sen}\frac{5\pi}{24} + \text{sen}\frac{\pi}{24}\right)$

Razonamiento y demostración

3. $C = \text{sen}20° + \text{sen}24° + \text{sen}28° + \text{sen}32°$

$C = (\text{sen}32° + \text{sen}20°) + (\text{sen}28° + \text{sen}24°)$
$C = (2\text{sen}26°\cos 6°) + (2\text{sen}26°\cos 2°)$
$C = 2\text{sen}26°(\cos 6° + \cos 2°)$
$C = 2\text{sen}26°(2\cos 4°\cos 2°)$
$\therefore C = 4\text{sen}26° \cdot \cos 4° \cdot \cos 2°$

Clave D

4. Por dato: A, B y C son los ángulos internos de un triángulo.

$\Rightarrow A + B + C = 180°$

Además:
$\text{sen}^2A + \text{sen}^2B + \text{sen}^2C = m + n\cos A\cos B\cos C$
Sea: $E = \text{sen}^2A + \text{sen}^2B + \text{sen}^2C$
Luego:
$2E = 2\text{sen}^2A + 2\text{sen}^2B + 2\text{sen}^2C$
$2E = (1 - \cos 2A) + (1 - \cos 2B) + (1 - \cos 2C)$
$2E = 3 - (\cos 2A + \cos 2B + \cos 2C)$
$2E = 3 - (-4\cos A\cos B\cos C - 1)$
$2E = 4 + 4\cos A\cos B\cos C$
$E = 2 + 2\cos A\cos B\cos C$

Entonces:
$2 + 2\cos A\cos B\cos C = m + n\cos A\cos B\cos C$
Comparando: $m = 2 \ \wedge \ n = 2$
Piden:
$m^2 + n^2 = (2)^2 + (2)^2 = 4 + 4$
$\therefore m^2 + n^2 = 8$

Clave B

5. Por dato:

$$(\sec\theta + \sec 3\theta)(\csc\theta - \csc 3\theta) = \frac{m\cos^2 n\theta}{\text{sen}p\theta}$$

$$\left(\frac{1}{\cos\theta} + \frac{1}{\cos 3\theta}\right)\left(\frac{1}{\text{sen}\theta} - \frac{1}{\text{sen}3\theta}\right) = \frac{m\cos^2 n\theta}{\text{sen}p\theta}$$

$$\left(\frac{\cos 3\theta + \cos\theta}{\cos\theta\cos 3\theta}\right)\left(\frac{\text{sen}3\theta - \text{sen}\theta}{\text{sen}\theta\,\text{sen}3\theta}\right) = \frac{m\cos^2 n\theta}{\text{sen}p\theta}$$

$$\left(\frac{2\cos 2\theta\cos\theta}{\cos\theta\cos 3\theta}\right)\left(\frac{2\cos 2\theta\,\text{sen}\theta}{\text{sen}\theta\,\text{sen}3\theta}\right) = \frac{m\cos^2 n\theta}{\text{sen}p\theta}$$

$$\frac{2.(4\cos^2 2\theta)}{2.(\text{sen}3\theta\cos 3\theta)} = \frac{m\cos^2 n\theta}{\text{sen}p\theta}$$

$$\Rightarrow \frac{8\cos^2 2\theta}{\text{sen}6\theta} = \frac{m\cos^2 n\theta}{\text{sen}p\theta}$$

Comparando: $m = 8$; $n = 2$; $p = 6$
Piden:
$C = (m + n)p = (8 + 2)6 = (10)6$
$\therefore C = 60$

Clave E

6. $$C = \frac{\text{sen}14° + 2\text{sen}18° + \text{sen}22°}{\cos^2 2°}$$

$$C = \frac{(\text{sen}18° + \text{sen}14°) + (\text{sen}22° + \text{sen}18°)}{\cos^2 2°}$$

$$C = \frac{(2\text{sen}16°\cos 2°) + (2\text{sen}20°\cos 2°)}{\cos^2 2°}$$

$$C = \frac{2\text{sen}16° + 2\text{sen}20°}{\cos 2°} = \frac{2(\text{sen}16° + \text{sen}20°)}{\cos 2°}$$

$$C = \frac{2(2\text{sen}18°\cos 2°)}{\cos 2°} = 4\text{sen}18°$$

$$C = 4\left(\frac{\sqrt{5} - 1}{4}\right) = \sqrt{5} - 1$$

$\therefore C = \sqrt{5} - 1$

Clave D

7. Piden:

$$C = \frac{\text{sen}48° + \text{sen}58°}{\text{sen}85°}$$

$$C = \frac{(\text{sen}58° + \text{sen}48°)}{\text{sen}85°} = \frac{(2\text{sen}53°\cos 5°)}{\text{sen}85°}$$

$$C = \frac{2\text{sen}53°\cos 5°}{\text{sen}(90° - 5°)} = \frac{2\text{sen}53°\cos 5°}{\cos 5°}$$

$$\Rightarrow C = 2\text{sen}53° = 2\left(\frac{4}{5}\right) = \frac{8}{5}$$

$\therefore C = \frac{8}{5} = 1{,}6$

Clave E

8. Piden:

$$C = \frac{\cos 5x + \cos 3x}{\text{sen}5x + \text{sen}3x}$$

$$C = \frac{2\cos\left(\frac{5x + 3x}{2}\right)\cos\left(\frac{5x - 3x}{2}\right)}{2\text{sen}\left(\frac{5x + 3x}{2}\right)\cos\left(\frac{5x - 3x}{2}\right)}$$

$$C = \frac{2\cos 4x\cos x}{2\text{sen}4x\cos x} = \frac{\cos 4x}{\text{sen}4x}$$

$\therefore C = \cot 4x$

Clave C

9. Piden:

$$L = \frac{\cos 12° - \cos 72°}{\text{sen}72° + \text{sen}12°}$$

$$L = \frac{-2\text{sen}42°\text{sen}(-30°)}{2\text{sen}42°\cos 30°} = -\frac{\text{sen}(-30°)}{\cos 30°}$$

$$L = -\frac{(-\text{sen}30°)}{\cos 30°} = \frac{\text{sen}30°}{\cos 30°}$$

$$\Rightarrow L = \tan 30° = \frac{\sqrt{3}}{3}$$

$\therefore L = \frac{\sqrt{3}}{3}$

Clave E

10. $E = \text{sen}7x\,\text{sen}2x - \text{sen}6x\,\text{sen}3x + \cos 6x\cos 3x$

Multiplicando por 2 a la expresión y empleando la transformación de producto a suma o diferencia, tenemos:

$2E = \cos 5x - \cos 9x - (\cos 3x - \cos 9x) + (\cos 9x + \cos 3x)$
$2E = \cos 5x - \cos 9x - \cos 3x + \cos 9x + \cos 9x + \cos 3x$
$2E = \cos 9x + \cos 5x$

$$2E = 2\cos\left(\frac{9x + 5x}{2}\right)\cos\left(\frac{9x - 5x}{2}\right)$$

$2E = 2\cos 7x\cos 2x$
$\therefore E = \cos 7x\cos 2x$

Clave C

11. Por dato:

- $D(\theta) = \cos 14\theta - \cos 16\theta$
  $D(\theta) = -2\text{sen}15\theta\,\text{sen}(-\theta)$
  $D(\theta) = -2\text{sen}15\theta(-\text{sen}\theta)$
  $\Rightarrow D(\theta) = 2\text{sen}15\theta\,\text{sen}\theta$

- $d(\theta) = \cos 10\theta + \dfrac{1}{2}$

  $d(\theta) = \dfrac{2\cos 10\theta + 1}{2}$

- $q(\theta)$: cociente
- $r = 0$: residuo

Se cumple que:
$D(\theta) = d(\theta)q(\theta) + r(\theta$
Entonces:

$$2\text{sen}15\theta\,\text{sen}\theta = \left(\frac{2\cos 10\theta + 1}{2}\right).q(\theta) + 0$$

$$\Rightarrow q(\theta) = \frac{4\text{sen}15\theta\,\text{sen}\theta}{(2\cos 10\theta + 1)}$$

Por identidad de ángulo triple:
$\text{sen}15\theta = \text{sen}5\theta(2\cos 10\theta + 1)$

$$\Rightarrow q(\theta) = \frac{4\text{sen}5\theta(2\cos 10\theta + 1)\text{sen}\theta}{(2\cos 10\theta + 1)}$$

$\therefore q(\theta) = 4\text{sen}\theta\ .\ \text{sen}5\theta$

Clave C

Resolución de problemas

12. Suma de los $(n + 1)$ es:

$S = \text{sen}\beta\ .\ \sec 3\beta + \text{sen}3\beta\ .\ \sec 9\beta + \ldots + \text{sen}3^n\beta\ .\ \sec 3^{n+1}\beta$

$$S = \frac{\text{sen}\beta}{\cos 3\beta} + \frac{\text{sen}3\beta}{\cos 9\beta} + \ldots + \frac{\text{sen}3^n\beta}{\cos 3^{n+1}\beta}$$

- Analizamos al primer término:

$$\frac{\text{sen}\beta}{\cos 3\beta} = \frac{\text{sen}\beta\cos\beta}{\cos 3\beta\cos\beta} = \frac{1}{2}\left(\frac{\text{sen}2\beta}{\cos 3\beta\cos\beta}\right)$$

$$= \frac{1}{2}\,\frac{\text{sen}(3\beta - \beta)}{\cos 3\beta\cos\beta} = \frac{1}{2}(\tan 3\beta - \tan\beta)$$

- De igual forma descomponemos el resto de términos:

$$\frac{\text{sen}\beta}{\cos 3\beta} = \frac{1}{2}(\tan 3\beta - \tan\beta)$$

$$\frac{\text{sen}3\beta}{\cos 9\beta} = \frac{1}{2}(\tan 9\beta - \tan 3\beta)$$

$$\frac{\text{sen}9\beta}{\cos 27\beta} = \frac{1}{2}(\tan 27\beta - \tan 9\beta)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\frac{\text{sen}3^n\beta}{\cos 3^{n+1}\beta} = \frac{1}{2}(\tan 3^{n+1}\beta - \tan 3^n\beta)$$

$$\Rightarrow S = \frac{1}{2}(\tan 3^{n+1}\beta - \tan\beta)$$

Clave C

13. Tenemos:

$a = \text{sen}5\alpha + \text{sen}3\alpha = 2\text{sen}4\alpha\ .\ \cos\alpha$

$b = \cos 3\alpha - \cos 5\alpha = 2\text{sen}4\alpha\ .\ \text{sen}\alpha$

$a^2 + b^2 = 4[\text{sen}^2 4\alpha(\cos^2\alpha + \text{sen}^2\alpha)]$

$a^2 + b^2 = 4\text{sen}^2 4\alpha$

$a^2 + b^2 = 4(2\text{sen}2\alpha\ .\ \cos 2\alpha)^2$

$a^2 + b^2 = 16\text{sen}^2 2\alpha\ .\ \cos^2 2\alpha\ \ldots (3)$

$\dfrac{a}{b} = \cot\alpha \Rightarrow \dfrac{b}{a} = \tan\alpha$

$$\text{sen}2\alpha = \frac{2\tan\alpha}{1 + \tan^2\alpha} = \frac{2\left(\frac{b}{a}\right)}{1 + \left(\frac{b}{a}\right)^2} = \frac{2ab}{a^2 + b^2}$$

$$\cos 2\alpha = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha} = \frac{1 - \left(\frac{b}{a}\right)^2}{1 + \left(\frac{b}{a}\right)^2} = \frac{a^2 - b^2}{a^2 + b^2}$$

- Reemplazamos en (3):

$$a^2 + b^2 = 16\left(\frac{2ab}{a^2 + b^2}\right)^2 . \left(\frac{a^2 - b^2}{a^2 + b^2}\right)^2$$

$$(a^2 + b^2)^5 = 64a^2b^2(a^2 - b^2)^2$$

Clave C

Nivel 2 (página 73) Unidad 3

Comunicación matemática

14. Sabemos que: $A + B + C = 180°$

- $\cos^2 A + \cos^2 B + \cos^2 C$
  $= 1 - 2\boxed{\cos A\cos B\cos C}$

- $\text{sen}^2\dfrac{A}{2} + \text{sen}^2\dfrac{B}{2} + \text{sen}^2\dfrac{C}{2}$

  $= 1 + \boxed{-2\text{sen}\dfrac{A}{2}\ \text{sen}\dfrac{B}{2}\ \text{sen}\dfrac{C}{2}}$

- $\cos 2A + \cos 2B - \cos 2C$
  $= -4\text{sen}A\ .\ \text{sen}B\ .\ \text{sen}C + \boxed{1}$

- $\text{sen}2A + \text{sen}2B + \text{sen}2C$
  $= 4\ \boxed{\text{sen}A\ \text{sen}B\ \text{sen}C}$

- Si: $A = 90°$

$$\tan B = \frac{\cos(B - C)}{\boxed{1} + \sec(C - B)}$$

15.

- $\text{sen}(a + b + c) + \text{sen}(a + b - c)$
  $= 2\text{sen}(a + b)\cos c$ (F)

- $2\text{sen}B\ .\ \text{sen}C = \cos(B - C) - \cos(B + C)$
  $= \cos(B - C) - (-\cos A)$
  $= \cos(B - C) + \cos A$

$$\Rightarrow \cos(B - C) + \cos A = 2\,\cancel{\text{sen}\frac{A}{2}}\cos\frac{A}{2}\left(\frac{\cos\frac{A}{2}}{\cancel{\text{sen}\frac{A}{2}}}\right)$$

$$\cos(B - C) + \cos A = 2\cos^2\frac{A}{2}$$

$\cos(B - C) + \cos A = 1 + \cos A$
$\cos(B - C) = 1$
$B = C \Rightarrow B - C = 0$
$\cos 0° = 1$ (V)

- $S = \underbrace{\cos^2\theta + \cos^2 2\theta + \cos^2 3\theta + \ldots}_{\text{"n" términos}}$

$2S = 2\cos^2\theta + 2\cos^2 2\theta + \ldots + 2\cos^2 2\theta$

$2S = 1 + \cos 2\theta + 1 + \cos 4\theta + \ldots + 1 + \cos 22\theta$

$2S = n + \underbrace{\cos 2\theta + \cos 4\theta + \cos 2n\theta}_{\text{razón} \Rightarrow r = 2\theta}$

\# de términos = n

$$2S = n + \frac{\text{sen}\left(\frac{n - 2\theta}{2}\right)}{\text{sen}\left(\frac{2\theta}{2}\right)} \times \cos\left(\frac{2\theta + 2n\theta}{2}\right)$$

$$2S = n + \frac{\text{sen}\,n\theta}{\text{sen}\theta} \times \cos(\theta + n\theta)$$

$$S = n + \frac{\text{sen}\,n\theta \times \cos(n + 1)\theta}{2\text{sen}\theta} \qquad (V)$$

Clave D

Razonamiento y demostración

16. $E = \sqrt{1 + \cos\dfrac{\alpha}{2}} + \sqrt{2} + \sqrt{2}\,\text{sen}\dfrac{\alpha}{4}$

$$E = \sqrt{\left(2\cos^2\frac{\alpha}{4}\right)} + \sqrt{2} + \sqrt{2}\,\text{sen}\frac{\alpha}{4}$$

$$E = \sqrt{2}\left|\cos\frac{\alpha}{4}\right| + \sqrt{2}\,\text{sen}\frac{\alpha}{4} + \sqrt{2}$$

Por dato: $\alpha \in \langle 0; 2\pi\rangle$

$\Rightarrow \dfrac{\alpha}{4} \in \left\langle 0; \dfrac{\pi}{2}\right\rangle \Rightarrow \dfrac{\alpha}{4} \in \text{IC}$

$\Rightarrow \cos\dfrac{\alpha}{4} > 0 \Rightarrow \left|\cos\dfrac{\alpha}{4}\right| = \cos\dfrac{\alpha}{4}$

Luego:

$$E = \sqrt{2}\cos\frac{\alpha}{4} + \sqrt{2}\,\text{sen}\frac{\alpha}{4} + \sqrt{2}$$

$$E = \sqrt{2}\left[\cos\frac{\alpha}{4} + \text{sen}\frac{\alpha}{4}\right] + \sqrt{2}$$

$$E = \sqrt{2}\left[\sqrt{2}\,\text{sen}\left(\frac{\pi}{4} + \frac{\alpha}{4}\right)\right] + \sqrt{2}$$

$$E = 2\text{sen}\left(\frac{\pi}{4} + \frac{\alpha}{4}\right) + 2\text{sen}\frac{\pi}{4}$$

$$E = 2\left[\text{sen}\left(\frac{\pi}{4} + \frac{\alpha}{4}\right) + \text{sen}\frac{\pi}{4}\right]$$

$$E = 2\left[2\text{sen}\left(\frac{\pi}{4} + \frac{\alpha}{8}\right).\ \cos\frac{\alpha}{8}\right]$$

$$\therefore E = 4\text{sen}\left(\frac{\pi}{4} + \frac{\alpha}{8}\right)\cos\frac{\alpha}{8}$$

Clave C

17. Piden:

$$P = \frac{\text{sen}6\beta - \text{sen}2\beta}{\cos 2\beta - \cos 6\beta} + \frac{\text{sen}14\beta - \text{sen}10\beta}{\cos 14\beta + \cos 10\beta}$$

$$P = \frac{2\cos 4\beta\,\text{sen}2\beta}{-2\text{sen}4\beta\,\text{sen}(-2\beta)} + \frac{2\cos 12\beta\,\text{sen}2\beta}{2\cos 12\beta\cos 2\beta}$$

$$P = \frac{\cos 4\beta\,\text{sen}2\beta}{-\text{sen}4\beta(-\text{sen}2\beta)} + \frac{\text{sen}2\beta}{\cos 2\beta}$$

$$P = \frac{\cos 4\beta \operatorname{sen} 2\beta}{\operatorname{sen} 4\beta \operatorname{sen} 2\beta} + \tan 2\beta$$

$P = \cot 4\beta + \tan 2\beta$

$P = \cot 4\beta + (\csc 4\beta - \cot 4\beta)$

$P = \csc 4\beta$

Como $\beta = \frac{\pi}{40}$, entonces:

$$P = \csc 4\left(\frac{\pi}{40}\right) = \csc \frac{\pi}{10} = \csc 18°$$

$$P = \frac{1}{\operatorname{sen} 18°} = \frac{1}{\left(\frac{\sqrt{5}-1}{4}\right)} = \frac{4}{\sqrt{5}-1}$$

$$P = \frac{4}{(\sqrt{5}-1)} \cdot \frac{(\sqrt{5}+1)}{(\sqrt{5}+1)} = \frac{4(\sqrt{5}+1)}{4}$$

$\therefore P = \sqrt{5} + 1$

Clave E

18. Por dato x e y son ángulos agudos y complementarios.

$\Rightarrow x + y = 90°$ ...(I)

Además:

$\sqrt{2}\operatorname{sen}(x-y) = \operatorname{sen}38° + \operatorname{sen}22° + \operatorname{sen}8°$

$\sqrt{2}\operatorname{sen}(x-y) = 2\operatorname{sen}30°\cos 8° + \operatorname{sen}8°$

$\sqrt{2}\operatorname{sen}(x-y) = 2\left(\frac{1}{2}\right)\cos 8° + \operatorname{sen}8°$

$\sqrt{2}\operatorname{sen}(x-y) = \cos 8° + \operatorname{sen}8°$

$\sqrt{2}\operatorname{sen}(x-y) = \operatorname{sen}82° + \operatorname{sen}8°$

$\sqrt{2}\operatorname{sen}(x-y) = 2\operatorname{sen}45°\cos 37°$

$\sqrt{2}\operatorname{sen}(x-y) = 2\left(\frac{\sqrt{2}}{2}\right)\cos 37°$

$\operatorname{sen}(x-y) = \cos 37° = \operatorname{sen}53°$

Entonces: $x - y = 53°$ ...(II)

De (I) y (II): $x = \frac{143°}{2} \wedge y = \frac{37°}{2}$

Piden:

$$2x + 4y = 2\left(\frac{143°}{2}\right) + 4\left(\frac{37°}{2}\right)$$

$\Rightarrow 2x + 4y = 143° + 74° = 217°$

$\therefore 2x + 4y = 217°$

Clave C

19. Sea:

$$H = \frac{\cos 21°(\cos 21° + \cos 147°)}{\cos 69°(\cos 21° - \cos 147°)}$$

$$H = \frac{\cos 21°(2\cos 84° \,.\, \cos 63°)}{\cos 69°(-2\operatorname{sen}84° \,.\, \operatorname{sen}(-63°))}$$

$$H = -\frac{2\cos 21° \,.\, \cos 63° \,.\, \cos 84°}{2\cos 69° \,.\, \operatorname{sen}84° \,.\, (-\operatorname{sen}63°)}$$

Pero: $\cos 69° = \operatorname{sen}21°$

$$\Rightarrow H = \frac{\cos 21° \,.\, \cos 63° \,.\, \cos 84°}{\operatorname{sen}21° \,.\, \operatorname{sen}63° \,.\, \operatorname{sen}84°}$$

$\Rightarrow H = \cot 21° \,.\, \cot 63° \,.\, \cot 84°$

$\Rightarrow H = \cot 21° \,.\, \cot 3(21°) \,.\, \cot 4(21°)$ ...(1)

Por dato:

$$\frac{\cos 21°(\cos 21° + \cos 147°)}{\cos 69°(\cos 21° - \cos 147°)} = \cot x \,.\, \cot 3x \,.\, \cot 4x$$

$\Rightarrow H = \cot x \,.\, \cot 3x \,.\, \cot 4x$ ...(2)

Comparando(1) y (2):

$\therefore x = 21°$

Clave C

20. Por dato: θ es agudo.

Además:

$\tan 60° \,.\, \operatorname{sen}\theta = \operatorname{sen}35° + \operatorname{sen}25° + \cos 55°$

$\tan 60° \,.\, \operatorname{sen}\theta = 2\operatorname{sen}30° \,.\, \cos 5° + \cos 55°$

$\tan 60° \,.\, \operatorname{sen}\theta = 2\left(\frac{1}{2}\right)\cos 5° + \cos 55°$

$\tan 60° \,.\, \operatorname{sen}\theta = \cos 5° + \cos 55°$

$\tan 60° \,.\, \operatorname{sen}\theta = 2\cos 30° \,.\, \cos 25°$

$(\sqrt{3}) \,.\, \operatorname{sen}\theta = 2\left(\frac{\sqrt{3}}{2}\right)\cos 25°$

$\operatorname{sen}\theta = \cos 25°$

$\Rightarrow \operatorname{sen}\theta = \operatorname{sen}65°$

$\therefore \theta = 65°$

Clave D

21. Sea:

$$E = \operatorname{sen}22° + \frac{\sqrt{2}}{5}\operatorname{sen}14° - \operatorname{sen}6°$$

$$E = (\operatorname{sen}22° - \operatorname{sen}6°) + \frac{\sqrt{2}}{5}\operatorname{sen}14°$$

$$E = (2\cos 14° \,.\, \operatorname{sen}8°) + \frac{\sqrt{2}}{5}\operatorname{sen}14°$$

Luego:

$$\operatorname{sen}8° = \sqrt{\frac{1 - \cos 16°}{2}} = \sqrt{\frac{1 - \left(\frac{24}{25}\right)}{2}}$$

$$\operatorname{sen}8° = \sqrt{\frac{1}{50}} = \frac{1}{5\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\Rightarrow \operatorname{sen}8° = \frac{\sqrt{2}}{10}$$

Reemplazando en E:

$$E = 2\cos 14° \,.\, \frac{\sqrt{2}}{10} + \frac{\sqrt{2}}{5}\operatorname{sen}14°$$

$$E = \frac{\sqrt{2}}{5}\cos 14° + \frac{\sqrt{2}}{5}\operatorname{sen}14°$$

$$E = \frac{\sqrt{2}}{5}(\cos 14° + \operatorname{sen}14°)$$

Pero: $\operatorname{sen}14° = \cos 76°$

Entonces:

$$E = \frac{\sqrt{2}}{5}(\cos 14° + \cos 76°)$$

$$E = \frac{\sqrt{2}}{5}(2\cos 45° \,.\, \cos 31°)$$

$$E = \frac{\sqrt{2}}{5}\left(2 . \frac{\sqrt{2}}{2} \,.\, \cos 31°\right)$$

$$E = \frac{2}{5}\cos 31°$$

$$\therefore \operatorname{sen}22° + \frac{\sqrt{2}}{5}\operatorname{sen}14° - \operatorname{sen}6° = \frac{2}{5}\cos 31°$$

Clave D

22. Piden:

$$A = \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$$

Por series trigonométricas:

Primer ángulo: $P = \frac{2\pi}{7}$

Último ángulo: $U = \frac{6\pi}{7}$

n.° de términos: $n = 3$

Razón: $r = \frac{2\pi}{7}$

$$A = \frac{\operatorname{sen}\left(\frac{nr}{2}\right)}{\operatorname{sen}\left(\frac{r}{2}\right)} \,.\, \cos\left(\frac{P+U}{2}\right)$$

Luego:

$$A = \frac{\operatorname{sen}\left(\frac{3.2\pi}{2.7}\right)}{\operatorname{sen}\left(\frac{2\pi}{2.7}\right)} \,.\, \cos\left(\frac{\frac{2\pi}{7} + \frac{6\pi}{7}}{2}\right)$$

$$A = \frac{\operatorname{sen}\frac{3\pi}{7}}{\operatorname{sen}\frac{\pi}{7}} \,.\, \cos\frac{4\pi}{7}$$

Pero: $\cos\frac{4\pi}{7} = -\cos\frac{3\pi}{7}$

$$\Rightarrow A = -\frac{\operatorname{sen}\frac{3\pi}{7} \,.\, \cos\frac{3\pi}{7}}{\operatorname{sen}\frac{\pi}{7}}$$

$$A = -\frac{2\operatorname{sen}\frac{3\pi}{7}\cos\frac{3\pi}{7}}{2\operatorname{sen}\frac{\pi}{7}} = -\frac{\operatorname{sen}\frac{6\pi}{7}}{2\operatorname{sen}\frac{\pi}{7}}$$

$$A = -\frac{\operatorname{sen}\left(\pi - \frac{\pi}{7}\right)}{2\operatorname{sen}\frac{\pi}{7}} = -\frac{\operatorname{sen}\frac{\pi}{7}}{2\operatorname{sen}\frac{\pi}{7}}$$

$$\therefore A = -\frac{1}{2}$$

Clave E

Resolución de problemas

23. Tenemos:

$$\operatorname{sen}(x+a)\operatorname{sen}(x+b) = \frac{1}{2}[\cos(x+a-x-b) - \cos(x+a+x+b)]$$

$$\cos(a-b) = \frac{1}{2}[\cos(a-b) - \cos(2x+a+b)]$$

$$-\cos(a-b) = \cos(2x+a+b)$$

$L = \cos(x+a) \,.\, \cos(x+b)$

$$L = \frac{1}{2}[\cos(x+a-x-b) + \cos(x+a+x+b)]$$

$$L = \frac{1}{2}[\cos(a-b) + \cos(2x+a+b)]$$

$L = \frac{1}{2}[\cos(a-b) - \cos(a-b)]$

$L = \frac{1}{2}(0) = 0$

Clave B

24. $2P = 2\text{sen}\frac{3\alpha}{2}\ \text{sen}\frac{\alpha}{2} + 2\cos^2\alpha$

$2P = \cos\alpha - \cos 2\alpha + 1 + \cos 2\alpha$

$2P = 1 + \cos\alpha = 2\cos^2\frac{\alpha}{2}$

$P = \cos^2\frac{\alpha}{2}$

Clave C

Nivel 3 (página 74) Unidad 3

Comunicación matemática

25.

I. $\text{sen}(2x + 10°)\ .\ \text{sen}(20° - 2x)$

$\frac{1}{2}[\cos(2x + 10° - 20° + 2x) - \cos(2x + 10° + 20° - 2x)]$

$\frac{1}{2}[\cos(4x - 10°) - \cos 30°]$

$\frac{1}{2}[\cos(4x - 10°) - \frac{\sqrt{3}}{2}]$

$-1 \leq \cos(4x - 10°) \leq 1$

$-1 - \frac{\sqrt{3}}{2} \leq \cos(4x - 10°) - \frac{\sqrt{3}}{2} \leq 1 - \frac{\sqrt{3}}{2}$

$\text{Máx.} = \frac{1}{2} - \frac{\sqrt{3}}{4}$ (V)

II. $2\text{sen}(3x + y)\ .\ \text{sen}(3x - y) - 2\text{sen}(x + y)\ .\ \text{sen}(x - y)$

$\cos 2y - \cos 6x - (\cos 2y - \cos 2x)$

$\cos 2x - \cos 6x = 2\text{sen}4x\ .\ \text{sen}2x$

(F)

III. $(\cos 3x - \text{sen}4x)^2 = (\text{sen}(90° - 3x) - \text{sen}4x)^2$

$\left[2\text{sen}\left(\frac{90° - 7x}{2}\right)\cos\left(\frac{90° + x}{2}\right)\right]^2$

$4\text{sen}^2\left(\frac{90° - 7x}{2}\right)\cos^2\left(\frac{90° + x}{2}\right)$

$4\left(\frac{1 - \cos(90° - 7x)}{2}\right)\left(\frac{1 + \cos(90° + x)}{2}\right)$

$(1 - \text{sen}7x)(1 - \text{sen}x)$ (V)

IV. $\cos 20° + \cos 100° + \cos 140°$

$\cos 20° + 2\cos 120°\cos 20°$

$\cos 20° + 2(-\cos 60°)\cos 20°$

$\cos 20° + 2(-1/2)\cos 20°$

$\therefore\ 0$ (F)

Clave E

Razonamiento y demostración

26. Por dato A, B y C son los ángulos internos de un triángulo.

$\Rightarrow A + B + C = 180° = \pi$ rad

Sea: $P = \text{sen}A + \text{sen}B - \text{sen}C$

Luego:

$P = 2\text{sen}\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) - \text{sen}C$

$P = 2\text{sen}\left(\frac{\pi - C}{2}\right)\cos\left(\frac{A-B}{2}\right) - \text{sen}C$

$P = 2\text{sen}\left(\frac{\pi}{2} - \frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) - \text{sen}C$

$P = 2\cos\frac{C}{2}\cos\left(\frac{A-B}{2}\right) - 2\text{sen}\frac{C}{2}\cos\frac{C}{2}$

$P = 2\cos\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) - \text{sen}\frac{C}{2}\right]$

Pero: $\text{sen}\frac{C}{2} = \cos\left(\frac{A+B}{2}\right)$

$\Rightarrow P = 2\cos\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right]$

$P = 2\cos\frac{C}{2}\left[-2\text{sen}\frac{A}{2}\text{sen}\left(-\frac{B}{2}\right)\right]$

$P = 2\cos\frac{C}{2}\left[-2\text{sen}\frac{A}{2}\left(-\text{sen}\frac{B}{2}\right)\right]$

$P = 4\text{sen}\frac{A}{2}\text{sen}\frac{B}{2}\cos\frac{C}{2}$

Piden:

$N = \dfrac{\text{sen}A + \text{sen}B - \text{sen}C}{\text{sen}\frac{A}{2}\text{sen}\frac{B}{2}\text{sen}\frac{C}{2}}$

$N = \dfrac{P}{\text{sen}\frac{A}{2}\text{sen}\frac{B}{2}\text{sen}\frac{C}{2}}$

$N = \dfrac{4\text{sen}\frac{A}{2}\text{sen}\frac{B}{2}\cos\frac{C}{2}}{\text{sen}\frac{A}{2}\text{sen}\frac{B}{2}\text{sen}\frac{C}{2}}$

$N = 4\left(\dfrac{\cos\frac{C}{2}}{\text{sen}\frac{C}{2}}\right) = 4\cot\frac{C}{2}$

$\therefore N = 4\cot\frac{C}{2}$

Clave E

27.

$N = \dfrac{\cos 3\alpha + \sqrt{103}\cos 2\alpha + \cos\alpha}{\text{sen}3\alpha + \sqrt{103}\,\text{sen}2\alpha + \text{sen}\alpha}$

$N = \dfrac{(\cos 3\alpha + \cos\alpha) + \sqrt{103}\cos 2\alpha}{(\text{sen}3\alpha + \text{sen}\alpha) + \sqrt{103}\,\text{sen}2\alpha}$

$N = \dfrac{\cos 2\alpha(2\cos\alpha + \sqrt{103})}{\text{sen}2\alpha(2\cos\alpha + \sqrt{103})}$

$\Rightarrow N = \dfrac{\cos 2\alpha}{\text{sen}2\alpha} = \cot 2\alpha$

$\therefore N = \cot 2\alpha$

Clave E

28. Por dato: $3\text{sen}x = \text{sen}(x + 2\theta)$

Sumando (senx) en ambos miembros de la igualdad:

$3\text{sen}x + \text{sen}x = \text{sen}(x + 2\theta) + \text{sen}x$

$4\text{sen}x = 2\text{sen}(x + \theta)\ .\ \cos\theta$

$\Rightarrow 2\text{sen}x = \text{sen}(x + \theta)\ .\ \cos\theta$ ...(I)

Restando (senx) en ambos miembros de la igualdad:

$3\text{sen}x - \text{sen}x = \text{sen}(x + 2\theta) - \text{sen}x$

$2\text{sen}x = 2\cos(x + \theta)\ .\ \text{sen}\theta$

$\Rightarrow \text{sen}x = \cos(x + \theta)\ .\ \text{sen}\theta$ ...(II)

Dividiendo (I) y (II), tenemos:

$\dfrac{2\text{sen}x}{\text{sen}x} = \dfrac{\text{sen}(x + \theta)\ .\ \cos\theta}{\cos(x + \theta)\ .\ \text{sen}\theta}$

$2 = \tan(x + \theta)\ .\ \cot\theta$

$2 = \tan(x + \theta)\ .\ \left(\dfrac{1}{\tan\theta}\right)$

$\therefore \dfrac{\tan(x + \theta)}{\tan\theta} = 2$

Clave A

29. Por dato: $2\cos x = \cos(x + 2\theta)$

Sumando (cosx) en ambos miembros de la igualdad:

$2\cos x + \cos x = \cos(x + 2\theta) + \cos x$

$3\cos x = 2\cos(x + \theta)\ .\ \cos\theta$

$\Rightarrow \frac{3}{2}\cos x = \cos(x + \theta)\ .\ \cos\theta$ ...(I)

Restando (cosx) en ambos miembros de la igualdad:

$2\cos x - \cos x = \cos(x + 2\theta) - \cos x$

$\cos x = -2\text{sen}(x + \theta)\ .\ \text{sen}\theta$

$\Rightarrow -\frac{1}{2}\cos x = \text{sen}(x + \theta)\ .\ \text{sen}\theta$ ...(II)

Dividiendo (I) y (II), tenemos:

$\dfrac{\frac{3}{2}\cos x}{-\frac{1}{2}\cos x} = \dfrac{\cos(x + \theta)\ .\ \cos\theta}{\text{sen}(x + \theta)\ .\ \text{sen}\theta}$

$-3 = \cot(x + \theta)\ .\ \cot\theta$

$\therefore \cot(x + \theta)\cot\theta = -3$

Clave A

30. $\dfrac{\text{sen}\theta - \text{sen}2\theta + \text{sen}3\theta + \text{sen}4\theta}{\text{sen}2\theta} = A + B\cos\theta + C\cos 2\theta$

Sea:

$H = \dfrac{\text{sen}\theta - \text{sen}2\theta + \text{sen}3\theta + \text{sen}4\theta}{\text{sen}2\theta}$

$H = \dfrac{(\text{sen}3\theta + \text{sen}\theta) + (\text{sen}4\theta - \text{sen}2\theta)}{\text{sen}2\theta}$

$H = \dfrac{(2\text{sen}2\theta\cos\theta) + (2\cos 3\theta\,\text{sen}\theta)}{\text{sen}2\theta}$

$H = \dfrac{2\text{sen}2\theta\cos\theta}{\text{sen}2\theta} + \dfrac{2\cos 3\theta\,\text{sen}\theta}{\text{sen}2\theta}$

$H = 2\cos\theta + \dfrac{2\cos 3\theta\,\text{sen}\theta}{2\text{sen}\theta\cos\theta}$

$H = 2\cos\theta + \dfrac{\cos 3\theta}{\cos\theta}$

$H = 2\cos\theta + \dfrac{\cos\theta(2\cos 2\theta - 1)}{\cos\theta}$

$\Rightarrow H = -1 + 2\cos\theta + 2\cos 2\theta$

Del enunciado:

$H = A + B\cos\theta + C\cos 2\theta$

Entonces:

$-1 + 2\cos\theta + 2\cos2\theta = A + B\cos\theta + C\cos2\theta$

Comparando:

$A = -1;\ B = 2;\ C = 2$

Piden:

$A + B + C = (-1) + (2) + (2) = 3 \qquad \therefore A + B + C = 3$

Clave E

31. Por dato: A, B y C son los ángulos internos de un triángulo.

$\Rightarrow A + B + C = 180° = \pi$ rad

Por propiedad:

$\text{sen}2A + \text{sen}2B + \text{sen}2C = 4\text{sen}A\text{sen}B\text{sen}C$

Sea: $H = \text{sen}2A - \text{sen}2B + \text{sen}2C$

$H = 2\cos\left(\dfrac{2A + 2B}{2}\right)\text{sen}\left(\dfrac{2A - 2B}{2}\right) + \text{sen}2C$

$H = 2\cos(A + B)\text{sen}(A - B) + \text{sen}2C$

$H = 2\cos(\pi - C)\text{sen}(A - B) + \text{sen}2C$

$H = 2(-\cos C)\text{sen}(A - B) + 2\text{sen}C\cos C$

$H = 2\cos C[\text{sen}C - \text{sen}(A - B)]$

Pero: $\text{sen}C = \text{sen}(A + B)$

$\Rightarrow H = 2\cos C[\text{sen}(A + B) - \text{sen}(A - B)]$

$\Rightarrow H = 2\cos C[2\cos A\text{sen}B]$

$\Rightarrow H = 4\cos A\text{sen}B\cos C$

Piden:

$L = \dfrac{\text{sen}2A + \text{sen}2B + \text{sen}2C}{\text{sen}2A - \text{sen}2B + \text{sen}2C}$

$\Rightarrow L = \dfrac{4\text{sen}A\text{sen}B\text{sen}C}{H}$

$L = \dfrac{4\text{sen}A\text{sen}B\text{sen}C}{4\cos A\text{sen}B\cos C}$

$L = \left(\dfrac{\text{sen}A}{\cos A}\right) . \left(\dfrac{\text{sen}C}{\cos C}\right) = (\tan A) . (\tan C)$

$\therefore L = \tan A . \tan C$

Clave C

32. $L = \cos4x + \cos8x + \cos12x + \cos16x$

$L = (\cos8x + \cos4x) + (\cos16x + \cos12x)$

$L = (2\cos6x\cos2x) + (2\cos14x\cos2x)$

$L = 2\cos2x(\cos14x + \cos6x)$

$L = 2\cos2x(2\cos10x\cos4x) \qquad \therefore L = 4\cos10x . \cos4x . \cos2x$

Clave B

33. $2\text{sen}^2\alpha + 2\cos^2(x - \alpha) + 2\text{sen}^2(x + \alpha) = 4$

$1 - \cos2\alpha + 1 + \cos(2x - 2\alpha) + 1 - \cos(2x + 2\alpha) = 4$

$\cos(2x - 2\alpha) - \cos(2x + 2\alpha) = 1 + \cos2\alpha$

$2\text{sen}2x . \text{sen}2\alpha = 1 + \cos2\alpha$

$\text{sen}2x = \dfrac{1 + \cos2\alpha}{2\text{sen}2\alpha}$

$\text{sen}2x = \dfrac{1}{2}\left(\dfrac{1 + \cos2\alpha}{\text{sen}2\alpha}\right)$

$\text{sen}2x = \dfrac{1}{2}\cot\alpha$

Clave A

34. Sea:

$M = \text{sen}^2\dfrac{\pi}{9} + \text{sen}^2\dfrac{2\pi}{9} + \text{sen}^2\dfrac{4\pi}{9}$

$2M = 2\text{sen}^2\dfrac{\pi}{9} + 2\text{sen}^2\dfrac{2\pi}{9} + 2\text{sen}^2\dfrac{4\pi}{9}$

$2M = 1 - \cos\dfrac{2\pi}{9} + 1 - \cos\dfrac{4\pi}{9} + 1 - \cos\dfrac{8\pi}{9}$

$2M = 3 - \cos\dfrac{2\pi}{9} - \left[\cos\dfrac{4\pi}{9} + \cos\dfrac{8\pi}{9}\right]$

$2M = 3 - \cos\dfrac{2\pi}{9} - 2\cos\dfrac{2\pi}{3}\cos\dfrac{2\pi}{9}$

$2M = 3 - \cos\dfrac{2\pi}{9} - 2\left(\dfrac{-1}{2}\right)\cos\dfrac{2\pi}{9}$

$2M = 3 - \cos\dfrac{2\pi}{9} + \cos\dfrac{2\pi}{9}$

$2M = 3$

$\therefore M = \dfrac{3}{2}$

Clave D

MARATÓN MATEMÁTICA (página 75) Unidad 3

1. $\dfrac{k + \cos\alpha}{\text{sen}\alpha} = \dfrac{\text{sen}\alpha}{1 - \cos\alpha}$

$k - \cos^2\alpha - k\cos\alpha + \cos\alpha = \text{sen}^2\alpha$

$k(1 - \cos\alpha) + \cos\alpha = \text{sen}^2\alpha + \cos^2\alpha$

$k(1 - \cos\alpha) = 1 - \cos\alpha$

$\therefore k = 1$

Clave A

2. Por condición:

$\cot\beta + \tan\beta = k$

$\dfrac{\cos\beta}{\text{sen}\beta} + \dfrac{\text{sen}\beta}{\cos\beta} = k \Rightarrow \dfrac{\cos^2\beta + \text{sen}^2\beta}{\text{sen}\beta . \cos\beta} = k$

$\csc\beta . \sec\beta = k$

$\text{sen}\beta . \cos\beta = 1/k$

Nos piden:

$(\text{sen}\beta + \cos\beta)^2 = \text{sen}^2\beta + 2\text{sen}\beta\cos\beta + \cos^2\beta$

$= 1 + 2\text{sen}\beta . \cos\beta$

$= 1 + 2/k = \dfrac{k + 2}{k}$

Clave E

3. $M = \cot40° + \sqrt{3}\tan10° . \cot40° + \tan10°$

$M = \tan50° + \tan10° + \tan60° . \tan10° . \tan50°$

$M = \tan50° + \tan10° + \tan(50° + 10°) . \tan10° . \tan50°$

$M = \tan50° + \tan10° + \dfrac{\tan50° + \tan10°}{1 - \tan50°\tan10°} \times \tan10° . \tan50°$

Factorizamos: $(\tan50° + \tan10°)$.

$M = (\tan50° + \tan10°)\left(1 + \dfrac{\tan50° . \tan10°}{1 - \tan50° . \tan10°}\right)$

$M = (\tan50° + \tan10°)\left(\dfrac{1 - \tan50° . \tan10° + \tan50° . \tan10°}{1 - \tan50° . \tan10°}\right)$

$M = \dfrac{\tan50° + \tan10°}{1 - \tan50° . \tan10°} = \tan(50° + 10°)$

$\therefore M = \tan60° = \sqrt{3}$

Clave B

4. Del dato tenemos:

$\dfrac{\tan 2x}{\tan x} = \sec 2x + 1 \ \wedge \ \tan 2x \ . \ \tan x = \sec 2x - 1$

$\underbrace{\sec 2x + 1 - \sec 2^2 x - 1 - \sec 2^3 x - 1 \ ...}_{\text{“10” términos}} \ = k$

$\sec 2x - \sec 2^2 x - \sec 2^3 x - \ ... \ - \sec 2^{10} x = k + 8$

$M = \sec 2x - 1 - (\sec 2^2 x - 1) - (\sec 2^3 x - 1) - \ ... \ - (\sec 2^{10} x - 1)$

$M = (\sec 2x - \sec 2^2 x - \sec 2^3 x - \ ... \ - \sec 2^{10} x - 1) + 8$

$M = k + 8 + 8$

$\therefore \ M = k + 16$

Clave C

5. $k = \dfrac{sen3\alpha}{\dfrac{sen\alpha}{\cos\alpha}} - 2\cos\alpha$

$k = (2\cos 2\alpha + 1)\cos\alpha - 2\cos\alpha$

$k = \cos\alpha(2\cos 2\alpha + 1 - 2)$

$k = \cos\alpha(2\cos 2\alpha - 1)$

$k = \cos 3\alpha$

Clave B

6. $A = \cos^2 25° + sen^2 5° - sen5° \ . \ \cos 25°$

$2A = 2\cos^2 25° + 2sen^2 5° - 2sen5° \ . \ \cos 25°$

$2A = 1 + \cos 50° + 1 - \cos 10° - sen30° + sen20°$

$2A = 2 + \cos 50° - \cos 10° - 1/2 + sen20°$

$2A = 3/2 - 2\cos 30° \ . \ sen20° + sen20°$

$2A = 3/2 - 2(1/2)sen20° + sen20°$

$\therefore \ 2A = 3/2 \ \Rightarrow \ A = 3/4$

Clave E

7. $M = sen(30° + x) - sen(30° - x)$

Transformamos a producto:

$M = 2\cos 30° senx = \sqrt{3} \ senx$

$N = sen(60° + x) - sen(60° - x)$

Transformamos a producto:

$N = 2\cos 60° senx$

Luego tenemos:

$M \times N = \sqrt{3} \ senx \ . \ senx = \sqrt{3} \ sen^2 x$

Clave D

8. $k = \dfrac{8sen^2 x + sen^2 2x}{4senx} - 3sen^3 x = \dfrac{8sen^2 x}{4senx} + \dfrac{sen^2 2x}{4senx} - 3sen^3 x$

$k = 2senx + \dfrac{4sen^2 x\cos^2 x}{4senx} - 3sen^3 x$

$k = 2senx + senx\cos^2 x - 3sen^3 x$

$k = senx(2 + \cos^2 x) - 3sen^3 x = senx(3 - sen^2 x) - 3sen^3 x$

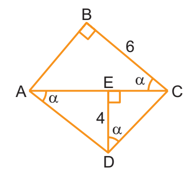
$k = 3senx - sen^3 x - 3sen^3 x$

$k = 3senx - 4sen^3 x$

$k = sen3x$

Clave B

9.



$AC = AE + EC$

$AC = 6\sec\alpha = 4\cot\alpha + 4\tan\alpha$

$6\sec\alpha = 4(\tan\alpha + \cot\alpha)$

$6\sec\alpha = 4\csc\alpha \ . \ \sec\alpha$

$sen\alpha = 4/6 \ \Rightarrow \ sen\alpha = 2/3$

$\therefore \ 1 - sen\alpha = 1/3$

Clave A

APLICAMOS LO APRENDIDO
Nivel 1 (página 78) Unidad 4

1. $f(x) = 7\cos^2x + 2$

Sabemos:

$-1 \le \cos x \le 1$

$0 \le \cos^2 x \le 1$

$0 \le 7\cos^2 x \le 7$

$2 \le \underbrace{7\cos^2 x + 2}_{f(x)} \le 9$

$\Rightarrow 2 \le f(x) \le 9$

Por lo tanto, el rango de f es: [2; 9]

Clave E

2. $f(x) = 5|\operatorname{sen}x| + 6$

Sabemos:

$-1 \le \operatorname{sen}x \le 1$

$0 \le |\operatorname{sen}x| \le 1$

$0 \le 5|\operatorname{sen}x| \le 5$

$6 \le \underbrace{5|\operatorname{sen}x| + 6}_{f(x)} \le 11$

$\Rightarrow 6 \le f(x) \le 11$

Por lo tanto, Ranf = [6; 11]

Clave A

3. $f(x) = \dfrac{15}{\cos x + 4}$

Sabemos:

$-1 \le \cos x \le 1$

$3 \le \cos x + 4 \le 5$

$\dfrac{1}{5} \le \dfrac{1}{\cos x + 4} \le \dfrac{1}{3}$

$\dfrac{15}{5} \le \underbrace{\dfrac{15}{\cos x + 4}}_{f(x)} \le \dfrac{15}{3}$

$\Rightarrow 3 \le f(x) \le 5$

Por lo tanto, Ranf = [3; 5]

Clave C

4. $M = 7\sec 3x + 2$

Para el dominio:

$3x \in \mathbb{R} - \{(2n+1)\frac{\pi}{2} / n \in \mathbb{Z}\}$

$\Rightarrow x \in \mathbb{R} - \{(2n+1)\frac{\pi}{6} / n \in \mathbb{Z}\}$

Por lo tanto, $\operatorname{Dom}M = \mathbb{R} - \{(2n+1)\frac{\pi}{6} / n \in \mathbb{Z}\}$

Clave A

5. $f(x) = (\operatorname{sen}x - 8)\operatorname{sen}x + 7$

$f(x) = \operatorname{sen}^2x - 8\operatorname{sen}x + 7$

$f(x) = \operatorname{sen}^2x - 2(\operatorname{sen}x)(4) + 4^2 + 7 - 4^2$

$f(x) = (\operatorname{sen}x - 4)^2 - 9$

Sabemos:

$-1 \le \operatorname{sen}x \le 1$

$-5 \le \operatorname{sen}x - 4 \le -3$

$9 \le (\operatorname{sen}x - 4)^2 \le 25$

$0 \le \underbrace{(\operatorname{sen}x - 4)^2 - 9}_{} \le 16$

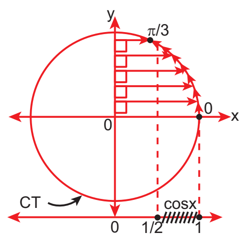
$0 \le f(x) \le 16$

Por lo tanto, el máximo valor de f(x) es 16.

Clave E

6. Por dato: $x \in [0; \frac{\pi}{3}]$

Analizando en la CT:



Entonces:

$\dfrac{1}{2} \le \cos x \le 1$

$1 \le 2\cos x \le 2$

$6 \le \underbrace{2\cos x + 5}_{} \le 7$

$6 \le f(x) \le 7$

Luego: Ranf = [6; 7]

Por dato: Ranf = [a; b]

$\Rightarrow a = 6 \ \wedge \ b = 7$

Piden:

$a + b = 6 + 7 = 13$

$\therefore\ a + b = 13$

Clave D

7. $f(x) = 5\sqrt{\cos x - 1}$

Para el dominio:

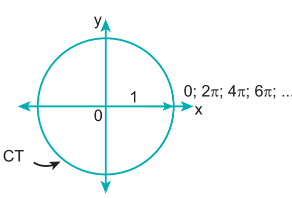
$\cos x - 1 \ge 0$

$\cos x \ge 1$

Pero: $\cos x \le 1$

Entonces: $\cos x = 1$

Analizando en la CT



Los valores que cumplen la condición tienen la forma: $\{2k\pi / k \in \mathbb{Z}\}$

Por lo tanto, $\operatorname{Dom}f = \{2k\pi / k \in \mathbb{Z}\}$

Clave A

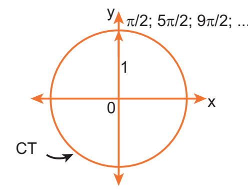
8. $f(x) = \dfrac{\cos x}{1 - \operatorname{sen}x}$

Para el dominio:

$1 - \operatorname{sen}x \ne 0$

$\Rightarrow \operatorname{sen}x \ne 1$

Analizando en la CT:



Entonces:

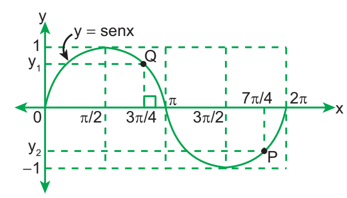
$x \ne \left\{\frac{\pi}{2}; \frac{5\pi}{2}; \frac{9\pi}{2}; ...\right\}$

$x \ne \left\{\frac{\pi}{2} + 2k\pi / k \in \mathbb{Z}\right\}$

Por lo tanto, $\operatorname{Dom}f = \mathbb{R} - \left\{\frac{\pi}{2} + 2k\pi / k \in \mathbb{Z}\right\}$

Clave A

9.



Las coordenadas del punto Q son:

$(x_1; y_1) = (\frac{3\pi}{4}; y_1)$

De donde: $y_1 = \operatorname{sen}x_1$

$\Rightarrow y_1 = \operatorname{sen}\frac{3\pi}{4} = \operatorname{sen}135° = \frac{\sqrt{2}}{2}$

Las coordenadas del punto P son:

$(x_2; y_2) = \left(\frac{7\pi}{4}; y_2\right)$

De donde: $y_2 = \operatorname{sen}x_2$

$\Rightarrow y_2 = \operatorname{sen}\frac{7\pi}{4} = \operatorname{sen}315° = -\frac{\sqrt{2}}{2}$

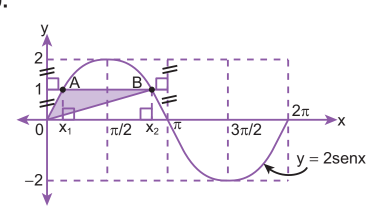
Piden:

$y_1 + y_2 = \left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)$

$\therefore y_1 + y_2 = 0$

Clave C

10.



Las coordenadas del punto A son:

$(x_1; y_1) = (x_1; 1)$

De donde: $y_1 = 2\text{sen}x_1;\ 0 < x_1 < \frac{\pi}{2}$

$1 = 2\text{sen}x_1$

$\Rightarrow \text{sen}x_1 = \frac{1}{2} \Rightarrow x_1 = 30° = \frac{\pi}{6}$

Las coordenadas del punto B son:
$(x_2; y_2) = (x_2; 1)$

De donde: $y_2 = 2\text{sen}x_2;\ \frac{\pi}{2} < x_2 < \pi$

$1 = 2\text{sen}x_2$

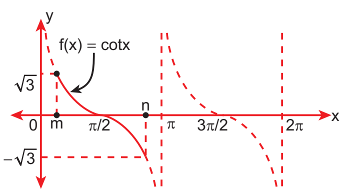
$\Rightarrow \text{sen}x_2 = \frac{1}{2} \Rightarrow x_2 = 150° = \frac{5\pi}{6}$

El área de la región triangular AOB será:

$$A_{\Delta AOB} = \frac{(AB)(1)}{2} = \frac{(x_2 - x_1)}{2}$$
$$= \frac{\frac{5\pi}{6} - \frac{\pi}{6}}{2} = \frac{\frac{2\pi}{3}}{2}$$
$$\therefore A_{\Delta AOB} = \frac{\pi}{3}$$

Clave C

11. Por dato: $f(x) = \cot x \wedge \text{Ran}f = \left[-\sqrt{3}; \sqrt{3}\right]$



Del gráfico:
$f(m) = \cot m = \sqrt{3};\ m \in \left\langle 0; \frac{\pi}{2}\right\rangle$

$\Rightarrow \cot m = \cot\frac{\pi}{6} \Rightarrow m = \frac{\pi}{6}$

$f(n) = \cot n = -\sqrt{3};\ n \in \left\langle \frac{\pi}{2}; \pi\right\rangle$

$\Rightarrow \cot n = \cot\frac{5\pi}{6} \Rightarrow n = \frac{5\pi}{6}$

Entonces: $\text{Dom}f = \left[\frac{\pi}{6}; \frac{5\pi}{6}\right]$

Piden:

$m + n = \frac{\pi}{6} + \frac{5\pi}{6} = \pi$

$\therefore m + n = \pi$

Clave B

12. Piden el rango de:
$f(x) = \text{sen}x + \cot x \,.\, \cos x - 1$

$f(x) = \text{sen}x + \frac{\cos x}{\text{sen}x} \,.\, \cos x - 1$

$\Rightarrow \text{sen}x \neq 0 \Rightarrow x \neq \{k\pi / k \in \mathbb{Z}\}$
$\Rightarrow \text{Dom}f = \mathbb{R} - \{k\pi / k \in \mathbb{Z}\}$

Luego:
$f(x) = \frac{\text{sen}^2x + \cos^2x}{\text{sen}x} - 1$

$f(x) = \frac{1}{\text{sen}x} - 1 = \csc x - 1$

A partir del dominio y analizando en la CT, tenemos:
$-\infty < \csc x \leq -1 \ \cup \ 1 \leq \csc x < +\infty$
$\underbrace{-\infty < \csc x - 1 \leq -2}_{-\infty < f(x) \leq -2} \ \cup \ \underbrace{0 \leq \csc x - 1 < +\infty}_{0 \leq f(x) < +\infty}$

$\Rightarrow f(x) \in \langle -\infty; -2] \cup [0; +\infty\rangle$

Un equivalente es: $f(x) \in \mathbb{R} - \langle -2; 0\rangle$

Clave D

13. $\sec 2x$: $2x \neq (2n+1)\frac{\pi}{2};\ n \in \mathbb{Z}$

$x \neq (2n+1)\frac{\pi}{4};\ n \in \mathbb{Z}$

$\csc 4x$: $4x \neq n\pi;\ n \in \mathbb{Z}$
$x \neq n\frac{\pi}{4};\ n \in \mathbb{Z}$

Luego:

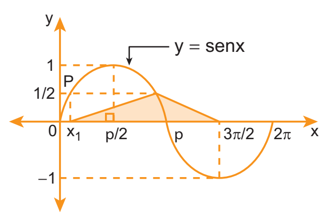
$$h(x) = \frac{\frac{1}{\cos 2x}}{\frac{1}{\text{sen}4x}}$$
$$= \frac{2\text{sen}2x\cos 2x}{\cos 2x}$$
$$= 2\text{sen}2x$$

$\therefore \text{Dom}h = \mathbb{R} - \left\{\frac{n\pi}{4}\right\};\ n \in \mathbb{Z}$

$\text{Ran}h = \langle -2; 2\rangle - \{0\}$

Clave E

14. En el gráfico:



Se observa: $P\left(x_1; \frac{1}{2}\right);\ 0 < x_1 < \frac{\pi}{2}$

Como la función y = senx pasa por el punto P, entonces se cumple:

$y = \frac{1}{2} = \text{sen}x_1;\ 0 < x_1 < \frac{\pi}{2}$

$\Rightarrow \text{sen}x_1 = \frac{1}{2} = \text{sen}\frac{\pi}{6}$

$\Rightarrow x_1 = \frac{\pi}{6}$

Piden el área de la región sombreada:

$$A_{\text{somb.}} = \frac{(\text{base})(\text{altura})}{2}$$
$$A_{\text{somb.}} = \frac{\left(\frac{3\pi}{2} - x_1\right)\left(\frac{1}{2}\right)}{2}$$
$$A_{\text{somb.}} = \frac{\left(\frac{3\pi}{2} - \frac{\pi}{6}\right)\left(\frac{1}{2}\right)}{2}$$
$$A_{\text{somb.}} = \frac{\left(\frac{4\pi}{3}\right)\left(\frac{1}{2}\right)}{2} = \frac{4\pi}{12}$$
$$\therefore A_{\text{somb.}} = \frac{\pi}{3}$$

Clave C

PRACTIQUEMOS
Nivel 1 (página 80) Unidad 4

Comunicación matemática

1.

2.

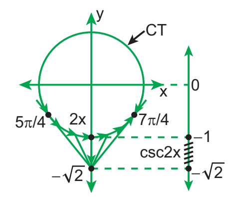
Razonamiento y demostración

3. $f(x) = \csc 2x$

Por dato: $\text{Dom}f = \left\langle \frac{5\pi}{8}; \frac{7\pi}{8}\right\rangle$

$\Rightarrow \frac{5\pi}{8} < x < \frac{7\pi}{8} \ \Rightarrow \ \frac{5\pi}{4} < 2x < \frac{7\pi}{4}$

Analizando en la C.T.:



Entonces: $-\sqrt{2} < \csc 2x \leq -1$
$-\sqrt{2} < f(x) \leq -1$
$\therefore \text{Ran}f = \langle -\sqrt{2}; -1]$

Clave D

4. Piden el rango de la función f.
$f(x) = (\text{sen}x + \cos x - 1)(\text{sen}x + \cos x + 1)$

De la función f se observa que aparecen las funciones seno y coseno, sabemos que están definidas en $\mathbb{R}$.
$\Rightarrow \text{Dom}f = \mathbb{R}$

Luego:
$f(x) = (\text{sen}x + \cos x)^2 - 1^2$
$f(x) = \text{sen}^2x + 2\text{sen}x\cos x + \cos^2x - 1$
$f(x) = (\text{sen}^2x + \cos^2x) + 2\text{sen}x\cos x - 1$
$f(x) = (1) + 2\text{sen}x\cos x - 1$
$f(x) = 2\text{sen}x\cos x = \text{sen}2x$
$\Rightarrow f(x) = \text{sen}2x$

A partir del dominio, tenemos:
$x \in \mathbb{R} \Rightarrow (2x) \in \mathbb{R}$
$\Rightarrow -1 \leq \text{sen}2x \leq 1 \Rightarrow -1 \leq f(x) \leq 1$
$\therefore \text{Ran}f = [-1; 1]$

Clave B

5. Piden el dominio y el rango de f.

$f(x) = \tan x \,.\, \frac{\cos x}{\text{sen}x}$

El dominio de f son todos los valores admisibles de x, entonces:

Por la función tanx: $x \neq (2k+1)\frac{\pi}{2};\ k \in \mathbb{Z}$

Además: $\text{sen}x \neq 0 \Rightarrow x \neq k\pi;\ k \in \mathbb{Z}$

De ambas restricciones deducimos:

$x \neq \frac{k\pi}{2};\ k \in \mathbb{Z}$

$\therefore \text{Dom}f = \mathbb{R} - \left\{\frac{k\pi}{2} / k \in \mathbb{Z}\right\}$

Una vez definido el dominio de f, simplificamos la expresión para obtener el rango de f.

$f(x) = \tan x \frac{\cos x}{\text{sen}x} = \left(\frac{\text{sen}x}{\cos x}\right)\frac{\cos x}{\text{sen}x}$

$\Rightarrow f(x) = 1$

$\therefore \text{Ran}f = \{1\}$

Clave B

6. Piden el período de las funciones:

I. $f(x) = 2\text{sen}3x + 1$

Sea T: el período de la función f.

$\Rightarrow f(x + T) = f(x)$

$2\text{sen}3(x + T) + 1 = 2\text{sen}3x + 1$

$\text{sen}(3x + 3T) = \text{sen}3x$

$\text{sen}(3T + 3x) = \text{sen}(2\pi + 3x)$

Comparando: $3T = 2\pi \Rightarrow T = \frac{2\pi}{3}$

II. $g(x) = 1 - \tan\frac{x}{3}$

Sea T: el período de la función g.

$\Rightarrow g(x + T) = g(x)$

$1 - \tan\frac{(x+T)}{3} = 1 - \tan\frac{x}{3}$

$\tan\left(\frac{x}{3} + \frac{T}{3}\right) = \tan\frac{x}{3}$

$\tan\left(\frac{T}{3} + \frac{x}{3}\right) = \tan\left(\pi + \frac{x}{3}\right)$

Comparando: $\frac{T}{3} = \pi \Rightarrow T = 3\pi$

III. $h(x) = 2\cos4x - 3$

Sea T: el período de la función h.

$\Rightarrow h(x + T) = h(x)$

$2\cos4(x + T) - 3 = 2\cos4x - 3$

$\cos(4x + 4T) = \cos4x$

$\cos(4T + 4x) = \cos(2\pi + 4x)$

Comparando: $4T = 2\pi \Rightarrow T = \frac{\pi}{2}$

Clave E

7. Piden el dominio de la función f.

$f(x) = 3\tan\left(4x + \frac{3\pi}{2}\right)$

Entonces:

$\left(4x + \frac{3\pi}{2}\right) \neq (2n + 1)\frac{\pi}{2};\ n \in \mathbb{Z}$

$4x + \frac{3\pi}{2} \neq n\pi + \frac{\pi}{2}$

$4x \neq n\pi + \frac{\pi}{2} - \frac{3\pi}{2}$

$4x \neq n\pi - \pi$

$x \neq (n - 1)\frac{\pi}{4}$

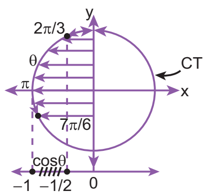
$\Rightarrow x \in \mathbb{R} - \left\{(n-1)\frac{\pi}{4} / n \in \mathbb{Z}\right\}$

$\therefore \text{Dom}f = \mathbb{R} - \left\{(n-1)\frac{\pi}{4} / n \in \mathbb{Z}\right\}$

Clave C

8. Piden el rango de la función: $f(\theta) = \cos\theta$

Analizando en la CT:



Entonces: $-1 \leq \cos\theta \leq -\frac{1}{2}$

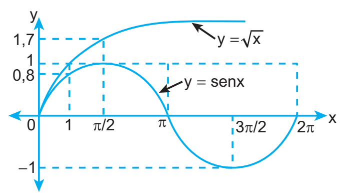
$-1 \leq f(\theta) \leq -\frac{1}{2}$

$\therefore \text{Ran}f = \left[-1; -\frac{1}{2}\right]$

Clave B

Resolución de problemas

9. Tabulando obtenemos:



Ambas gráficas se intersectan en un solo punto: (0; 0)

Clave B

10. El sinusoide de x, está representado por la regla de correspondencia: $y = \text{sen}x$

A) $\left(\frac{5\pi}{2};1\right) = (x;\ y) \Rightarrow x = \frac{5\pi}{2} \wedge y = 1$

$\Rightarrow y = \text{sen}\frac{5\pi}{2} = \text{sen}\frac{\pi}{2} = 1$ (V)

B) $(4\pi;\ 0) = (x;\ y) \Rightarrow x = 4\pi \wedge y = 0$

$\Rightarrow y = \text{sen}4\pi = \text{sen}2\pi = 0$ (V)

C) $\left(-\frac{3\pi}{2};1\right) = (x;\ y) \Rightarrow x = -\frac{3\pi}{2} / y = 1$

$\Rightarrow y = \text{sen}\left(-\frac{3\pi}{2}\right) = -\text{sen}\frac{3\pi}{2} = -(-1) = 1$ (V)

D) $\left(\frac{7\pi}{6};\frac{1}{2}\right) = (x;\ y) \Rightarrow x = \frac{7\pi}{6} \wedge y = \frac{1}{2}$

$\Rightarrow y = \text{sen}\frac{7\pi}{6} = -\text{sen}\frac{\pi}{6} = -\frac{1}{2}$ (F)

E) $\left(-\frac{11\pi}{4}; -\frac{\sqrt{2}}{2}\right) = (x;\ y)$

$\Rightarrow x = -\frac{11\pi}{4} \wedge y = -\frac{\sqrt{2}}{2}$

$\Rightarrow y = \text{sen}\left(-\frac{11\pi}{4}\right)$

$= -\text{sen}\frac{11\pi}{4} = -\frac{\sqrt{2}}{2}$ (V)

Clave D

Nivel 2 (página 80) Unidad 4

Comunicación matemática

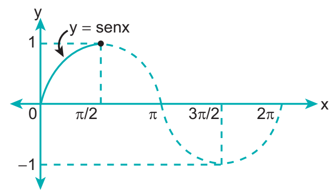
11.

12.

Razonamiento y demostración

13. Por dato: $x \in \left\langle 0;\ \frac{\pi}{2}\right\rangle$

Además: $\text{sen}x = \frac{a}{2} - 1$



De la gráfica: $0 < \text{sen}x < 1$

Entonces:

$0 < \frac{a}{2} - 1 < 1$

$1 < \frac{a}{2} < 2$

$2 < a < 4$

$\therefore a \in \langle 2;\ 4\rangle$

Clave A

14. Por dato:

$f(x) = 2|\cos x| + 3;\ \forall\ x \in \mathbb{R}$

Piden: Ranf

Como: $x \in \mathbb{R} \Rightarrow -1 \leq \cos x \leq 1$

$\Rightarrow 0 \leq |\cos x| \leq 1$

$0 \leq 2|\cos x| \leq 2$

$3 \leq 2|\cos x| + 3 \leq 5$

$3 \leq f(x) \leq 5$

$\Rightarrow f(x) \in [3;\ 5]$

$\therefore \text{Ran}f = [3;\ 5]$

Clave C

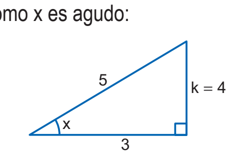
15. Por dato:

$A(x;\ y) = A\left(x;\ \frac{3}{5}\right);\ 0 < x < \frac{\pi}{2} \Rightarrow y = \frac{3}{5}$

Además: el punto A es un punto que pertenece al gráfico del cosx.

$\Rightarrow y = \cos x \Rightarrow \frac{3}{5} = \cos x$

Luego como x es agudo:



Por el teorema de Pitágoras: k = 4

$\Rightarrow \tan x = \frac{k}{3} = \frac{4}{3}$

Piden: $M = \tan x + \cos^2 x$

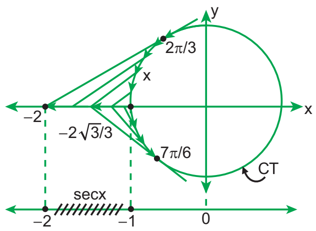
$M = \left(\frac{4}{3}\right) + \left(\frac{3}{5}\right)^2 = \frac{4}{3} + \frac{9}{25}\quad \therefore M = \frac{127}{75}$

Clave B

16. Por dato:

$f(x) = \sqrt{3}\,|\sec x|;\ \dfrac{2\pi}{3} \le x \le \dfrac{7\pi}{6}$

Analizando en la CT:



Observamos que la secx no presenta restricciones en el intervalo dado.

Además: $-2 \le \sec x \le -1$

$\Rightarrow 1 \le |\sec x| \le 2$

$\sqrt{3} \le \underbrace{\sqrt{3}\,|\text{sen}x|}_{} \le 2\sqrt{3}$

$\sqrt{3} \le f(x) \le 2\sqrt{3}$

$\Rightarrow f(x) \in \left[\sqrt{3};\ 2\sqrt{3}\right]$

$\therefore \text{Ranf} = \left[\sqrt{3};\ 2\sqrt{3}\right]$

Clave E

17. $f(x) = 2 + 4\csc^2\left(\dfrac{x}{3}\right)$

Piden: el rango de la función f.

Donde: $\dfrac{x}{3} \ne n\pi;\ n \in \mathbb{Z}$

$\Rightarrow x \ne 3n\pi \Rightarrow \text{Domf} = \mathbb{R} - \{3n\pi \,/\, n \in \mathbb{Z}\}$

Luego a partir del dominio obtenemos:

$-\infty < \csc\left(\dfrac{x}{3}\right) \le -1 \lor 1 \le \csc\left(\dfrac{x}{3}\right) < +\infty$

Al elevar al cuadrado se tiene:

$1 \le \csc^2\left(\dfrac{x}{3}\right) < +\infty \Rightarrow 4 \le 4\csc^2\left(\dfrac{x}{3}\right) < +\infty$

$\Rightarrow 6 \le \underbrace{2 + 4\csc^2\left(\dfrac{x}{3}\right)}_{f(x)} < +\infty$

$\Rightarrow f(x) \in [6;\ +\infty\rangle$

$\therefore \text{Ranf} = [6;\ +\infty\rangle$

Clave C

18. Por dato:

$f(x) = \dfrac{1 + \text{sen}x}{2 + \text{sen}x};\ \forall\ x \in \left[\dfrac{\pi}{2};\ \dfrac{3\pi}{2}\right]$

De la función f se observa que aparece la función seno y sabemos que está definida en los $\mathbb{R}$, además el denominador no afecta al dominio dado ya que $(2 + \text{sen}x)$ es siempre diferente de cero para todo $x \in \mathbb{R}$.

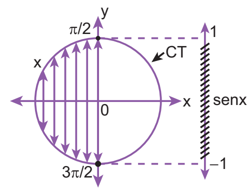
Luego:

$f(x) = \dfrac{1 + \text{sen}x}{2 + \text{sen}x} = \dfrac{(2 + \text{sen}x) - 1}{2 + \text{sen}x}$

$f(x) = \dfrac{(2 + \text{sen}x)}{2 + \text{sen}x} - \dfrac{1}{2 + \text{sen}x}$

$\Rightarrow f(x) = 1 - \dfrac{1}{2 + \text{sen}x}$

Analizando en la CT:



Entonces: $-1 \le \text{sen}x \le 1$

$1 \le \text{sen}x + 2 \le 3$

Luego:

$\dfrac{1}{3} \le \dfrac{1}{2 + \text{sen}x} \le 1$

$-1 \le -\dfrac{1}{2 + \text{sen}x} \le -\dfrac{1}{3}$

$0 \le \underbrace{1 - \dfrac{1}{2 + \text{sen}x}}_{} \le \dfrac{2}{3}$

$0 \le f(x) \le \dfrac{2}{3}$

$\therefore \text{Ranf} = \left[0;\ \dfrac{2}{3}\right]$

Clave E

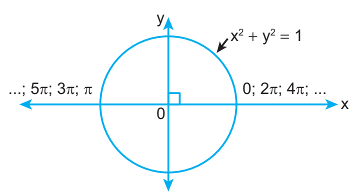
Resolución de problemas

19. Cuando una función interseca al eje x, los puntos de intersección tienen la forma (x; 0), donde $x \in \mathbb{R}$, es decir la ordenada (y) vale cero.

Por dato: y = senx

$\Rightarrow \text{sen}x = 0$

Analizando en la CT:



$\Rightarrow x = \{0;\ \pi;\ 2\pi;\ 3\pi;\ ...\}$

En general: $x = \{n\pi \,/\, n \in \mathbb{Z}\}$

Luego nos piden en el intervalo:

$\left\langle -\dfrac{7\pi}{4};\ \dfrac{5\pi}{2} \right\rangle$

$\therefore -\dfrac{7\pi}{4} < x < \dfrac{5\pi}{2} \Rightarrow -\dfrac{7\pi}{4} < n\pi < \dfrac{5\pi}{2}$

$-\dfrac{7}{4} < n < \dfrac{5}{2} \Rightarrow -1{,}75 < n < 2{,}5$

$\Rightarrow n = \{-1;\ 0;\ 1;\ 2\}$

Por cada valor de n se presenta un punto de intersección de la función con el eje x.

Por lo tanto, hay 4 puntos de intersección.

Clave D

20. $H(x) = \text{sen}x - \cos x$

Piden: las coordenadas de los puntos de intersección de H con el eje x, en $\langle 0;\ 2\pi\rangle$

Sabemos:

$y = H(x) = \text{sen}x - \cos x$

Del ejercicio anterior deducimos que para hallar los puntos de intersección con el eje x, la ordenada debe ser cero.

$\Rightarrow y = 0$

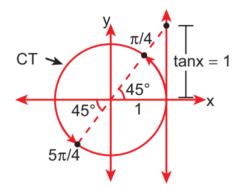
Luego:

$\text{sen}x - \cos x = 0$

$\text{sen}x = \cos x$

$\Rightarrow \tan x = 1$

Analizando en la CT:



Entonces:

$x = \dfrac{\pi}{4} \lor x = \dfrac{5\pi}{4}$; en $\langle 0;\ 2\pi\rangle$

Por lo tanto, las coordenadas de los puntos serán: $\left(\dfrac{\pi}{4}; 0\right); \left(\dfrac{5\pi}{4}; 0\right)$

Clave A

Nivel 3 (página 81) Unidad 4

Comunicación matemática

21.

22.

Razonamiento y demostración

23. Piden el máximo valor de la función:

$f(x) = \text{sen}x(\text{sen}x - 6) + 4$

$f(x) = \text{sen}^2x - 6\text{sen}x + 4$

$f(x) = \text{sen}^2x - 2(\text{sen}x)(3) + 3^2 - 3^2 + 4$

$f(x) = (\text{sen}x - 3)^2 - 9 + 4$

$\Rightarrow f(x) = (\text{sen}x - 3)^2 - 5$

Como x no presenta restricciones, entonces:

$-1 \le \text{sen}x \le 1$

$-4 \le \text{sen}x - 3 \le -2$

$4 \le (\text{sen}x - 3)^2 \le 16$

$-1 \le \underbrace{(\text{sen}x - 3)^2 - 5}_{} \le 11$

$-1 \le f(x) \le 11$

$\Rightarrow f(x) \in [-1;\ 11]$

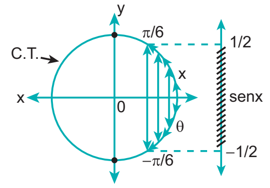
$\therefore f(x)_{\text{máx.}} = 11$

Clave A

24. Por dato: $f(x) = \sec\left(\frac{\pi}{2}\text{sen}x\right)$

Además: $x \in \left[-\frac{\pi}{6}; \frac{\pi}{6}\right]$

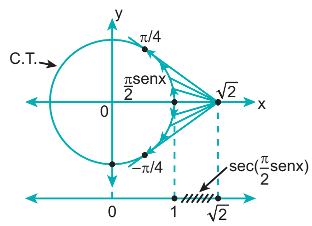
Analizando en la CT, tenemos:



Entonces: $-\frac{1}{2} \leq \text{sen}x \leq \frac{1}{2}$

$\Rightarrow -\frac{\pi}{4} \leq \frac{\pi}{2}\text{sen}x \leq \frac{\pi}{4}$

Analizando nuevamente en la C.T.:



Entonces: $\underbrace{1 \leq \sec\left(\frac{\pi}{2}\text{sen}x\right) \leq \sqrt{2}}$

$1 \leq f(x) \leq \sqrt{2}$

Además que todos los valores de $x \in \left[-\frac{\pi}{6}; \frac{\pi}{6}\right]$ son admisibles para la función f.

Piden:

$f(x)_{máx.} + f(x)_{mín.} = \left(\sqrt{2}\right) + (1)$

$\therefore f(x)_{máx.} + f(x)_{mín.} = \sqrt{2} + 1$

Clave A

25. Piden el rango de la función f.

$f(x) = \sec^2 2x + |2\sec 2x| + |\cot^2 x - \csc^2 x|$

Luego:

Para la función sec2x:

$2x \neq (2k+1)\frac{\pi}{2}; k \in \mathbb{Z}$

$\Rightarrow x \neq (2k+1)\frac{\pi}{4}; k \in \mathbb{Z}$

Para las funciones cotx y cscx:

$x \neq k\pi; k \in \mathbb{Z}$

Entonces:

$x \neq \left\{(2k+1)\frac{\pi}{4}; k\pi\right\}; k \in \mathbb{Z}$

$\Rightarrow \text{Dom}f = \mathbb{R} - \left\{(2k+1)\frac{\pi}{4}; k\pi\right\}; k \in \mathbb{Z}$

Reduciendo la función f tenemos:

$f(x) = \sec^2 2x + |2\sec 2x| + |-1|$

$f(x) = |\sec 2x|^2 + 2|\sec 2x| + 1$

$f(x) = (|\sec 2x| + 1)^2$

Analizando en la CT y teniendo en cuenta el dominio de la función, se tiene:

$-\infty < \sec 2x \leq -1 \vee 1 < \sec 2x < +\infty$

Al tomar el valor absoluto:

$\Rightarrow 1 \leq |\sec 2x| < +\infty$

$2 \leq |\sec 2x| + 1 < +\infty$

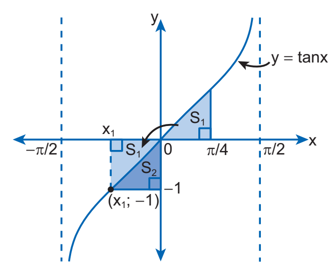
$4 \leq (|\sec 2x| + 1)^2 < +\infty$

$4 \leq f(x) < +\infty$

$\therefore \text{Ran}f = [4; +\infty\rangle$

Clave A

26.



Como la función y = tanx es impar, entonces su gráfica es simétrica con respecto al origen de coordenadas.
Luego, trasladamos el área S_1 por simetría.

Piden el área de la región sombreada.

$A_{somb.} = S_1 + S_2 = |x_1| \cdot |-1|$

$\Rightarrow A_{somb.} = |x_1|(1) = |x_1|$

Como la función tangente pasa por el punto $(x_1; -1)$, entonces se cumple:

$y = \tan x_1 = -1; x \in \left\langle -\frac{\pi}{2}; 0\right\rangle$

$\Rightarrow \tan x_1 = \tan\left(-\frac{\pi}{4}\right) \Rightarrow x_1 = -\frac{\pi}{4}$

Entonces:

$A_{somb.} = \left|-\frac{\pi}{4}\right| = -\left(-\frac{\pi}{4}\right)$

$\therefore A_{somb.} = \frac{\pi}{4}$

Clave B

27. Piden el dominio y el rango de f.

$f(x) = \frac{\text{sen}2x}{\tan x}$

El dominio de f son todos los valores admisibles de x, entonces:

Por la función tanx: $x \neq (2k+1)\frac{\pi}{2}; k \in \mathbb{Z}$

Además: $\tan x \neq 0 \Rightarrow x \neq k\pi; k \in \mathbb{Z}$

De ambas restricciones deducimos:

$x \neq \left\{\frac{k\pi}{2}\right\}; k \in \mathbb{Z}$

$\therefore \text{Dom}f = \left\{\mathbb{R} - \frac{k\pi}{2} / k \in \mathbb{Z}\right\}$

Una vez definido el dominio de f, simplificamos la expresión para obtener el rango de f.

$$f(x) = \frac{\text{sen}2x}{\tan x} = \frac{2\text{sen}x\cos x}{\left(\frac{\text{sen}x}{\cos x}\right)}$$

$\Rightarrow f(x) = 2\cos^2 x$

Luego analizando en la CT y teniendo en cuenta el dominio de f, tenemos:

$-1 < \cos x < 0 \ \vee \ 0 < \cos x < 1$

Al elevar al cuadrado se tiene:

$0 < \cos^2 x < 1 \Rightarrow 0 < 2\cos^2 x < 2$

$\Rightarrow 0 < f(x) < 2$

$\therefore \text{Ran}f = \langle 0; 2\rangle$

Clave E

28. Piden el rango de:

$h(x) = \cot x - \tan x - 2\tan 2x$

Donde: $x \neq k\pi \ \wedge \ x \neq (2k+1)\frac{\pi}{2}$

Además: $2x \neq (2k+1)\frac{\pi}{2} \Rightarrow x \neq (2k+1)\frac{\pi}{4}$

Se deduce: $x \in \mathbb{R} - \left\{\frac{k\pi}{4} / k \in \mathbb{Z}\right\}$

Reduciendo la función h:

$h(x) = (2\cot 2x) - 2\tan 2x$

$h(x) = 2(\cot 2x - \tan 2x) = 2(2\cot 4x)$

$\Rightarrow h(x) = 4\cot 4x$

Por dato: $x \in \left\langle -\frac{\pi}{16}; \frac{\pi}{24}\right] - \{0\}$

$\Rightarrow 4x \in \left\langle -\frac{\pi}{4}; \frac{\pi}{6}\right] - \{0\}$

Teniendo en cuenta el dominio y analizando en la C.T., tenemos:

$-\infty < \cot 4x < -1 \ \cup \ \sqrt{3} \leq \cot 4x < +\infty$

$-\infty < \underbrace{4\cot 4x} < -4 \ \cup \ 4\sqrt{3} \leq \underbrace{4\cot 4x} < +\infty$

$-\infty < h(x) < -4 \ \cup \ 4\sqrt{3} \leq h(x) < +\infty$

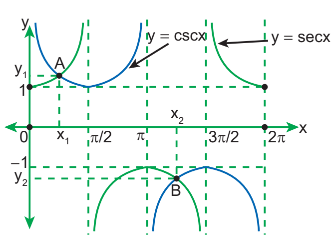
$\Rightarrow h(x) \in \langle -\infty; -4\rangle \cup [4\sqrt{3}; +\infty\rangle$

Un equivalente es: $h(x) \in \mathbb{R} - [-4; 4\sqrt{3}\rangle$

Clave B

Resolución de problemas

29.



Para ambos puntos se cumple:

$y_1 = \sec x_1 = \csc x_1; \ 0 < x_1 < \frac{\pi}{2}$

$\Rightarrow \sec x_1 = \csc x_1$

$$\frac{1}{\cos x_1} = \frac{1}{\text{sen}x_1} \Rightarrow \frac{\text{sen}x_1}{\cos x_1} = 1$$

$$\Rightarrow \tan x_1 = 1$$

Sabemos: $\tan\frac{\pi}{4} = 1$

$\Rightarrow x_1 = \frac{\pi}{4} \wedge y_1 = \sec\frac{\pi}{4} = \sqrt{2}$

$y_2 = \sec x_2 = \csc x_2; \ \pi < x_2 < \frac{3\pi}{2}$

$\Rightarrow \frac{1}{\cos x_2} = \frac{1}{\text{sen}x_2} \Rightarrow \tan x_2 = 1$

Sabemos: $\tan\frac{5\pi}{4} = 1$

$\Rightarrow x_2 = \frac{5\pi}{4} \ \wedge \ y_2 = \sec\frac{5\pi}{4} = -\sqrt{2}$

Piden:

$$(x_1 + x_2) + (y_1 + y_2) = \left(\frac{\pi}{4} + \frac{5\pi}{4}\right) + (\sqrt{2} - \sqrt{2})$$

$\therefore (x_1 + x_2) + (y_1 + y_2) = \frac{3\pi}{2}$

Clave A

30. La función $y = \cot x$ presenta:

$\text{Dom}(\cot x) = \mathbb{R} - \{n\pi / n \in \mathbb{Z}\}$

$\text{Ran}(\cot x) = \mathbb{R}$

Entonces sus asíntotas presentan la forma:

$x = \{n\pi / n \in \mathbb{Z}\}$

Luego, nos piden el número de asíntotas en el intervalo $\left\langle -\frac{7\pi}{2}; \frac{9\pi}{4}\right\rangle$

$\Rightarrow -\frac{7\pi}{2} < x < \frac{9\pi}{4}$

$-\frac{7\pi}{2} < n\pi < \frac{9\pi}{4}$

$-\frac{7}{2} < n < \frac{9}{4}$

$-3{,}5 < n < 2{,}25$

$\Rightarrow n = \{-3; -2; -1; 0; 1; 2\}$

Por cada valor de n se presenta una asíntota en la gráfica.

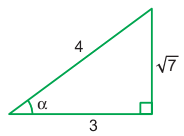
Por lo tanto, la gráfica presentará 6 asíntotas en el intervalo $\left\langle -\frac{7\pi}{2}; \frac{9\pi}{4}\right\rangle$.

Clave D

APLICAMOS LO APRENDIDO
Nivel 1 (página 82) Unidad 4

1. Por dato: $\alpha = \arctan\frac{\sqrt{7}}{3}$

Entonces:
$\tan\alpha = \frac{\sqrt{7}}{3}$



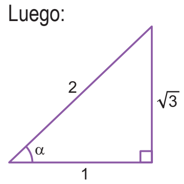
Piden: $\cos\alpha$
$\therefore\ \cos\alpha = \frac{3}{4}$

Clave E

2. Por dato: $\alpha = \text{arcsec}2$

Entonces: $\sec\alpha = \frac{2}{1}$

Luego:



Piden:

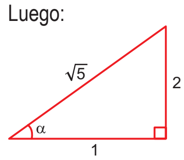
$\text{sen}\alpha\cos\alpha = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{4}$

$\therefore \text{sen}\alpha\cos\alpha = \frac{\sqrt{3}}{4}$

Clave B

3. Haciendo: $\arctan 2 = \alpha \Rightarrow \tan\alpha = 2 = \frac{2}{1}$

Luego:

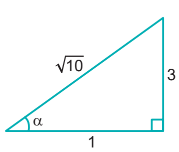


$N = \csc(\alpha) = \frac{\sqrt{5}}{2}$

Clave C

4. Haciendo: $\arctan 3 = \alpha$

$\Rightarrow \tan\alpha = 3 = \frac{3}{1}$



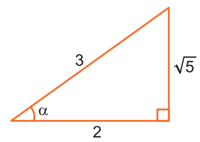
$E = \cos(2\alpha) = 2\cos^2\alpha - 1$

$E = 2\left(\frac{1}{\sqrt{10}}\right)^2 - 1 = \frac{2}{10} - 1 = \frac{2-10}{10}$

$\Rightarrow\ E = -\frac{4}{5}$

Clave C

5. Haciendo: $\arccos\frac{2}{3} = \alpha \Rightarrow \cos\alpha = \frac{2}{3}$



Luego:
$\text{arcsen}x = \alpha \Rightarrow \text{sen}\alpha = x$

$\therefore x = \frac{\sqrt{5}}{3}$

Clave A

6. $M = \arctan\left(\frac{2+4}{1-\underbrace{2\ .\ 4}_{2\,.\,4>1 \Rightarrow n=1}}\right) + n\pi$

Luego:
$M = \arctan\left(\frac{6}{1-8}\right) + \pi = \arctan\left(\frac{6}{-7}\right) + \pi$

$\Rightarrow\ M = -\arctan\frac{6}{7} + \pi$

Clave E

7. $\text{arcsen}x + \underbrace{\text{arcsen}x + \arccos x}_{\frac{\pi}{2}} = \frac{5\pi}{6}$

$\text{arcsen}x + \frac{\pi}{2} = \frac{5\pi}{6} \Rightarrow \text{arcsen}x = \frac{\pi}{3}$

$\Rightarrow x = \text{sen}\left(\frac{\pi}{3}\right) \Rightarrow x = \frac{\sqrt{3}}{2}$

Clave D

8. $A = \text{sen}(\arctan\frac{3}{5} + \arctan\frac{1}{4})$

$A = \text{sen}\left(\arctan\left(\frac{\frac{3}{5}+\frac{1}{4}}{1-\frac{3}{5}\cdot\frac{1}{4}}\right) + k\pi\right)$

De donde: $\frac{3}{5}\cdot\frac{1}{4} < 1 \Rightarrow k = 0$

Luego:
$A = \text{sen}(\underbrace{\arctan 1}_{\frac{\pi}{4}}) = \text{sen}\frac{\pi}{4}$

$A = \text{sen}\frac{\pi}{4} = \text{sen}45° = \frac{\sqrt{2}}{2}$

$\therefore\ A = \frac{\sqrt{2}}{2}$

Clave C

9. $B = \tan(\arctan 1 - \arctan\frac{1}{2})$

Sea:

$\arctan 1 = \alpha \Rightarrow \tan\alpha = 1$

$\arctan\frac{1}{2} = \beta \Rightarrow \tan\beta = \frac{1}{2}$

Luego:
$B = \tan(\alpha - \beta)$

$B = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$B = \frac{1-\left(\frac{1}{2}\right)}{1+1\left(\frac{1}{2}\right)} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$

$\therefore B = \frac{1}{3}$

Clave C

10. Piden:

$Q = \frac{\text{arcsen}\frac{1}{3}}{\text{arcsen}\left(-\frac{1}{3}\right)}$

Sabemos:
$\text{arcsen}(-x) = -\text{arcsen}x$, si: $x \in [-1; 1]$

Como $-\frac{1}{3} \in [-1; 1]$, entonces:

$\text{arcsen}\left(-\frac{1}{3}\right) = -\text{arcsen}\ \frac{1}{3}$

Reemplazando en la expresión Q:

$\Rightarrow Q = \frac{\text{arcsen}\frac{1}{3}}{-\text{arcsen}\frac{1}{3}} = -1$

$\therefore Q = -1$

Clave B

11. Piden:

$Q = \frac{\arctan 1}{\arccos\frac{1}{2}}$

Sea:
$\alpha = \arctan 1 \Rightarrow \tan\alpha = 1$

$\Rightarrow \alpha = \frac{\pi}{4}$

$\beta = \arccos\frac{1}{2} \quad \Rightarrow \cos\beta = \frac{1}{2}$

$\Rightarrow \beta = \frac{\pi}{3}$

Entonces:

$Q = \frac{\alpha}{\beta} = \frac{\left(\frac{\pi}{4}\right)}{\left(\frac{\pi}{3}\right)} = \frac{3}{4}$

$\therefore Q = \frac{3}{4}$

Clave E

12. $B = \sec^2(\arctan 3) + \csc^2(\text{arccot}5)$

$B = \tan^2(\arctan 3) + 1 + \cot^2(\text{arccot}5) + 1$

$B = 2 + (\tan(\arctan 3))^2 + (\cot(\text{arccot}5))^2$

$B = 2 + (3)^2 + (5)^2$

$B = 2 + 9 + 25 = 36$

$\therefore B = 36$

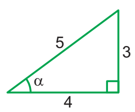
Clave D

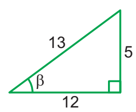
13. $\arccos\frac{4}{5} + \arccos\frac{12}{13} = \arccos\frac{33}{x}$

Sea:

$\alpha = \arccos\frac{4}{5}$ $\qquad$ $\beta = \arccos\frac{12}{13}$

$\cos\alpha = \frac{4}{5}$ $\qquad$ $\cos\beta = \frac{12}{13}$





Entonces:

$\arccos\frac{33}{x} = \alpha + \beta$

$\Rightarrow \frac{33}{x} = \cos(\alpha + \beta)$

$\frac{33}{x} = \cos\alpha\cos\beta - \mathrm{sen}\alpha\,\mathrm{sen}\beta$

$\frac{33}{x} = \frac{4}{5}\cdot\frac{12}{13} - \frac{3}{5}\cdot\frac{5}{13}$

$\frac{33}{x} = \frac{33}{65}$

$\therefore x = 65$

Clave C

14. $\arccos\frac{\sqrt{8}}{3} = \mathrm{arcsen}x$

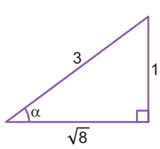
$\mathrm{sen}\left(\arccos\frac{\sqrt{8}}{3}\right) = \mathrm{sen}(\mathrm{arcsen}x)$

$\mathrm{sen}(\underbrace{\arccos\frac{\sqrt{8}}{3}}_{\alpha}) = x$

$\Rightarrow x = \mathrm{sen}\alpha$

Luego: $\alpha = \arccos\frac{\sqrt{8}}{3}$

$\cos\alpha = \frac{\sqrt{8}}{3}$



Entonces:

$x = \mathrm{sen}\alpha = \frac{1}{3}$

$\therefore x = \frac{1}{3}$

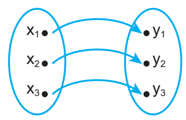
Clave E

PRACTIQUEMOS
Nivel 1 (página 84) Unidad 4

Comunicación matemática

1.

I.



Varios elementos en el dominio
$\Rightarrow$ I es falso

II. $\forall\ y \in \mathrm{Ran}(f)\ \exists\ \mathrm{Dom}(f) \in x$: $f(x) = y$
$\Rightarrow$ II es verdadero

III. Una función es biyectiva si es inyectiva y sobreyectiva
$\Rightarrow$ III es falso

IV. Si una función f es biyectiva, entonces su función inversa f^{-1} existe y es también biyectiva.
$\Rightarrow$ IV es verdadero

$\therefore$ FVFV

Clave C

2.

Función	Dominio
y = senx	$\left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$
y = tanx	$\left\langle-\frac{\pi}{2}; \frac{\pi}{2}\right\rangle$
y = secx	$\left[0; \frac{\pi}{2}\right\rangle \cup \left\langle\frac{\pi}{2}; \pi\right]$
y = cscx	$\left[-\frac{\pi}{2}; 0\right\rangle \cup \left\langle 0; \frac{\pi}{2}\right]$
y = cosx	$[0; \pi]$

Razonamiento y demostración

3. Por dato: $\cos\left(\theta + \frac{\pi}{3}\right) = m$

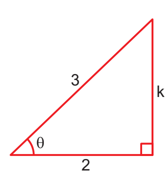
$\Rightarrow \left(\theta + \frac{\pi}{3}\right) = \arccos m$

$\therefore \theta = \arccos m - \frac{\pi}{3}$

Clave A

4. Por dato: $\theta = \arccos\frac{2}{3}$

$\Rightarrow \cos\theta = \frac{2}{3}$



Por el teorema de Pitágoras:

$2^2 + k^2 = 3^2$

$\Rightarrow k = \sqrt{5}$

Piden:

$\tan\theta = \frac{k}{2} = \frac{\sqrt{5}}{2}$

$\therefore \tan\theta = \frac{\sqrt{5}}{2}$

Clave B

5. Por dato: $\alpha = \arctan\frac{2}{3}$

$\Rightarrow \tan\alpha = \frac{2}{3}$

Piden:

$P = \mathrm{sen}\alpha\cos\alpha = \frac{2\mathrm{sen}\alpha\cos\alpha}{2}$

$P = \frac{\mathrm{sen}2\alpha}{2} = \frac{1}{2}\left(\frac{2\tan\alpha}{1 + \tan^2\alpha}\right)$

$P = \frac{1}{2}\left(\frac{2\left(\frac{2}{3}\right)}{1 + \left(\frac{2}{3}\right)^2}\right) = \frac{1}{2}\left(\frac{12}{13}\right)$

$\therefore P = \frac{6}{13}$

Clave A

6. Por dato: $\mathrm{sen}\frac{3\theta}{2} = x$

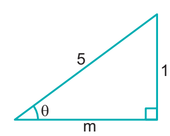
$\Rightarrow \frac{3\theta}{2} = \mathrm{arcsen}x$

$\therefore \theta = \frac{2}{3}\mathrm{arcsen}x$

Clave B

7. Por dato: $\theta = \mathrm{arcsen}\frac{1}{5}$

$\Rightarrow \mathrm{sen}\theta = \frac{1}{5}$



Por el teorema de Pitágoras:

$m^2 + 1^2 = 5^2$

$\Rightarrow m = 2\sqrt{6}$

Piden: $\cot\theta = \frac{m}{1} = \frac{2\sqrt{6}}{1}$

$\therefore \cot\theta = 2\sqrt{6}$

Clave B

8. Por dato: $\theta = \arctan\frac{3}{2}$

$\Rightarrow \tan\theta = \frac{3}{2}$

Piden:

$\mathrm{sen}2\theta = \frac{2\tan\theta}{1 + \tan^2\theta}$

$\mathrm{sen}2\theta = \frac{2\left(\frac{3}{2}\right)}{1 + \left(\frac{3}{2}\right)^2} = \frac{3}{\left(\frac{13}{4}\right)}$

$\therefore \mathrm{sen}2\theta = \frac{12}{13}$

Clave A

9. Piden:

$P = \tan(\arctan 4 - \arctan 3)$

Sea: $\alpha = \arctan 4 - \arctan 3$

Sabemos:
$\arctan(-x) = -\arctan x$, si: $x \in \mathbb{R}$

Como $3 \in \mathbb{R}$, entonces:
$\arctan(-3) = -\arctan 3$

Luego:
$\alpha = \arctan 4 + \arctan(-3)$

Por propiedad: $\alpha = \arctan\left(\dfrac{4+(-3)}{1-4(-3)}\right) + k\pi$

Como: $4(-3) = -12 < 1 \Rightarrow k = 0$

$\alpha = \arctan\left(\dfrac{1}{13}\right) + (0)\pi$

$\alpha = \arctan\dfrac{1}{13} \Rightarrow \tan\alpha = \dfrac{1}{13}$

Entonces: $P = \tan(\alpha) = \dfrac{1}{13}$

$\therefore P = \dfrac{1}{13}$

Clave E

10. Sea:

$\alpha = \arccos\dfrac{1}{2} \Rightarrow \cos\alpha = \dfrac{1}{2}$

$\Rightarrow \alpha = \dfrac{\pi}{3}$

$\theta = \arccos\left(-\dfrac{1}{2}\right)$

$\theta = \pi - \arccos\dfrac{1}{2} = \pi - \left(\dfrac{\pi}{3}\right)$

$\Rightarrow \theta = \dfrac{2\pi}{3}$

$\beta = \arccos\left(-\dfrac{\sqrt{2}}{2}\right)$

$\beta = \pi - \arccos\dfrac{\sqrt{2}}{2} = \pi - \left(\dfrac{\pi}{4}\right)$

$\Rightarrow \beta = \dfrac{3\pi}{4}$

Piden:

$$R = \dfrac{\arccos\dfrac{1}{2} + \arccos\left(-\dfrac{1}{2}\right)}{\arccos\left(-\dfrac{\sqrt{2}}{2}\right)}$$

$$R = \dfrac{\alpha+\theta}{\beta} = \dfrac{\left(\dfrac{\pi}{3}\right)+\left(\dfrac{2\pi}{3}\right)}{\left(\dfrac{3\pi}{4}\right)}$$

$$R = \dfrac{\pi}{\left(\dfrac{3\pi}{4}\right)} = \dfrac{4}{3}$$

$\therefore R = \dfrac{4}{3}$

Clave D

Resolución de problemas

11. Sabemos:

$\operatorname{arcsen} x \Rightarrow x \in [-1; 1]$

Entonces:
$0 \le \cos^4\alpha + \operatorname{sen}^4\alpha \le 1$ ...(I)

Recordemos:

$\dfrac{1}{2^{n-1}} \le \operatorname{sen}^{2n}\alpha + \cos^{2n}\alpha \le 1;\ n \in \mathbb{Z}^+$

$\Rightarrow \dfrac{1}{2} \le \operatorname{sen}^4\alpha + \cos^4\alpha \le 1$ ... (II)

Intersecamos (I) y (II):

$\dfrac{1}{2} \le \operatorname{sen}^4\alpha + \cos^4\alpha \le 1$

$\operatorname{arcsen}\left(\dfrac{1}{2}\right) \le \operatorname{arcsen}[\operatorname{sen}^4\alpha + \cos^4\alpha] \le \operatorname{arcsen}(1)$

$\dfrac{\pi}{6} \le M(\alpha) \le \dfrac{\pi}{2}$

$\therefore \operatorname{Ran}(M) = \left[\dfrac{\pi}{6}; \dfrac{\pi}{2}\right]$

Clave A

12. El dominio es definido por:

$\operatorname{arcsen} k \Leftrightarrow k \in [-1; 1]$

$\Rightarrow -1 \le 4x - 9 \le 1$

$8 \le 4x \le 10$

$2 \le x \le \dfrac{5}{2}$

$\therefore \operatorname{Dom}(f) = \left[2; \dfrac{5}{2}\right]$

Definimos el rango:

$-\dfrac{\pi}{2} \le \operatorname{arcsen} k \le \dfrac{\pi}{2}$

$-\dfrac{\pi}{2} \le \operatorname{arcsen}(4x-9) \le \dfrac{\pi}{2}$

$-2\pi \le 4\operatorname{arcsen}(4x-9) \le 2\pi$

$-\pi \le 4\operatorname{arcsen}(4x-9) + \pi \le 3\pi$

$-\pi \le F(x) \le 3\pi$

$\therefore \operatorname{Ran}(F) = [-\pi; 3\pi]$

Clave D

Nivel 2 (página 85) Unidad 4

Comunicación matemática

13.

Función	Dominio	Rango
$y = \operatorname{arcsen} x$	$[-1; 1]$	$\left[-\dfrac{\pi}{2}; \dfrac{\pi}{2}\right]$
$y = \operatorname{arcsec} x$	$\langle -\infty; -1] \cup [1; +\infty\rangle$	$\left[0; \dfrac{\pi}{2}\right\rangle \cup \left\langle \dfrac{\pi}{2}; \pi\right]$
$y = \arctan x$	$\mathbb{R}$	$\left\langle -\dfrac{\pi}{2}; \dfrac{\pi}{2}\right\rangle$
$y = \arccos x$	$[-1; 1]$	$[0; \pi]$
$y = \operatorname{arccsc} x$	$\langle -\infty; -1] \cup [1; +\infty\rangle$	$\left[-\dfrac{\pi}{2}; 0\right\rangle \cup \left\langle 0; \dfrac{\pi}{2}\right]$

14. Por propiedad, si $x \in [-1; 1]$

$\Rightarrow$ arcsenx + arccosx $= \dfrac{\pi}{2}$ (V)

Por propiedad; si $x \in \mathbb{R} - \langle -1; 1\rangle$

$\Rightarrow$ arccscx + arcsecx $= \dfrac{\pi}{2}$ (F)

Por propiedad:

$$\arctan(a) + \arctan(b) = \arctan\left(\frac{a+b}{1-ab}\right) + k\pi$$

Si: $ab > 1$; $a < 0 \ \wedge \ b < 0$

$\Rightarrow k = -1$

$\Rightarrow \arctan(a) + \arctan(b)$

$= \arctan\left(\dfrac{a+b}{1-ab}\right) - \pi$ (F)

Por definición:

$\theta = \text{arcsen}x \Leftrightarrow \text{sen}\theta = x \wedge \theta \in \left[-\dfrac{\pi}{2}; \dfrac{\pi}{2}\right]$ (V)

Por definición:

$\theta = \arccos x \Leftrightarrow \cos\theta = x \wedge \theta \in [0; \pi]$ (F)

$\therefore$ Dos son verdaderas.

Clave B

Razonamiento y demostración

15. Por dato: $\alpha = \arctan\dfrac{1}{3}$

$\Rightarrow \tan\alpha = \dfrac{1}{3}$

Piden:

$$P = \text{sen}\alpha\cos\alpha = \frac{2\text{sen}\alpha\cos\alpha}{2}$$

$$P = \frac{\text{sen}2\alpha}{2} = \frac{1}{2}\left(\frac{2\tan\alpha}{1+\tan^2\alpha}\right)$$

$$P = \frac{1}{2}\left[\frac{2\left(\frac{1}{3}\right)}{1+\left(\frac{1}{3}\right)^2}\right] = \frac{1}{2}\left(\frac{3}{5}\right)$$

$$\therefore P = \frac{3}{10} = 0{,}3$$

Clave C

16. Por dato: $\alpha = \text{arcsec}2\sqrt{2}$

$\Rightarrow \sec\alpha = 2\sqrt{2} \Rightarrow \cos\alpha = \dfrac{1}{2\sqrt{2}}$

Piden:

$\cos2\alpha = 2\cos^2\alpha - 1$

$$\cos2\alpha = 2\left(\frac{1}{2\sqrt{2}}\right)^2 - 1 = -\frac{3}{4}$$

$$\therefore \cos2\alpha = -\frac{3}{4}$$

Clave B

17. Sea:

$\alpha = \text{arcsen}\dfrac{1}{2} \Rightarrow \text{sen}\alpha = \dfrac{1}{2} \Rightarrow \alpha = \dfrac{\pi}{6}$

$\theta = \arccos\dfrac{\sqrt{2}}{2} \Rightarrow \cos\theta = \dfrac{\sqrt{2}}{2} \Rightarrow \theta = \dfrac{\pi}{4}$

$\beta = \arctan\sqrt{3} \Rightarrow \tan\beta = \sqrt{3}$

$\Rightarrow \beta = \dfrac{\pi}{3}$

Piden:

$$Q = \frac{\text{arcsen}\frac{1}{2} + \arccos\frac{\sqrt{2}}{2}}{\arctan\sqrt{3}}$$

$$Q = \frac{\alpha + \theta}{\beta} = \frac{\left(\frac{\pi}{6}\right) + \left(\frac{\pi}{4}\right)}{\left(\frac{\pi}{3}\right)}$$

$$Q = \frac{\left(\frac{5\pi}{12}\right)}{\left(\frac{\pi}{3}\right)} = \frac{5}{4}$$

$$\therefore Q = \frac{5}{4}$$

Clave E

18. Sea:

$\alpha = \arctan(-1)$

$\alpha = -(\arctan1) = -\left(\dfrac{\pi}{4}\right)$

$\Rightarrow \alpha = -\dfrac{\pi}{4}$

$\theta = \arccos\dfrac{\sqrt{3}}{2} \Rightarrow \cos\theta = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \theta = \dfrac{\pi}{6}$

$\beta = \text{arcsen}\left(-\dfrac{1}{2}\right)$

$\beta = -\left(\text{arcsen}\dfrac{1}{2}\right) = -\left(\dfrac{\pi}{6}\right)$

$\Rightarrow \beta = -\dfrac{\pi}{6}$

Piden:

$$R = \frac{\arctan(-1) + \arccos\frac{\sqrt{3}}{2}}{\text{arcsen}\left(-\frac{1}{2}\right)}$$

$$R = \frac{\alpha + \theta}{\beta} = \frac{\left(-\frac{\pi}{4}\right) + \left(\frac{\pi}{6}\right)}{\left(-\frac{\pi}{6}\right)}$$

$$R = \frac{\left(-\frac{\pi}{12}\right)}{\left(-\frac{\pi}{6}\right)} = \frac{1}{2}$$

$$\therefore R = \frac{1}{2}$$

Clave A

19. Piden el valor de x.

Por dato:

$\arctan(\text{sen}^2(\arctan\sqrt{3})) = \text{arcsen}(2x - 1)$

Sabemos: $\arctan\sqrt{3} = \dfrac{\pi}{3}$

Entonces.

$$\arctan\left(\text{sen}^2\frac{\pi}{3}\right) = \text{arcsen}(2x-1)$$

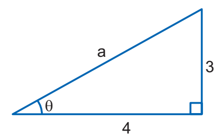
$$\arctan\left[\left(\frac{\sqrt{3}}{2}\right)^2\right] = \text{arcsen}(2x-1)$$

$$\arctan\frac{3}{4} = \text{arcsen}(2x-1)$$

Sea:

$\theta = \arctan\dfrac{3}{4} = \text{arcsen}(2x - 1)$

$\Rightarrow \tan\theta = \dfrac{3}{4} \ \wedge \ \text{sen}\theta = 2x - 1$



Por el teorema de Pitágoras:

$3^2 + 4^2 = a^2$

$\Rightarrow a = 5$

$\Rightarrow \text{sen}\theta = \dfrac{3}{a} = 2x - 1$

$\Rightarrow \dfrac{3}{5} = 2x - 1 \Rightarrow 2x = \dfrac{8}{5}$

$\therefore x = \dfrac{4}{5}$

Clave D

20. Piden:

$S = \tan(2\arctan x)\cos^2(\text{arcsen}x)$

Sean:

$\arctan x = \alpha \Rightarrow \tan\alpha = x$

$\text{arcsen}x = \theta \Rightarrow \text{sen}\theta = x$

Entonces:

$S = \tan(2\alpha)\cos^2(\theta)$

$S = \tan2\alpha\cos^2\theta$

$$S = \left(\frac{2\tan\alpha}{1-\tan^2\alpha}\right)(1 - \text{sen}^2\theta)$$

$$S = \left(\frac{2(x)}{1-(x)^2}\right)(1-(x)^2)$$

$$S = \frac{(2x)}{(1-x^2)}(1-x^2) = 2x$$

$\therefore S = 2x$

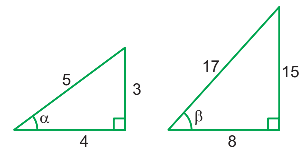
Clave B

21. Piden: $\cos\left(\text{arcsen}\dfrac{3}{5} - \arccos\dfrac{8}{17}\right)$

Sea:

$\alpha = \text{arcsen}\dfrac{3}{5} \Rightarrow \text{sen}\alpha = \dfrac{3}{5}$

$\beta = \arccos\dfrac{8}{17} \Rightarrow \cos\beta = \dfrac{8}{17}$



Entonces:

$\cos(\text{arcsen}\dfrac{3}{5} - \arccos\dfrac{8}{17}) = \cos(\alpha - \beta)$

Luego:

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \text{sen}\alpha\text{sen}\beta$

$$\cos(\alpha - \beta) = \left(\frac{4}{5}\right)\left(\frac{8}{17}\right) + \left(\frac{3}{5}\right)\left(\frac{15}{17}\right)$$

$$\cos(\alpha - \beta) = \frac{77}{85}$$

$$\therefore \cos\left(\text{arcsen}\frac{3}{5} - \text{arccos}\frac{8}{17}\right) = \frac{77}{85}$$

Clave D

22. Sea:

$$E = \text{arcsen}\left(\text{sen}\frac{8\pi}{9}\right) + \text{arccos}\left(\cos\frac{19\pi}{18}\right)$$

Por propiedad:

$\text{arcsen}(\text{sen}x) = x$; si: $x \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

$\text{arccos}(\cos x) = x$; si: $x \in [0; \pi]$

Observamos que $8\pi/9$ y $19\pi/18$ no se encuentran en los intervalos para aplicar la propiedad respectiva, para ello buscamos los equivalentes de:

$$\text{sen}\frac{8\pi}{9} = \text{sen}\left(\pi - \frac{\pi}{9}\right) = \text{sen}\frac{\pi}{9}$$

$$\Rightarrow \text{sen}\frac{8\pi}{9} = \text{sen}\frac{\pi}{9}$$

$$\cos\frac{19\pi}{18} = \cos\left(2\pi - \frac{17\pi}{18}\right) = \cos\frac{17\pi}{18}$$

$$\Rightarrow \cos\frac{19\pi}{18} = \cos\frac{17\pi}{18}$$

Luego:

$$E = \text{arcsen}\left(\text{sen}\frac{\pi}{9}\right) + \text{arcos}\left(\cos\frac{17\pi}{18}\right)$$

Ahora $\frac{\pi}{9}$ y $\frac{17\pi}{18}$ si se encuentran en los intervalos para aplicar la propiedad respectiva, entonces:

$$E = \left(\frac{\pi}{9}\right) + \left(\frac{17\pi}{18}\right) = \frac{19\pi}{18}$$

$$\Rightarrow E = \frac{19\pi}{18}$$

$$\therefore \text{arcsen}\left(\text{sen}\frac{8\pi}{9}\right) + \text{arccos}\left(\cos\frac{19\pi}{18}\right) = \frac{19\pi}{18}$$

Clave A

23. Piden:

$$M = \tan\left(\frac{\pi}{4} - \text{arccot}\,3\right)$$

Sea: $\theta = \text{arccot}3$

$$\Rightarrow \cot\theta = 3 \Rightarrow \tan\theta = \frac{1}{3}$$

Entonces:

$$M = \tan\left(\frac{\pi}{4} - \theta\right)$$

$$M = \frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta}$$

$$M = \frac{(1) - \left(\frac{1}{3}\right)}{1 + (1)\left(\frac{1}{3}\right)} = \frac{\left(\frac{2}{3}\right)}{\left(\frac{4}{3}\right)}$$

$$\therefore M = \frac{1}{2}$$

Clave B

Resolución de problemas

24. De las ecuaciones:

$$\left.\begin{aligned} \text{sen}x + \cos y &= \frac{16}{21} \\ \cos y - \text{sen}x &= \frac{2}{21} \end{aligned}\right\}(+)$$

$$2\cos y = \frac{18}{21}$$

$$\cos y = \frac{9}{21} = \frac{3}{7}$$

$$\text{sen}x = \frac{7}{21} = \frac{1}{3}$$

Si $\cos y = \frac{3}{7}$:

$$\therefore y = 2\pi - \text{arccos}\left(\frac{3}{7}\right)$$

Si $\text{sen}x = \frac{1}{3}$

$$\therefore x = \text{arcsen}x\left(\frac{1}{3}\right)$$

Clave x

25. En la función:

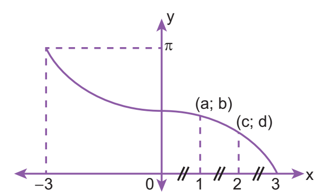
$$y = \text{arccos}\left(\frac{x}{3}\right)$$

$$-1 \le \frac{x}{3} \le 1$$

$$-3 \le x \le 3$$

$$0 \le \text{arccos}\left(\frac{x}{3}\right) \le \pi$$

En el gráfico tenemos:



$$\Rightarrow a = 1; c = 2$$

$$b = \text{arccos}\left(\frac{1}{3}\right)$$

Nos piden: $(a + c) - b$

$$= (1 + 2) - \text{arccos}\left(\frac{1}{3}\right)$$

$$3 - \text{arcos}\left(\frac{1}{3}\right)$$

Clave B

Nivel 3 (página 86) Unidad 4

Comunicación matemática

26.

Por propiedad sabemos:

$$\text{arccos}x + \text{arcsen}x = \frac{\pi}{2}, \quad -1 \le x \le 1$$

$$\text{arctan}x + \text{arccot}x = \frac{\pi}{2}, \quad x \in \mathbb{R}$$

Entonces:

$$\left.\begin{aligned} \text{arccos}\left(\frac{2}{3}\right) + \text{arcsen}\left(\frac{2}{3}\right) &= \frac{\pi}{2} \\ \text{arccos}\left(\frac{1}{3}\right) + \text{arcsen}\left(\frac{1}{3}\right) &= \frac{\pi}{2} \end{aligned}\right\}(+)$$

$$M = \pi$$

$$\left.\begin{aligned} \text{arctan}\left(\frac{4}{5}\right) + \text{arccot}\left(\frac{4}{5}\right) &= \frac{\pi}{2} \\ \text{arctan}\left(\frac{5}{4}\right) + \text{arccot}\left(\frac{5}{4}\right) &= \frac{\pi}{2} \end{aligned}\right\}(+)$$

$$N = \pi$$

$$\therefore M = N$$

Clave D

27. $\text{arcsen}[\text{sen}(x)] \Leftrightarrow -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\Rightarrow \text{arcsen}[\text{sen}(\pi)] \nexists$

$\text{arctan}[\tan(x)] \Leftrightarrow -\frac{\pi}{2} < x < \frac{\pi}{2}$

$\Rightarrow \text{arctan}\left[\tan\left(-\frac{\pi}{3}\right)\right]$ existe

$\text{arcsec}[\sec(x)] \Leftrightarrow x \in [0; \pi] - \left\{\frac{\pi}{2}\right\}$

$\Rightarrow \text{arcsec}\left[\sec\left(\frac{\pi}{2}\right)\right] \nexists$

$\text{arccos}[\cos(x)] \Leftrightarrow 0 \le x \le \pi$

$\Rightarrow \text{arccos}[\cos(0)]$ existe

$\text{arccsc}[\csc(x)] \Leftrightarrow x \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] - \{0\}$

$\Rightarrow \text{arccsc}\left[\csc\left(\frac{2\pi}{5}\right)\right]$ existe

$\therefore$ Existen tres proposiciones.

Clave E

Razonamiento y demostración

28. Piden:

$$T = (\text{arctan}2 + \text{arccot}2)(\text{arcsec}3 + \text{arccsc}3)$$

Por propiedad:

$\text{arctan}x + \text{arccot}x = \frac{\pi}{2}$, si: $x \in \mathbb{R}$

Como $2 \in \mathbb{R}$, entonces:

$$\text{arctan}2 + \text{arccot}2 = \frac{\pi}{2}$$

$\text{arcsec}x + \text{arccsc}x = \frac{\pi}{2}$; si $x \in \mathbb{R} - \langle -1; 1\rangle$

Como $3 \in \mathbb{R} - \langle -1; 1\rangle$, entonces:

$$\text{arcsec}3 + \text{arccsc}3 = \frac{\pi}{2}$$

Reemplazando en la expresión T:

$$T = \left(\frac{\pi}{2}\right)\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4}$$

$$\therefore T = \frac{\pi^2}{4}$$

Clave B

29. Por dato: 2arcsenx = 3arccosx

Por propiedad:

$\text{arcsenx} + \text{arccosx} = \frac{\pi}{2}; x \in [-1; 1]$

Entonces:

$2\text{arcsenx} = 3(\frac{\pi}{2} - \text{arcsenx})$

$2\text{arcsenx} = \frac{3\pi}{2} - 3\text{arcsenx}$

$5\text{arcsenx} = \frac{3\pi}{2}\text{s}$

$\text{arcsenx} = \frac{3\pi}{10} \Rightarrow x = \text{sen}\frac{3\pi}{10}$

$\Rightarrow x = \text{sen}54° = \text{sen}(90° - 36°)$

$\Rightarrow x = \cos 36° = \left(\frac{\sqrt{5}+1}{4}\right) \in [-1; 1]$

$\therefore x = \frac{\sqrt{5}+1}{4}$

Clave B

30. Sea:

$$H = \frac{\text{arcsen}\left(\frac{1}{2}\right)\text{arccos}\,x}{\text{arccos}\left(\frac{1}{2}\right)} + \frac{\text{arcsen}\left(\frac{\sqrt{2}}{2}\right)\text{arcsenx}}{\text{arcsen}(1)}$$

$$H = \frac{\left(\frac{\pi}{6}\right)\text{arccos}\,x}{\left(\frac{\pi}{3}\right)} + \frac{\left(\frac{\pi}{4}\right)\text{arcsenx}}{\left(\frac{\pi}{2}\right)}$$

$H = \frac{1}{2}\text{arccosx} + \frac{1}{2}\text{arcsenx}$

$H = \frac{1}{2}(\text{arccosx} + \text{arcsenx}) = \frac{1}{2}\left(\frac{\pi}{2}\right)$

$\Rightarrow H = \frac{\pi}{4}$

$$\therefore \frac{\text{arcsen}\left(\frac{1}{2}\right)\text{arccos}\,x}{\text{arccos}\left(\frac{1}{2}\right)} + \frac{\text{arcsen}\left(\frac{\sqrt{2}}{2}\right)\text{arcsenx}}{\text{arcsen}(1)} = \frac{\pi}{4}$$

Clave B

31. Piden:

$$E = \frac{\tan(3\text{arcsenx} + 2\,\text{arccos}\,x)}{\tan(3\text{arcsenx} + 4\,\text{arccos}\,x)}$$

Sea:

$\alpha = 3\text{arcsenx} + 2\text{arccosx}$ ...(I)

$\theta = 3\text{arcsenx} + 4\text{arccosx}$ ...(II)

Sumando (I) y (II):

$\alpha + \theta = 6\text{arcsenx} + 6\text{arccosx}$

$\alpha + \theta = 6(\text{arcsenx} + \text{arccosx})$

$\alpha + \theta = 6\left(\frac{\pi}{2}\right) = 3\pi$

$\Rightarrow \alpha = 3\pi - \theta \Rightarrow \tan\alpha = \tan(3\pi - \theta)$

$\Rightarrow \tan\alpha = \tan(2\pi + \pi - \theta) = \tan(\pi - \theta)$

$\Rightarrow \tan\alpha = -\tan\theta$

Entonces:

$E = \frac{\tan(\alpha)}{\tan(\theta)}$

$E = \frac{(-\tan\theta)}{\tan\theta} = -1$

$\therefore E = -1$

Clave B

32. Sea:

$H = \arctan\frac{1}{6} + \arctan\frac{5}{7}$

Por propiedad:

$$H = \arctan\left(\frac{\frac{1}{6}+\frac{5}{7}}{1-\frac{1}{6}\cdot\frac{5}{7}}\right) + k\pi$$

Como:

$\frac{1}{6}\cdot\frac{5}{7} < 1 \Rightarrow k = 0$

$\Rightarrow H = \arctan(1) + (0)\pi$

$\Rightarrow H = \arctan 1 = \frac{\pi}{4}$

$\therefore \arctan\frac{1}{6} + \arctan\frac{5}{7} = \frac{\pi}{4}$

Clave C

33. Por dato:

arctan2 + arctan3 = arcsecx

Por propiedad:

$\arctan 2 + \arctan 3 = \arctan\left(\frac{2+3}{1-2\cdot 3}\right) + k\pi$

Como:

$2\cdot 3 > 1$ y $2 > 0 \land 3 > 0 \Rightarrow k = 1$

Luego:

$\arctan 2 + \arctan 3 = \arctan(-1) + (1)\pi$

$\arctan 2 + \arctan 3 = \left(-\frac{\pi}{4}\right) + \pi$

$\Rightarrow \arctan 2 + \arctan 3 = \frac{3\pi}{4}$

Entonces:

$\frac{3\pi}{4} = \text{arcsecx} \Rightarrow x = \sec\frac{3\pi}{4}$

$\Rightarrow x = \sec 135° = -\sqrt{2}$

$\therefore x = -\sqrt{2}$

Clave E

34. $A = \text{sen}^2\left(\text{arccos}\frac{1}{2}\right) + \cos^4\left(\text{arcsen}\frac{\sqrt{2}}{2}\right)$

$A = \text{sen}^2\left(\frac{\pi}{3}\right) + \cos^4\left(\frac{\pi}{4}\right)$

$A = \left(\text{sen}\frac{\pi}{3}\right)^2 + \left(\cos\frac{\pi}{4}\right)^4$

$A = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^4 = \frac{3}{4} + \frac{1}{4}$

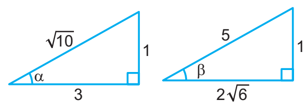
$\Rightarrow A = 1$

$$B = \csc^2(\text{arccot}3) - \sqrt{6}\cot\left(\text{arcsen}\frac{1}{5}\right)$$

Sea:

$\text{arccot}3 = \alpha \Rightarrow \cot\alpha = 3$

$\text{arcsen}\frac{1}{5} = \beta \Rightarrow \text{sen}\beta = \frac{1}{5}$



$B = \csc^2(\alpha) - \sqrt{6}\cot(\beta)$

$B = (\csc\alpha)^2 - \sqrt{6}\,(\cot\beta)$

$$B = \left(\frac{\sqrt{10}}{1}\right)^2 - \sqrt{6}\left(\frac{2\sqrt{6}}{1}\right) = 10 - 12$$

$\Rightarrow B = -2$

Piden:

$A + B = (1) + (-2) = -1$

$\therefore A + B = -1$

Clave B

35. Piden: sen(2arctan2)

Sea: $\arctan 2 = \theta$

$\Rightarrow \tan\theta = 2$

Luego:

$\text{sen}(2\arctan 2) - \text{sen}(2\theta)$

$$\text{sen}(2\arctan 2) = \frac{2\tan\theta}{1 + \tan^2\theta}$$

$$\text{sen}(2\arctan 2) = \frac{2(2)}{1+(2)^2} = \frac{4}{5}$$

$$\therefore \text{sen}(2\arctan 2) = \frac{4}{5}$$

Clave A

Resolución de problemas

36. Sabemos:

$\text{arcsen}x \Leftrightarrow x \in [-1; 1]$ ...(I)

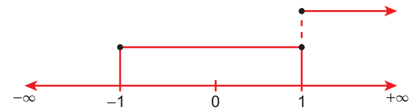
$$\sqrt{\arctan x - \frac{\pi}{4}} \Rightarrow \arctan x - \frac{\pi}{4} \geq 0$$

$$\arctan x \geq \frac{\pi}{4}$$

$$\tan[\arctan(x)] \geq \tan\frac{\pi}{4}$$

$x \geq 1$ ...(II)

Intersecamos (I) y (II):



Se intersecan en un solo punto:

$\therefore x = 1$

Reemplazamos en T(x):

$$T(x) = \sqrt{\arctan x - \frac{\pi}{4}} + \text{arcsen}x$$

$$T(x) = \sqrt{\arctan(1) - \frac{\pi}{4}} + \text{arcsen}(1)$$

$$T(x) = \sqrt{\frac{\pi}{4} - \frac{\pi}{4}} + \frac{\pi}{2} = \frac{\pi}{2}$$

$$\therefore \text{Ran}T(x) = \left\{\frac{\pi}{2}\right\}$$

Clave A

37. Sabemos:

$\text{arcsen}x \Leftrightarrow x \in [-1; 1] \quad \wedge \quad \text{arccos}x \Leftrightarrow x \in [-1; 1]$

Entonces, tenemos un dominio definido:

$$-\frac{\pi}{2} \leq \text{arcsen}x \leq \frac{\pi}{2}$$

$$0 \leq |\text{arcsen}x| \leq \frac{\pi}{2} \quad \text{... (1)}$$

$$0 \leq \text{arccos}x \leq \pi$$

$$0 \leq 2\text{arccos}x \leq 2\pi \quad \text{... (2)}$$

De (1) y (2):

$$0 \leq |\text{arcsen}x| + 2\text{arccos}x \leq \frac{5\pi}{2}$$

$$0 \leq ||\text{arcsen}x| + 2\text{arccos}x| \leq \frac{5\pi}{2}$$

Pero:

$$\text{arcsen}x + \text{arccos}x = \frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{2} \leq A(x) \leq 5\frac{\pi}{2}$$

Clave B

APLICAMOS LO APRENDIDO

Nivel 1 (página 87) Unidad 4

1. $\cos\left(\frac{x}{6}\right) = 1$

Entonces: VP = arccos1 = 0°

Luego:

$E_G = 2k\pi \pm VP;\ k \in \mathbb{Z}$

$E_G = 2k\pi \pm 0$

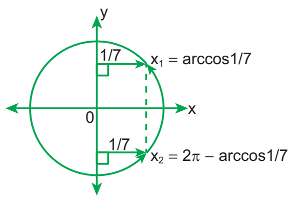
$\left(\frac{x}{6}\right) = 2k\pi$

$\therefore x \in 12k\pi;\ k \in \mathbb{Z}$

Clave D

2. $\cos x = \frac{1}{7}$

Analizando las dos primeras soluciones positivas en la CT:



Nos piden:

$x_1 + x_2 = \arccos\frac{1}{7} + \left(2\pi - \arccos\frac{1}{7}\right)$

$x_1 + x_2 = 2\pi = 360°$

$\therefore x_1 + x_2 = 360°$

Clave E

3. $x + y = 90°$ ...(I)

$\operatorname{sen}x = \sqrt{3}\operatorname{sen}y$ ...(II)

De (I):

$y = 90° - x$

$\operatorname{sen}y = \operatorname{sen}(90° - x)$

$\Rightarrow \operatorname{sen}y = \cos x$

Reemplazando en (II):

$\operatorname{sen}x = \sqrt{3}(\cos x)$

$\frac{\operatorname{sen}x}{\cos x} = \sqrt{3} \Rightarrow \tan x = \sqrt{3}$

$\Rightarrow x = 60°$

Reemplazando en (I):

$60° + y = 90°$

$\Rightarrow y = 30°$

Entonces: $x = 60° \wedge y = 30°$

Clave C

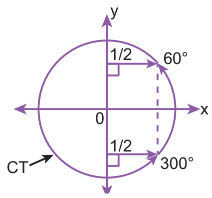
4. $\frac{\operatorname{sen}2x}{\operatorname{sen}x} = 1$

$\frac{2\operatorname{sen}x\cos x}{\operatorname{sen}x} = 1$

$2\cos x = 1$

$\Rightarrow \cos x = \frac{1}{2}$

Analizando en la CT:



Entonces las soluciones positivas de x son: {60°; 300°; 420°; ...}

Por lo tanto, la segunda solución positiva es 300°.

Clave A

5. $\cos3x + 2\cos x = 0$

$\underbrace{\cos3x + \cos x} + \cos x = 0$

$2\cos2x \cdot \cos x + \cos x = 0$

$\cos x(2\cos2x + 1) = 0$

$\Rightarrow \cos x = 0 \ \vee\ \cos2x = -\frac{1}{2}$

Para: $\cos x = 0$

$x = \{90°; 270°; ...\}$

Para: $\cos2x = -\frac{1}{2}$

$2x = \{120°; 240°; ...\}$

$\Rightarrow x = \{60°; 120°; ...\}$

Por lo tanto, una solución de la ecuación es 120°.

Clave C

6. $\operatorname{sen}x + \cos2x = 0$

$\operatorname{sen}x + (1 - 2\operatorname{sen}^2x) = 0$

$2\operatorname{sen}^2x - \operatorname{sen}x - 1 = 0$

$$\begin{array}{ll} 2\operatorname{sen}x & 1 \\ \operatorname{sen}x & -1 \end{array}$$

$(2\operatorname{sen}x + 1)(\operatorname{sen}x - 1) = 0$

$\Rightarrow \operatorname{sen}x = -\frac{1}{2} \ \vee\ \operatorname{sen}x = 1$

$x = 210°$ $\qquad x = 90°$

$x = 330°$ $\qquad x = 450°$

Las soluciones positivas para x serían:

$x \in \{90°; 210°; 330°; 450°; ...\}$

Por lo tanto, la tercera solución positiva es 330°.

Clave E

7. $\operatorname{sen}^4x + \cos^4x = \frac{5}{8}$

$1 - 2\operatorname{sen}^2x\cos^2x = \frac{5}{8}$

$2\operatorname{sen}^2x\cos^2x = \frac{3}{8}$

$4\operatorname{sen}^2x\cos^2x = \frac{3}{4}$

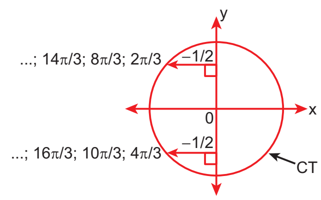
$(2\operatorname{sen}x\cos x)^2 = \frac{3}{4}$

$\operatorname{sen}^2 2x = \frac{3}{4}$

$\frac{1 - \cos4x}{2} = \frac{3}{4}$

$\Rightarrow \cos4x = -\frac{1}{2}$

Analizando en la CT:



Entonces:

$4x \in \left\{\frac{2\pi}{3}; \frac{4\pi}{3}; \frac{8\pi}{3}; \frac{10\pi}{3}; \frac{14\pi}{3}; \frac{16\pi}{3}; ...\right\}$

$\Rightarrow x \in \left\{\frac{\pi}{6}; \frac{\pi}{3}; \frac{2\pi}{3}; \frac{5\pi}{6}; \frac{7\pi}{6}; \frac{4\pi}{3}; ...\right\}$

En el intervalo de $[0; \pi]$ las soluciones serían:

$\left\{\frac{\pi}{6}; \frac{\pi}{3}; \frac{2\pi}{3}; \frac{5\pi}{6}\right\}$

Por lo tanto, el número de soluciones es 4.

Clave D

8. $\tan x + \cot x = 2$

$(\tan x)\tan x + (\tan x)\cot x = (\tan x)2$

$\tan^2x + 1 = 2\tan x$

$\tan^2x - 2\tan x + 1 = 0$

$(\tan x - 1)^2 = 0$

$\Rightarrow \tan x = 1$

Entonces:

$VP = \arctan 1 = \frac{\pi}{4}$

Luego:

$E_G = k\pi + VP;\ k \in \mathbb{Z}$

$\Rightarrow x = k\pi + \frac{\pi}{4};\ k \in \mathbb{Z}$

Las dos primeras soluciones positivas serán:

$x_1 = (0)\pi + \frac{\pi}{4} = \frac{\pi}{4}$

$x_2 = (1)\pi + \frac{\pi}{4} = \frac{5\pi}{4}$

Nos piden:

$x_1 + x_2 = \frac{\pi}{4} + \frac{5\pi}{4} = \frac{6\pi}{4} = \frac{3\pi}{2} = 270°$

$\therefore x_1 + x_2 = 270°$

Clave C

9. $2(\text{senx} + \cos x) = \sec x$

$$2\text{senx} + 2\cos x = \frac{1}{\cos x}$$

$$2\text{senx}\cos x + 2\cos^2 x = 1$$
$$\text{sen}2x + (\cos 2x + 1) = 1$$
$$\text{sen}2x + \cos 2x = 0$$
$$\sqrt{2}\,\text{sen}\left(2x + \frac{\pi}{4}\right) = 0$$
$$\text{sen}\left(2x + \frac{\pi}{4}\right) = 0$$

Entonces: VP = arcsen0 = 0

Luego:
$$E_G = k\pi + (-1)^k \underbrace{VP}_{0};\ k \in \mathbb{Z}$$
$$E_G = k\pi$$
$$\left(2x + \frac{\pi}{4}\right) = k\pi$$
$$2x = k\pi - \frac{\pi}{4}$$
$$\therefore x = \frac{k\pi}{2} - \frac{\pi}{8};\ k \in \mathbb{Z}$$

Clave A

10. $\text{sen}(5x - 10°) = \frac{\sqrt{2}}{2}$

Entonces: $VP = \text{arcsen}\,\frac{\sqrt{2}}{2} = \frac{\pi}{4}$

Luego:
$E_G = k\pi + (-1)^k \frac{\pi}{4}$

Para: k = 0
$E_G = \frac{\pi}{4} = 45°$

$(5x_1 - 10°) = 45° \Rightarrow x_1 = 11°$

Para: k = 1
$E_G = \pi + \frac{\pi}{4} = 135°$

$(5x_2 - 10°) = 135° \Rightarrow x_2 = 29°$

Para: k = 2
$E_G = 2\pi + \frac{\pi}{4} = 405°$

$(5x_3 - 10°) = 405° \Rightarrow x_3 = 83°$

Nos piden:
$$x_1 + x_2 + x_3 = 11° + 29° + 83° = 123°$$
$$\therefore x_1 + x_2 + x_3 = 123°$$

Clave B

11. Piden, la menor solución positiva de la ecuación:
$$2\tan 2x \tan x = 1 - \tan^2 x$$
$$\tan 2x\left(\frac{2\tan x}{1 - \tan^2 x}\right) = 1$$
$$\tan 2x(\tan 2x) = 1$$
$$\tan^2 2x = 1$$
$$\tan 2x = \pm 1$$
$$\Rightarrow \tan 2x = 1 \ \vee \ \tan 2x = -1$$

Empleando la expresión general para la tangente en ambos casos se tiene:
$$2x = k\pi + \arctan 1 \vee 2x = k\pi + \arctan(-1)$$
$$\Rightarrow x = \frac{k\pi}{2} + \frac{\pi}{8} \quad \vee \quad x = \frac{k\pi}{2} - \frac{\pi}{8};\ (k \in \mathbb{Z})$$

Evaluando:

Para: $k = 0 \Rightarrow x = \frac{\pi}{8} \ \vee \ x = -\frac{\pi}{8}$

Para: $k = 1 \Rightarrow x = \frac{5\pi}{8} \ \vee \ x = \frac{3\pi}{8}$

Por lo tanto, la menor solución positiva que satisface la igualdad original es $\frac{\pi}{8}$.

Clave B

12. Piden, la solución general de la ecuación:
$\tan 8x - \tan 4x = 0$

Empleando las identidades del ángulo doble:
$$\frac{2\tan 4x}{1 - \tan^2 4x} - \tan 4x = 0$$
$$\tan 4x\left[\frac{2}{1 - \tan^2 4x} - 1\right] = 0$$
$$\tan 4x\left[\frac{1 + \tan^2 4x}{1 - \tan^2 4x}\right] = 0$$
$$\tan 4x(\sec 8x) = 0$$
$$\frac{\tan 4x}{\cos 8x} = 0$$
$$\Rightarrow \tan 4x = 0;\ \cos 8x \neq 0 \Rightarrow x \neq (2k + 1)\frac{\pi}{16};\ k \in \mathbb{Z}$$

Empleando la expresión general para la tangente:
$E_G = k\pi + VP;\ k \in \mathbb{Z}$
$E_G = k\pi + \arctan 0$
$4x = k\pi + 0 \Rightarrow 4x = k\pi$
$\therefore x \in \frac{k\pi}{4};\ k \in \mathbb{Z}$

Clave B

13. Por dato:
$$\sec^2 x = \sqrt{3}\tan x + 1$$
$$\Rightarrow 1 + \tan^2 x = \sqrt{3}\tan x + 1$$
$$\tan^2 x = \sqrt{3}\tan x$$
$$\Rightarrow \tan x(\tan x - \sqrt{3}) = 0$$
$$\Rightarrow \tan x = 0 \ \vee \ \tan x = \sqrt{3}$$

Empleando la expresión general para la tangente en ambos casos se tiene:
$x = k\pi + \arctan 0 \quad \vee \quad x = k\pi + \arctan\sqrt{3}$
$x = k\pi \quad \vee \quad x = k\pi + \frac{\pi}{3};\ (k \in \mathbb{Z})$

Evaluando:

Para: $k = 0 \Rightarrow x = 0 \ \vee \ x = \frac{\pi}{3}$

Para: $k = 1 \Rightarrow x = \pi \ \vee \ x = \frac{4\pi}{3}$

Para: $k = 2 \Rightarrow x = 2\pi \ \vee \ x = \frac{7\pi}{3}$

Luego, las dos primeras soluciones positivas son: $\frac{\pi}{3}$ y π.

Piden la suma de las dos primeras soluciones positivas.
$$\Rightarrow \frac{\pi}{3} + \pi = \frac{4\pi}{3} = 240°$$

Clave B

14. Piden, la solución principal de la ecuación:
$$\text{sen}2x + \text{sen}4x + \text{senx} = 0$$
$$\text{sen}2x + 2\text{sen}2x\cos 2x + \text{senx} = 0$$
$$\text{sen}2x(1 + 2\cos 2x) + \text{senx} = 0$$
$$2\text{senx}\cos x(1 + 2\cos 2x) + \text{senx} = 0$$
$$\text{senx}[2\cos x(1 + 2\cos 2x) + 1] = 0$$

Al igualar cada factor a cero, se tiene:
senx = 0

Empleando la expresión general para el seno:
$E_G = k\pi + (-1)^k VP;\ k \in \mathbb{Z}$
$E_G = k\pi + (-1)^k \text{arcsen}0$
$E_G = k\pi + (-1)^k(0)$
$\Rightarrow x \in \{k\pi;\ k \in \mathbb{Z}\}$

Evaluando:
Para: $k = -1 \Rightarrow x = -\pi$
Para: $k = 0 \Rightarrow x = 0$
Para: $k = 1 \Rightarrow x = \pi$

Observamos que el cero forma parte de la solución de la ecuación y satisface la igualdad original, además es el menor valor real no negativo. Por lo tanto, la solución principal de la ecuación es 0.

Clave A

PRACTIQUEMOS

Nivel 1 (página 89) unidad 4

Comunicación matemática

1.

2.

Razonamiento y demostración

3. $\text{sen}6x = \frac{\sqrt{3}}{2}$

Entonces: $VP = \text{arcsen}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

Usando la expresión general para el seno:
$$x_G = k\pi + (-1)^k VP;\ k \in \mathbb{Z}$$
$$\Rightarrow \underbrace{x_G}_{} = k\pi + (-1)^k \cdot \frac{\pi}{3}$$
$$6x = k\pi + (-1)^k \cdot \frac{\pi}{3}$$
$$\therefore x \in \left\{\frac{k\pi}{6} + (-1)^k\frac{\pi}{18} / k \in \mathbb{Z}\right\}$$

Clave E

4. $\text{sen}4x = \frac{\sqrt{2}}{2}$

Entonces: $VP = \text{arcsen}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

Empleando la expresión general para el seno:

$x_G = k\pi + (-1)^k VP;\ k \in \mathbb{Z}$

$\underbrace{x_G} = k\pi + (-1)^k \frac{\pi}{4}$

$4x = k\pi + (-1)^k \frac{\pi}{4}$

$\therefore\ x \in \left\{\frac{k\pi}{4} + (-1)^k \frac{\pi}{16} / k \in \mathbb{Z}\right\}$

Clave D

5. $\cos 4x = \frac{1}{2}$

Entonces: $VP = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Usando la expresión general para el coseno:

$x_G = 2k\pi \pm VP;\ k \in \mathbb{Z}$

$\underbrace{x_G} = 2k\pi \pm \frac{\pi}{3}$

$4x = 2k\pi \pm \frac{\pi}{3}$

$\therefore\ x \in \left\{\frac{k\pi}{2} \pm \frac{\pi}{12} / k \in \mathbb{Z}\right\}$

Clave D

6. $\cos 8x = \frac{\sqrt{2}}{2}$

Entonces: $VP = \arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

Usando la expresión general para el coseno:

$x_G = 2k\pi \pm VP;\ k \in \mathbb{Z}$

$\underbrace{x_G} = 2k\pi \pm \frac{\pi}{4}$

$8x = 2k\pi \pm \frac{\pi}{4}$

$\therefore\ x \in \left\{\frac{k\pi}{4} \pm \frac{\pi}{32} / k \in \mathbb{Z}\right\}$

Clave D

7. $\tan 2x = \frac{\sqrt{3}}{3}$

Entonces:

$VP = \arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

Usando la expresión general para la tangente:

$x_G = k\pi + VP;\ k \in \mathbb{Z}$

$\underbrace{x_G} = k\pi + \frac{\pi}{6}$

$2x = k\pi + \frac{\pi}{6}$

$\therefore x \in \left\{\frac{k\pi}{2} + \frac{\pi}{12} / k \in \mathbb{Z}\right\}$

Clave C

8. $\tan 5x = 0$

Entonces: $VP = \arctan(0) = 0$

Usando la expresión general para la tangente:

$x_G = k\pi + VP;\ k \in \mathbb{Z}$

$\underbrace{x_G} = k\pi + 0$

$5x = k\pi$

$\therefore\ x \in \left\{\frac{k\pi}{5} / k \in \mathbb{Z}\right\}$

Clave B

9. $\text{sen}3x = -\frac{\sqrt{2}}{2}$

Entonces: $VP = \text{arcsen}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

Usando la expresión general para el seno:

$x_G = k\pi + (-1)^k VP;\ k \in \mathbb{Z}$

$\underbrace{x_G} = k\pi + (-1)^k\left(-\frac{\pi}{4}\right)$

$3x = k\pi - (-1)^k \frac{\pi}{4}$

$\Rightarrow x \in \left\{\frac{k\pi}{3} - (-1)^k \cdot \frac{\pi}{12} / k \in \mathbb{Z}\right\}$

Evaluando:

$k = 0 \Rightarrow x = -\frac{\pi}{12} = -15°$

$k = 1 \Rightarrow x = \frac{5\pi}{12} = 75°$

$k = 2 \Rightarrow x = \frac{7\pi}{12} = 105°$

$k = 3 \Rightarrow x = \frac{13\pi}{12} = 195°$

Piden la suma de las tres primeras soluciones positivas.

$\Rightarrow 75° + 105° + 195° = 375°$

Clave B

10. $\cos 3x = -\frac{\sqrt{2}}{2}$

Entonces: $VP = \arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

Empleando la expresión general para el coseno:

$x_G = 2k\pi \pm VP;\ k \in \mathbb{Z}$

$\underbrace{x_G} = 2k\pi \pm \frac{3\pi}{4}$

$3x = 2k\pi \pm \frac{3\pi}{4}$

$\Rightarrow x \in \left\{\frac{2k\pi}{3} \pm \frac{\pi}{4} / k \in \mathbb{Z}\right\}$

Evaluando:

$k = 0 \Rightarrow x = -\frac{\pi}{4} \quad \vee \quad x = \frac{\pi}{4}$

$k = 1 \Rightarrow x = \frac{5\pi}{12} \quad \vee \quad x = \frac{11\pi}{12}$

$k = 2 \Rightarrow x = \frac{13\pi}{12} \quad \vee \quad x = \frac{19\pi}{12}$

Piden la suma de las tres primeras soluciones positivas.

$\Rightarrow \frac{\pi}{4} + \frac{5\pi}{12} + \frac{11\pi}{12} = \frac{19\pi}{12} = 285°$

Clave D

Nivel 2 (página 90) Unidad 4

Comunicación matemática

11.

12.

Razonamiento y demostración

13. $\tan 4x = -\sqrt{3}$

Entonces: $VP = \arctan(-\sqrt{3}) = -\frac{\pi}{3}$

Usando la expresión general para la tangente:

$x_G = k\pi + VP;\ k \in \mathbb{Z}$

$\underbrace{x_G} = k\pi + \left(-\frac{\pi}{3}\right)$

$\Rightarrow 4x = k\pi - \frac{\pi}{3}$

$\Rightarrow x \in \left\{\frac{k\pi}{4} - \frac{\pi}{12} / k \in \mathbb{Z}\right\}$

Evaluando para obtener las soluciones positivas:

$k = 1 \Rightarrow x = \frac{\pi}{6} = 30°$

$k = 2 \Rightarrow x = \frac{5\pi}{12} = 75°$

$k = 3 \Rightarrow x = \frac{2\pi}{3} = 120°$

Ordenando las soluciones positivas tenemos:

$x \in \{30°; 75°; 120°; ...\}$

Piden la suma de las tres primeras soluciones positivas.

$\Rightarrow 30° + 75° + 120° = 225°$

Clave E

14. $\tan\left(4x - \frac{\pi}{3}\right) = 0$

Entonces: $VP = \arctan(0) = 0$

Usando la expresión general para la tangente:

$x_G = k\pi + VP;\ k \in \mathbb{Z}$

$\Rightarrow x_G = k\pi + 0$

$\Rightarrow \left(4x - \frac{\pi}{3}\right) = k\pi$

$\therefore\ x \in \left\{\frac{k\pi}{4} + \frac{\pi}{12} / k \in \mathbb{Z}\right\}$

Clave B

15. $\text{sen}(2x - 10°) = \frac{1}{2}$

Entonces: $VP = \text{arcsen}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

Empleando la expresión general para el seno:

$x_G = k\pi + (-1)^k VP;\ k \in \mathbb{Z}$

$x_G = k\pi + (-1)^k\left(\frac{\pi}{6}\right)$

$\Rightarrow \left(2x - \frac{\pi}{18}\right) = k\pi + (-1)^k \frac{\pi}{6}$

$\Rightarrow x \in \left\{\frac{k\pi}{2} + (-1)^k . \frac{\pi}{12} + \frac{\pi}{36} / k \in \mathbb{Z}\right\}$

Evaluando para obtener las soluciones positivas:

$k = 0 \Rightarrow x = \frac{\pi}{9} = 20°$

$k = 1 \Rightarrow x = \frac{4\pi}{9} = 80°$

$k = 2 \Rightarrow x = \frac{10\pi}{9} = 200°$

$k = 3 \Rightarrow x = \frac{13\pi}{9} = 260°$

Piden la suma de las cuatro primeras soluciones positivas.

$\Rightarrow 20° + 80° + 200° + 260° = 560°$

Clave C

16. $\text{sen}(5x - 10°) = \frac{\sqrt{3}}{2}$

Entonces: $VP = \text{arcsen}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

Usando la expresión general para el seno:

$x_G = k\pi + (-1)^k VP; k \in \mathbb{Z}$

$x_G = k\pi + (-1)^k\left(\frac{\pi}{3}\right)$

$\Rightarrow \left(5x - \frac{\pi}{18}\right) = k\pi + (-1)^k \frac{\pi}{3}$

$\Rightarrow x \in \left\{\frac{k\pi}{5} + (-1)^k \frac{\pi}{15} + \frac{\pi}{90} / k \in \mathbb{Z}\right\}$

Evaluando:

$k = -1 \Rightarrow x = -\frac{23\pi}{90} = -46°$

$k = 0 \Rightarrow x = \frac{7\pi}{90} = 14°$

$k = 1 \Rightarrow x = \frac{13\pi}{90} = 26°$

$k = 2 \Rightarrow x = \frac{43\pi}{90} = 86°$

$k = 3 \Rightarrow x = \frac{49\pi}{90} = 98°$

Piden la suma de las cuatro primeras soluciones positivas.

$\Rightarrow 14° + 26° + 86° + 98° = 224°$

Clave B

17. $\cos(2x - 14°) = \frac{1}{2}$

Entonces: $VP = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Usando la expresión general para el coseno:

$x_G = 2k\pi \pm VP; k \in \mathbb{Z}$

$x_G = 2k\pi \pm \frac{\pi}{3}$

$\Rightarrow \left(2x - \frac{7\pi}{90}\right) = 2k\pi \pm \frac{\pi}{3}$

$\Rightarrow x \in \left\{k\pi \pm \frac{\pi}{6} + \frac{7\pi}{180} / k \in \mathbb{Z}\right\}$

Evaluando para obtener las soluciones positivas:

$k = 0 \Rightarrow x = -23° \vee x = 37°$

$k = 1 \Rightarrow x = 157° \vee x = 217°$

$k = 2 \Rightarrow x = 337° \vee x = 397°$

Ordenando las soluciones positivas tenemos:

$x = \{37°; 157°; 217°; 337°; 397°; ...\}$

Piden la suma de las tres primeras soluciones positivas.

$\Rightarrow 37° + 157° + 217° = 411°$

Clave C

18. $\cos\left(3x + \frac{\pi}{8}\right) = \frac{\sqrt{2}}{2}$

Entonces: $VP = \arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

Empleando la expresión general para el coseno:

$x_G = 2k\pi \pm VP; k \in \mathbb{Z}$

$x_G = 2k\pi \pm \frac{\pi}{4}$

$\Rightarrow \left(3x + \frac{\pi}{8}\right) = 2k\pi \pm \frac{\pi}{4}$

$\Rightarrow x \in \left\{\frac{2k\pi}{3} \pm \frac{\pi}{12} - \frac{\pi}{24} / k \in \mathbb{Z}\right\}$

Evaluando para obtener las soluciones positivas:

$k = 0 \Rightarrow x = -\frac{\pi}{8} \quad \vee \quad x = \frac{\pi}{24}$

$k = 1 \Rightarrow x = \frac{13\pi}{24} \quad \vee \quad x = \frac{17\pi}{24}$

$k = 2 \Rightarrow x = \frac{29\pi}{24} \quad \vee \quad x = \frac{11\pi}{8}$

Ordenando las soluciones positivas tenemos:

$x \in \left\{\frac{\pi}{24}; \frac{13\pi}{24}; \frac{17\pi}{24}; \frac{29\pi}{24}; \frac{11\pi}{8}; ...\right\}$

Piden la suma de las tres primeras soluciones positivas:

$\Rightarrow \frac{\pi}{24} + \frac{13\pi}{24} + \frac{17\pi}{24} = \frac{31\pi}{24}$

Clave C

19. $\tan(5x + 20°) = \frac{\sqrt{3}}{3}$

Entonces: $VP = \arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

Empleando la expresión general para la tangente:

$x_G = k\pi + VP; k \in \mathbb{Z}$

$x_G = k\pi + \frac{\pi}{6}$

$\Rightarrow \left(5x + \frac{\pi}{9}\right) = k\pi + \frac{\pi}{6}$

$\Rightarrow x \in \left\{\frac{k\pi}{5} + \frac{\pi}{90} / k \in \mathbb{Z}\right\}$

Evaluando para obtener las soluciones positivas:

$k = 0 \Rightarrow x = \frac{\pi}{90} = 2°$

$k = 1 \Rightarrow x = \frac{19\pi}{90} = 38°$

$k = 2 \Rightarrow x = \frac{37\pi}{90} = 74°$

$k = 3 \Rightarrow x = \frac{11\pi}{18} = 110°$

Ordenando tenemos:

$x \in \{2°; 38°; 74°; 110°; ...\}$

Piden la suma de las cuatro primeras soluciones positivas.

$\Rightarrow 2° + 38° + 74° + 110° = 224°$

Clave C

20. $\tan(5x - 20°) = \sqrt{3}$

Entonces: $VP = \arctan(\sqrt{3}) = \frac{\pi}{3}$

Empleando la expresión general para la tangente:

$x_G = k\pi + VP; k \in \mathbb{Z}$

$x_G = k\pi + \frac{\pi}{3}$

$\Rightarrow \left(5x - \frac{\pi}{9}\right) = k\pi + \frac{\pi}{3}$

$\Rightarrow x \in \left\{\frac{k\pi}{5} + \frac{4\pi}{45} / k \in \mathbb{Z}\right\}$

Evaluando para obtener las soluciones positivas:

$k = 0 \Rightarrow x = \frac{4\pi}{45} = 16°$

$k = 1 \Rightarrow x = \frac{13\pi}{45} = 52°$

$k = 2 \Rightarrow x = \frac{22\pi}{45} = 88°$

$k = 3 \Rightarrow x = \frac{31\pi}{45} = 124°$

Piden la suma de las cuatro primeras soluciones positivas.

$\Rightarrow 16° + 52° + 88° + 124° = 280°$

Clave B

Nivel 3 (página 90) Unidad 4

Comunicación matemática

21.

22.

Razonamiento y demostración

23. $\tan(4x - 25°) = 2 - \sqrt{3}$

Sabemos: $\tan 15° = 2 - \sqrt{3}$

$\tan\frac{\pi}{12} = 2 - \sqrt{3}$

$\Rightarrow \arctan(2 - \sqrt{3}) = \frac{\pi}{12}$

Entonces: $VP = \arctan(2 - \sqrt{3}) = \frac{\pi}{12}$

Usando la expresión general para la tangente:

$x_G = k\pi + VP;\ k \in \mathbb{Z}$

$\Rightarrow x_G = k\pi + \frac{\pi}{12}$

$\Rightarrow \left(4x - \frac{5\pi}{36}\right) = k\pi + \frac{\pi}{12}$

$\therefore x \in \left\{\frac{k\pi}{4} + \frac{\pi}{18} / k \in \mathbb{Z}\right\}$

Clave B

24. $4\text{sen}^2x - 4\text{sen}x + 1 = 0$

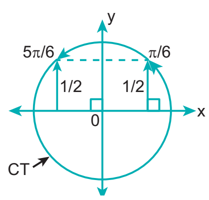
$2\text{sen}x \quad -1$
$2\text{sen}x \quad -1$

$(2\text{sen}x - 1)^2 = 0$

$2\text{sen}x - 1 = 0 \Rightarrow \text{sen}x = \frac{1}{2}$

Piden: la segunda solución positiva.

Analizando en la CT:



Observamos:

1.ª solución positiva $= \frac{\pi}{6} = 30°$

2.ª solución positiva $= \frac{5\pi}{6} = 150°$

Clave C

25. $2s2\cos^2x + 2 = 5\cos x$

$\Rightarrow 2\cos^2x - 5\cos x + 2 = 0$

$(2\cos x - 1)(\cos x - 2) = 0$

$\Rightarrow \cos x = \frac{1}{2} \lor \cos x = 2$

Sabemos: $-1 \leq \cos x \leq 1$

Entonces, en $\cos x = 2$ no existe solución en los $\mathbb{R}$.

Luego: $\cos x = \frac{1}{2}$

$\Rightarrow Vp = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Usando la expresión general para el coseno:

$x_G = 2k\pi \pm VP;\ k \in \mathbb{Z}$

$\Rightarrow x_G = 2k\pi \pm \frac{\pi}{3}$

$\Rightarrow x = \left\{2k\pi \pm \frac{\pi}{3} / k \in \mathbb{Z}\right\}$

Evaluando:

$k = 0 \Rightarrow x = -\frac{\pi}{3} \lor x = \frac{\pi}{3}$

$k = 1 \Rightarrow x = \frac{5\pi}{3} \lor x = \frac{7\pi}{3}$

$k = 2 \Rightarrow x = \frac{11\pi}{3} \lor x = \frac{13\pi}{3}$

Ordenando las soluciones positivas tenemos:

$x = \left\{\frac{\pi}{3}; \frac{5\pi}{3}; \frac{7\pi}{3}; \frac{11\pi}{3}; \frac{13\pi}{3}; ...\right\}$

Piden la tercera solución positiva.

$\Rightarrow x = \frac{7\pi}{3} = \frac{7(180°)}{3} = 420°$

Clave C

26. $10\cos^2x + 4 = 13\cos x$

$10\cos^2x - 13\cos x + 4 = 0$

$5\cos x \quad -4$
$2\cos x \quad -1$

$(5\cos x - 4)(2\cos x - 1) = 0$

$\Rightarrow \cos x = \frac{4}{5} \lor \cos x = \frac{1}{2}$

Piden la tercera solución positiva.

Analizando en los cuadrantes donde el coseno es positivo:

En el IC: $\cos 37° = \frac{4}{5} \land \cos 60° = \frac{1}{2}$

$\Rightarrow x = 37° \lor x = 60°$

En el IVC: $\cos 323° = \frac{4}{5} \land \cos 300° = \frac{1}{2}$

$\Rightarrow x = 323° \lor x = 300°$

Ordenando las soluciones positivas tenemos:

$x \in \{37°; 60°; 300°; 323°; ...\}$

Por lo tanto, la tercera solución positiva es 300°.

Clave D

27. $\tan^2x - \tan x = 0$

$\tan x(\tan x - 1) = 0$

$\Rightarrow \tan x = 0 \lor \tan x = 1$

Si: $\tan x = 0 \Rightarrow VP = \arctan(0) = 0$

$\Rightarrow x = k\pi + VP;\ k \in \mathbb{Z}$

$\Rightarrow x = k\pi + 0 \Rightarrow x \in \{k\pi / k \in \mathbb{Z}\}$ ...(I)

Si: $\tan x = 1 \Rightarrow VP = \arctan(1) = \frac{\pi}{4}$

$\Rightarrow x = k\pi + VP;\ k \in \mathbb{Z}$

$\Rightarrow x \in \left\{k\pi + \frac{\pi}{4} / k \in \mathbb{Z}\right\}$ ...(II)

Luego, la solución de la ecuación será: (I) ∪ (II)

$\Rightarrow x \in \{k\pi\} \cup \left\{k\pi + \frac{\pi}{4}\right\};\ k \in \mathbb{Z}$

Evaluando para obtener las soluciones positivas:

$k = 0 \Rightarrow x = 0 \quad \vee \quad x = \frac{\pi}{4}$

$k = 1 \Rightarrow x = \pi \quad \vee \quad x = \frac{5\pi}{4}$

$k = 2 \Rightarrow x = 2\pi \quad \vee \quad x = \frac{9\pi}{4}$

Ordenando las soluciones positivas tenemos:

$x \in \left\{\frac{\pi}{4}; \pi; \frac{5\pi}{4}; 2\pi; \frac{9\pi}{4}; ...\right\}$

Piden la segunda solución positiva.

$\Rightarrow x = \pi = 180°$

Clave B

28. $\tan x + \cot x = 2$

Observamos que $x \neq \frac{k\pi}{2}$; $k \in \mathbb{Z}$, luego:

$$\tan x + \frac{1}{\tan x} = 2$$

$$\tan^2 x + 1 = 2\tan x$$

$$\tan^2 x - 2\tan x + 1 = 0$$

$$(\tan x - 1)^2 = 0$$

$$\tan x - 1 = 0$$

$$\Rightarrow \tan x = 1$$

Entonces: $VP = \arctan(1) = \frac{\pi}{4}$

Empleando la expresión general para la tangente:

$x_G = k\pi + VP; k \in \mathbb{Z}$

$\Rightarrow x_G = k\pi + \frac{\pi}{4}$

$\Rightarrow x \in \left\{k\pi + \frac{\pi}{4} / k \in \mathbb{Z}\right\}$

Evaluando para obtener las soluciones positivas:

$k = 0 \Rightarrow x = \frac{\pi}{4} = 45°$

$k = 1 \Rightarrow x = \frac{5\pi}{4} = 225°$

$k = 2 \Rightarrow x = \frac{9\pi}{4} = 405°$

Piden la segunda solución positiva:

$\therefore x = 225°$

Clave C

29. Por dato: $x + y = 90°$

Entonces: $\operatorname{sen}x = \cos y$

Además: $(\operatorname{sen}x)^{\cos y} = \sqrt[5]{0,216}$

$$(\operatorname{sen}x)^{(\operatorname{sen}x)} = \sqrt[5]{\frac{27}{125}}$$

$$(\operatorname{sen}x)^{(\operatorname{sen}x)} = \sqrt[5]{\left(\frac{3}{5}\right)^3}$$

$$(\operatorname{sen}x)^{(\operatorname{sen}x)} = \left(\frac{3}{5}\right)^{\left(\frac{3}{5}\right)}$$

Comparando: $\operatorname{sen}x = \frac{3}{5}$

$\Rightarrow x = 37° \wedge y = 53°$

Piden:

$y - x = 53° - 37° = 16°$

$\therefore \quad y - x = 16°$

Clave C

30. $\underbrace{\operatorname{sen}2x + \cos 2x}_{} = \sqrt{2}\operatorname{sen}x$

$\sqrt{2}\operatorname{sen}\left(2x + \frac{\pi}{4}\right) = \sqrt{2}\operatorname{sen}x$

$\Rightarrow \operatorname{sen}\left(2x + \frac{\pi}{4}\right) - \operatorname{sen}x = 0$

Empleando las transformaciones trigonométricas:

$2\operatorname{sen}\left(\frac{x}{2} + \frac{\pi}{8}\right)\cos\left(\frac{3x}{2} + \frac{\pi}{8}\right) = 0$

$\Rightarrow \operatorname{sen}\left(\frac{x}{2} + \frac{\pi}{8}\right) = 0 \vee \cos\left(\frac{3x}{2} + \frac{\pi}{8}\right) = 0$

Analizando en la CT, se obtiene:

Si: $\operatorname{sen}\theta = 0 \Rightarrow \theta = k\pi; k \in \mathbb{Z}$

Si: $\cos\theta = 0 \Rightarrow \theta = (2k + 1)\frac{\pi}{2}; k \in \mathbb{Z}$

Entonces:

$\left(\frac{x}{2} + \frac{\pi}{8}\right) = k\pi \Rightarrow x = \left\{2k\pi - \frac{\pi}{4} / k \in \mathbb{Z}\right\}$

$\left(\frac{3x}{2} + \frac{\pi}{8}\right) = (2k + 1)\frac{\pi}{2} \Rightarrow x = \left\{\frac{2k\pi}{3} + \frac{\pi}{4} / k \in \mathbb{Z}\right\}$

$\Rightarrow x \in \left\{2k\pi - \frac{\pi}{4}\right\} \cup \left\{\frac{2k\pi}{3} + \frac{\pi}{4}\right\}; k \in \mathbb{Z}$

Evaluando:

$k = 0 \Rightarrow x = -\frac{\pi}{4} \quad \vee \quad x = \frac{\pi}{4}$

$k = 1 \Rightarrow x = \frac{7\pi}{4} \quad \vee \quad x = \frac{11\pi}{12}$

Piden la solución principal, que es la menor solución positiva.

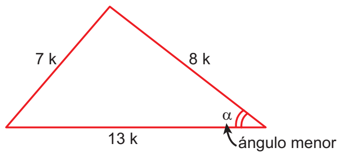
$\therefore x = \frac{\pi}{4} = 45°$

Clave D

APLICAMOS LO APRENDIDO

Nivel 1 (página 91) Unidad 4

1. Sea:



Por ley de cosenos:

$(7k)^2 = (13k)^2 + (8k)^2 - 2(13k)(8k)(\cos\alpha)$

$49k^2 = 169k^2 + 64k^2 - (2)(13k)(8k)\cos\alpha$

$(2)(13k)(8k)\cos\alpha = 184k^2$

$\cos\alpha = \dfrac{23}{26}$

Clave B

2. Por ley de senos tenemos:

$\dfrac{a}{\text{sen}A} = \dfrac{b}{\text{sen}B} = \dfrac{c}{\text{sen}C} = 2R$

$\Rightarrow \left.\begin{array}{l} a = 2R\text{sen}A \\ b = 2R\text{sen}B \\ c = 2R\text{sen}C \end{array}\right\}$ R radio de la circunferencia circunscrita.

Reemplazamos en la igualdad:

$\dfrac{2R\text{sen}A}{\text{sen}A} = \dfrac{2R\text{sen}B}{\cos B} = \dfrac{2R\text{sen}C}{\cos C}$

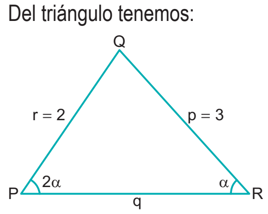
$\Rightarrow 1 = \tan B = \tan C$

$n\angle B = m\angle C = 45°$

$\therefore m\angle A = 90°$

Clave C

3. Del triángulo tenemos:



Por ley de senos tenemos:

$\dfrac{2}{\text{sen}\alpha} = \dfrac{3}{\text{sen}2\alpha} \Rightarrow \dfrac{2}{\text{sen}\alpha} = \dfrac{3}{2\text{sen}\alpha\cos\alpha}$

$\cos\alpha = \dfrac{3}{4}$

Aplicamos ley de cosenos:

$r^2 = p^2 + q^2 - 2pq\cos R$

$(2)^2 = (3)^2 + (q)^2 - 2(3)(q)\cos\alpha$

$4 = 9 + q^2 - 6q\left(\dfrac{3}{4}\right)$

$0 = 2q^2 - 9q + 10$

$2q \quad -5 \Rightarrow 2q - 5 = 0 ;\ q = \dfrac{5}{2}$

$q \quad -2 \Rightarrow q - 2 = 0 ;\ q = 2$

$\therefore q = 2$

Clave C

4. Aplicamos la ley de senos:

$\dfrac{x+1}{\text{sen}74°} = \dfrac{x-1}{\text{sen}37°}$

$\dfrac{x+1}{2\text{sen}37°.\cos 37°} = \dfrac{x-1}{\text{sen}37°}$

$\dfrac{x+1}{2\left(\frac{4}{5}\right)} = x - 1 \Rightarrow 5x + 5 = 8x - 8$

$13 = 3x$

$\therefore x = \dfrac{13}{3}$

Clave D

5. Por ley de senos, tenemos:

$\dfrac{a}{\text{sen}A} = \dfrac{c}{\text{sen}C}$

$\dfrac{a}{c} = \dfrac{\text{sen}A}{\text{sen}C}$

$\dfrac{2}{9} = \dfrac{\text{sen}A}{\text{sen}C} \Rightarrow \dfrac{9}{2} = \dfrac{\text{sen}C}{\text{sen}A} = k$

$\therefore k = \dfrac{9}{2}$

Clave A

6. Por ley de senos tenemos:

$\dfrac{3k}{\text{sen}B} = \dfrac{4k}{\text{sen}A}$

$\dfrac{\text{sen}B}{3} = \dfrac{\text{sen}A}{4} = m \Rightarrow \begin{cases} \text{sen}B = 3m \\ \text{sen}A = 4m \end{cases}$

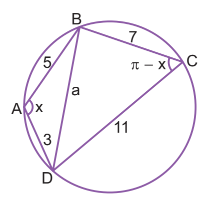
Reemplazamos en M:

$M = \dfrac{\text{sen}A + \text{sen}B}{\text{sen}A - \text{sen}B} = \dfrac{4m + 3m}{4m - 3m} = \dfrac{7m}{m}$

$\therefore M = 7$

Clave C

7.



Por cuadrilátero inscrito: $m\angle C = \pi - x$

En el ΔDAB, por ley de cosenos:

$a^2 = 5^2 + 3^2 - 2(5)(3)\cos x$

$a^2 = 25 + 9 - 30\cos x$

$a^2 = 34 - 30\cos x \qquad ...(I)$

En el ΔBCD, por ley de cosenos:

$a^2 = 7^2 + 11^2 - 2(7)(11)\cos(\pi - x)$

$a^2 = 49 + 121 - 154(-\cos x)$

$a^2 = 170 + 154\cos x \qquad ...(II)$

Igualando (I) y (II):

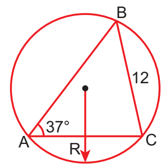
$34 - 30\cos x = 170 + 154\cos x$

$-184\cos x = 136$

$\therefore \cos x = -\dfrac{17}{23}$

Clave A

8. Del enunciado:



De la ley de senos:

$\dfrac{12}{\text{sen}A} = 2R$

$\dfrac{12}{\text{sen}37°} = 2R \Rightarrow \dfrac{12}{\left(\frac{3}{5}\right)} = 2R$

Luego:

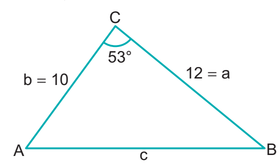
$2R = 5(4)$

$2R = 20$

$\therefore R = 10$

Clave D

9. Del enunciado, tenemos:



Por ley de cosenos:

$c^2 = 12^2 + 10^2 - 2(10)(12)\cos 53°$

$c^2 = 144 + 100 - 240\left(\dfrac{3}{5}\right)$

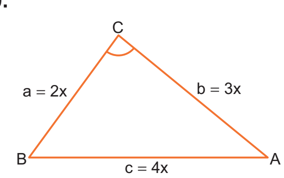
$c^2 = 244 - 144 = 100$

$c = \sqrt{100} = 10$

$\therefore c = 10$

Clave A

10.



De la ley de senos:

$\dfrac{a}{\text{sen}A} = \dfrac{b}{\text{sen}B} = \dfrac{c}{\text{sen}C}$

$\dfrac{2x}{\text{sen}A} = \dfrac{3x}{\text{sen}B} = \dfrac{4x}{\text{sen}C}$

$\dfrac{\text{sen}A}{2} = \dfrac{\text{sen}B}{3} = \dfrac{\text{sen}C}{4} = k$

$\Rightarrow \text{sen}A = 2k;\ \text{sen}B = 3k;\ \text{sen}C = 4k$

Piden:

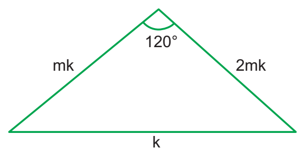
$M = \dfrac{\text{sen}A}{\text{sen}B} + \dfrac{\text{sen}B}{\text{sen}C} + \dfrac{\text{sen}A}{\text{sen}C}$

$M = \dfrac{2k}{3k} + \dfrac{3k}{4k} + \dfrac{2k}{4k}$

$M = \dfrac{2}{3} + \dfrac{3}{4} + \dfrac{2}{4} = \dfrac{23}{12} \quad \therefore M = \dfrac{23}{12}$

Clave C

11. En el triángulo tenemos:



Por ley de cosenos tenemos:

$k^2 = (mk)^2 + (2mk)^2 - 2(mk)(2mk)\cos 120°$

$k^2 = 5m^2k^2 - 4m^2k^2\left(-\frac{1}{2}\right)$

$1 = 5m^2 + 2m^2 \Rightarrow 7m^2 = 1 \Rightarrow m^2 = \frac{1}{7}$

$\therefore m = \sqrt{\frac{1}{7}} = \frac{\sqrt{7}}{7}$

Clave B

12. Reducimos la expresión.

$M = \left[\frac{a}{b} + \frac{(b+c)(b-c)}{ab}\right]\sec C$

$M = \left[\frac{a^2}{ab} + \frac{b^2-c^2}{ab}\right]\sec C = \frac{a^2+b^2-c^2}{ab\cos C}$

Por ley de cosenos tenemos:

$c^2 = a^2 + b^2 - 2ab\cos C$

$2ab\cos C = a^2 + b^2 - c^2$

Reemplazamos en m:

$M = \frac{2ab\cos C}{ab\cos C}$

$\therefore M = 2$

Clave E

13. Por dato:

$\frac{a+b}{a+c} = \frac{c-a}{b}$

$(a+b)b = (c+a)(c-a)$

$ab + b^2 = c^2 - a^2$

$\Rightarrow c^2 = a^2 + b^2 + ab \quad \text{... (I)}$

De la ley de cosenos:

$c^2 = a^2 + b^2 - 2ab\,.\,\cos C \quad \text{...(II)}$

Reemplazando (I) en (II):

$a^2 + b^2 + ab = a^2 + b^2 - 2ab\,.\,\cos C$

$ab = -2ab\cos C$

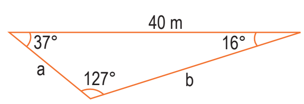
$1 = -2\cos C$

$\Rightarrow \cos C = -\frac{1}{2}$

$\therefore m\angle C = 120°$

Clave E

14.



De la ley senos:

$\frac{a}{\text{sen}16°} = \frac{40}{\text{sen}127°} = \frac{b}{\text{sen}37°}$

$\frac{a}{\frac{7}{25}} = \frac{40}{\frac{4}{5}} = \frac{b}{\frac{3}{5}}$

$\Rightarrow a = 14 \quad \wedge \quad b = 30$

Nos piden:

$a + b + 40 = 84$ m

Clave E

PRACTIQUEMOS

Nivel 1 (página 93) Unidad 4

Comunicación matemática

1. a) Ley de senos
b) Ley de cosenos o ley de proyecciones
c) Ley de cosenos
d) Ley de senos
e) Ley de senos

2.

I) R: circunradio
$\Rightarrow x = 2R\,\text{sen}A$

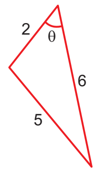
II) Ley de cosenos:
$x^2 = c^2 + b^2 - 2bc\cos A$
$x = \sqrt{c^2 + b^2 - 2bc\cos A}$

III) R: circunradio; A = 90°
$\Rightarrow x = 2R\,\text{sen}A$
$x = 2R\,\text{sen}90°$
$\therefore x = 2R$

IV) Ley de senos:
$\Rightarrow \frac{x}{\text{sen}B} = \frac{a}{\text{sen}A}$
$\therefore x = a\frac{\text{sen}B}{\text{sen}A}$

Razonamiento y demostración

3. Piden: $\cos\theta$



Por ley de cosenos:

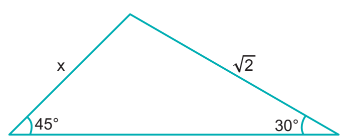
$5^2 = 2^2 + 6^2 - 2(2)(6)\cos\theta$

$\Rightarrow 24\cos\theta = 15$

$\therefore \cos\theta = \frac{5}{8}$

Clave C

4.



Por ley de senos:

$\frac{x}{\text{sen}30°} = \frac{\sqrt{2}}{\text{sen}45°} \Rightarrow \frac{x}{\left(\frac{1}{2}\right)} = \frac{\sqrt{2}}{\left(\frac{\sqrt{2}}{2}\right)}$

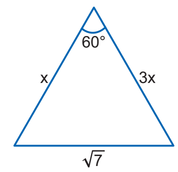
Resolviendo, tenemos:

$2x = 2$

$\therefore x = 1$

Clave A

5.



Por ley de cosenos:

$(\sqrt{7})^2 = x^2 + (3x)^2 - 2(x)(3x)\cos 60°$

$7 = x^2 + 9x^2 - 6x^2\left(\frac{1}{2}\right)$

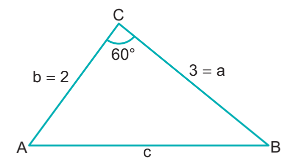
$7 = 7x^2$

$\Rightarrow x^2 = 1$

$\therefore x = 1$

Clave A

6. Por dato:



En el $\triangle ABC$, por ley de cosenos:

$c^2 = 2^2 + 3^2 - 2(2)(3)\cos 60°$

$c^2 = 4 + 9 - 12\left(\frac{1}{2}\right)$

$c^2 = 7$

$\therefore c = \sqrt{7}$

Clave E

7. Piden:

$N = a\,\text{sen}B - b\,\text{sen}A$

Por ley de senos:

$a = 2R\,\text{sen}A; \qquad b = 2R\,\text{sen}B$

Entonces:

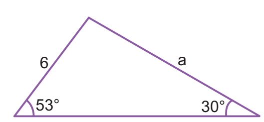
$N = (2R\,\text{sen}A)\text{sen}B - (2R\,\text{sen}B)\text{sen}A$

$N = 2R\,\text{sen}A\,\text{sen}B - 2R\,\text{sen}A\,\text{sen}B$

$\therefore N = 0$

Clave C

8.



Por ley de senos:

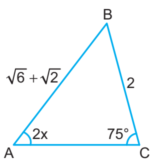
$\frac{a}{\text{sen}53°} = \frac{6}{\text{sen}30°} \Rightarrow \frac{a}{\left(\frac{4}{5}\right)} = \frac{6}{\left(\frac{1}{2}\right)}$

$\Rightarrow \frac{5a}{4} = 12 \Rightarrow 5a = 48$

$\therefore a = \frac{48}{5}$

Clave B

9.



Del gráfico: BC < AB

Entonces por correspondencia triangular se cumple:

$2x < 75° \Rightarrow x < 37{,}5°$

En el $\triangle ABC$ por la ley de senos:

$$\frac{2}{\text{sen}2x} = \frac{\sqrt{6}+\sqrt{2}}{\text{sen}75°}$$

$$\frac{2}{\text{sen}2x} = \frac{\sqrt{6}+\sqrt{2}}{\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)}$$

$$\frac{2}{\text{sen}2x} = 4 \Rightarrow \text{sen}2x = \frac{1}{2} \qquad ...(I)$$

De (I):

$2x = 30° \quad \vee \quad 2x = 150°$

$x = 15° \quad \vee \quad x = 75°$

Como $x < 37{,}5°$; entonces: $x = 15°$

Piden:

$\tan x = \tan 15°$

$\tan x = \csc 30° - \cot 30°$

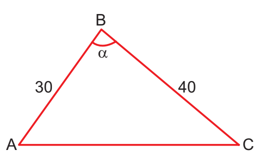
$\Rightarrow \tan x = (2) - (\sqrt{3})$

$\therefore \tan x = 2 - \sqrt{3}$

Clave E

Resolución de problemas

10. Tenemos:



Por dato:

$\log(\text{sen}\alpha) = -0{,}30103$

$\text{sen}\alpha = 10^{-0{,}30103}$

$\text{sen}\alpha = (10^{0{,}30103})^{-1}$

$\text{sen}\alpha = (2)^{-1} \Rightarrow \text{sen}\alpha = \frac{1}{2}$

Piden $A_{\triangle ABC}$:

$A_{\triangle ABC} = 30(40)\text{sen}\alpha$

$A_{\triangle ABC} = 30(40)\left(\frac{1}{2}\right)$

$\therefore A_{\triangle ABC} = 600\ \text{cm}^2$

Clave D

11. En Q tenemos:

$$Q = m(\cos\alpha \,.\, \cos N + \text{sen}\alpha \,.\, \text{sen}N) + n(\cos\alpha \,.\, \cos M - \text{sen}\alpha \,.\, \text{sen}M)$$

$$Q = \cos\alpha\underbrace{(m\cos N + n\cos M)}_{\text{Ley de proyecciones}} + \text{sen}\alpha\underbrace{(m\,\text{sen}N - n\,\text{sen}M)}_{\text{Ley de senos}}$$

$Q = \cos\alpha(p) + \text{sen}\alpha(0)$

$\therefore Q = p\cos\alpha$

Clave D

Nivel 2 (página 94) Unidad 4

Comunicación matemática

12. Por ley de proyecciones:

$a = b\cos C + c\cos B$ (V)

$b = a\cos C + c\cos B$ (F)

Por ley de senos, tenemos:

$0 = a\,\text{sen}B - b\,\text{sen}C$ (F)

$0 = b\,\text{sen}C - c\,\text{sen}B$ (V)

Por ley de cosenos:

$2ac\cos B = b^2 - a^2 - c^2$ (F)

Clave B

13. Tomamos los datos de I:

$x^2 = AB^2 + AC^2 - 2(AB)(AC)\cos A$

$x^2 = 5^2 + 3^2 - 2(5)(3)\left(-\frac{4}{5}\right)$

$x^2 = 25 + 9 + 24 \Rightarrow x = \sqrt{58}$

Tomamos los datos de II:

(Ley de senos)

$$\frac{x}{\text{sen}D} = \frac{BD}{\text{sen}C}$$

$$\frac{x}{\text{sen}37°} = \frac{\frac{5}{3}\sqrt{29}}{\text{sen}45°}$$

$$\frac{x}{\frac{3}{5}} = \frac{\frac{5}{3}\sqrt{29}}{\frac{\sqrt{2}}{2}}$$

$$x = \frac{3}{5} \times \frac{5}{3} \times \sqrt{29} \times \frac{2}{\sqrt{2}}$$

$x = \sqrt{29} \times \sqrt{2}$

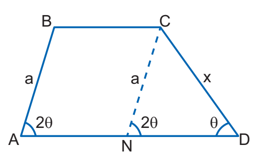
$x = \sqrt{58}$

$\therefore$ I o II

Clave D

Razonamiento y demostración

14. Por dato: ABCD es un trapecio.



Trazamos $\overline{CN}$ // $\overline{BA}$, entonces se forma el paralelogramo ABCN.

Luego en el $\triangle NCD$ por ley de senos:

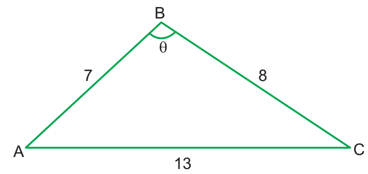
$$\frac{x}{\text{sen}2\theta} = \frac{a}{\text{sen}\theta} \Rightarrow x = \frac{a\,\text{sen}2\theta}{\text{sen}\theta}$$

$$\Rightarrow x = \frac{a(2\,\text{sen}\theta\cos\theta)}{\text{sen}\theta} = 2a\cos\theta$$

$\therefore x = 2a\cos\theta$

Clave B

15.



Por ley de cosenos:

$13^2 = 7^2 + 8^2 - 2(7)(8)\cos\theta$

$169 = 49 + 64 - 112\cos\theta$

$\Rightarrow 112\cos\theta = -56$

$\cos\theta = -\frac{56}{112} = -\frac{1}{2}$

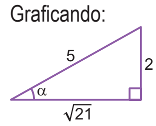
$\cos\theta = -\frac{1}{2}$

$\therefore \theta = 120°$

Clave E

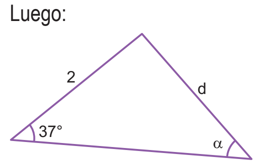
16. $\cos\alpha = \frac{\sqrt{21}}{5}$

Graficando:



$\Rightarrow \text{sen}\alpha = \frac{2}{5}$

Luego:



Por ley de senos:

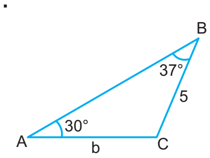
$$\frac{d}{\text{sen}37°} = \frac{2}{\text{sen}\alpha}$$

$$d = \frac{2\,\text{sen}37°}{\text{sen}\alpha} = \frac{2\left(\frac{3}{5}\right)}{\frac{2}{5}} = \frac{30}{10} = 3$$

$\therefore d = 3$

Clave C

17.



Por ley de senos:

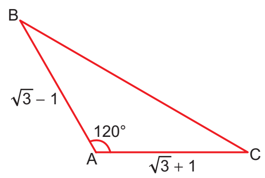
$$\frac{b}{\text{sen}37°} = \frac{5}{\text{sen}30°}$$

$$b = \frac{5\,\text{sen}37°}{\text{sen}30°} = \frac{5\left(\frac{3}{5}\right)}{\left(\frac{1}{2}\right)}$$

$\therefore b = 6$

Clave C

18.



Por la ley de cosenos:

$$(BC)^2 = (\sqrt{3}-1)^2 + (\sqrt{3}+1)^2 - 2(\sqrt{3}-1)(\sqrt{3}+1)\cos 120°$$

$$(BC)^2 = 4 - 2\sqrt{3} + 4 + 2\sqrt{3} - 2(3-1)\left(-\frac{1}{2}\right)$$

$$(BC)^2 = 8 + 2 = 10$$

$\therefore BC = \sqrt{10}$

Clave D

19. Por dato:

$$\frac{\text{sen}A}{2} = \frac{\text{sen}B}{3} = \frac{\text{sen}C}{4} \qquad ...(I)$$

Por ley de senos:

$2R\text{sen}A = a \Rightarrow \text{sen}A = \dfrac{a}{2R}$

$2R\text{sen}B = b \Rightarrow \text{sen}B = \dfrac{b}{2R}$

$2R\text{sen}C = c \Rightarrow \text{sen}C = \dfrac{c}{2R}$

Reemplazando en (I):

$$\frac{\left(\frac{a}{2R}\right)}{2} = \frac{\left(\frac{b}{2R}\right)}{3} = \frac{\left(\frac{c}{2R}\right)}{4}$$

$$\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = k$$

$\Rightarrow a = 2k;\ b = 3k;\ c = 4k$

Piden:

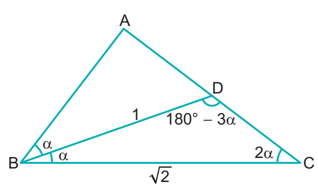
$$J = \frac{b^2+c^2}{b^2-a^2} = \frac{(3k)^2+(4k)^2}{(3k)^2-(2k)^2}$$

$$J = \frac{9k^2+16k^2}{9k^2-4k^2} = \frac{25k^2}{5k^2}$$

$\therefore J = 5$

Clave D

20. Del enunciado:



En el ΔBDC por la ley de senos:

$$\frac{\sqrt{2}}{\text{sen}(180°-3\alpha)} = \frac{1}{\text{sen}2\alpha} \Rightarrow \frac{\sqrt{2}}{\text{sen}3\alpha} = \frac{1}{\text{sen}2\alpha}$$

$$\Rightarrow \frac{\text{sen}3\alpha}{\text{sen}2\alpha} = \sqrt{2}$$

Empleando identidades trigonométricas se tiene:

$$\frac{\text{sen}\alpha(2\cos 2\alpha + 1)}{2\text{sen}\alpha\cos\alpha} = \sqrt{2}$$

$$2\cos 2\alpha + 1 = 2\sqrt{2}\cos\alpha$$

$$2(2\cos^2\alpha - 1) + 1 = 2\sqrt{2}\cos\alpha$$

$$4\cos^2\alpha - 1 = 2\sqrt{2}\cos\alpha$$

$$\Rightarrow 4\cos^2\alpha - 2\sqrt{2}\cos\alpha - 1 = 0$$

$$\cos\alpha = \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(4)(-1)}}{2(4)}$$

$$\cos\alpha = \frac{2\sqrt{2} \pm \sqrt{24}}{8} = \frac{2\sqrt{2} \pm 2\sqrt{6}}{8}$$

$$\Rightarrow \cos\alpha = \frac{\sqrt{2}+\sqrt{6}}{4} \ \vee\ \cos\alpha = \frac{\sqrt{2}-\sqrt{6}}{4}$$

Resolviendo:

$\alpha = 15° \qquad \vee \qquad \alpha = 105°$

Como el ΔABC es isósceles:

$2\alpha < 90° \Rightarrow \alpha < 45°$

$\Rightarrow \alpha = 15°$

Piden: $m\angle A$ y $m\angle B$

$m\angle A = 180° - 4\alpha = 180° - 4(15°)$

$\Rightarrow m\angle A = 120°$

$m\angle B = 2\alpha = 2(15°)$

$\Rightarrow m\angle B = 30°$

Clave D

21. En un triángulo ABC:

$$\frac{a}{\cos A} + \frac{b}{\cos B} + \frac{c}{\cos C} = R$$

De la ley de senos:

$a = 2R\text{sen}A;\ b = 2R\text{sen}B;\ c = 2R\text{sen}C$

Reemplazando en la expresión:

$$\frac{2R\text{sen}A}{\cos A} + \frac{2R\text{sen}B}{\cos B} + \frac{2R\text{sen}C}{\cos C} = R$$

$$2(\tan A + \tan B + \tan C) = 1$$

$$\Rightarrow \tan A + \tan B + \tan C = \frac{1}{2}$$

Como: $A + B + C = \pi$

Se cumple:

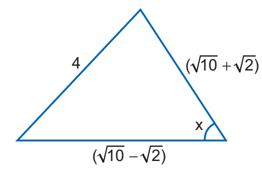
$$\underbrace{\tan A\tan B\tan C}_{E} = \underbrace{\tan A + \tan B + \tan C}_{\frac{1}{2}}$$

$\therefore E = \dfrac{1}{2}$

Clave C

Resolución de problemas

22. Graficamos el triángulo:



Por ley de cosenos, tenemos:

$$(4)^2 = (\sqrt{10}+\sqrt{2})^2 + (\sqrt{10}-\sqrt{2})^2 - 2(\sqrt{10}+\sqrt{2})(\sqrt{10}-\sqrt{2})\cos x$$

$$16 = 10 + 2\sqrt{20} + 2 + 10 - 2\sqrt{20} + 2 - 2(10-2)\cos x$$

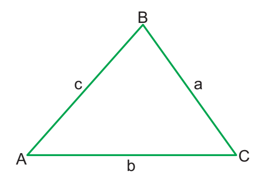
$$16 = 24 - 2(8)\cos x$$

$$\cos x = \frac{8}{2(8)} \Rightarrow \cos x = \frac{1}{2}$$

$\therefore x = 60°$

Clave E

23.



Perímetro: $2p = a + b + c$

Sabemos:

$$\text{sen}\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} \wedge \cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}}$$

Por ley de cosenos:

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

Reemplazamos en:

$$\text{sen}\left(\frac{A}{2}\right) = \sqrt{\frac{1-\frac{b^2+c^2-a^2}{2bc}}{2}}$$

$$\text{sen}\left(\frac{A}{2}\right) = \sqrt{\frac{2bc-b^2-c^2+a^2}{4bc}}$$

$$\text{sen}\left(\frac{A}{2}\right) = \sqrt{\frac{a^2-(b-c)^2}{4bc}}$$

$$\text{sen}\left(\frac{A}{2}\right) = \sqrt{\frac{(a+c-b)(a+b-c)}{4bc}}$$

$$\text{sen}\left(\frac{A}{2}\right) = \sqrt{\frac{(2p-b-b)(2p-c-c)}{4bc}}$$

$$\text{sen}\left(\frac{A}{2}\right) = \sqrt{\frac{(p-b)(p-c)}{bc}}$$

$$\cos\left(\frac{A}{2}\right)=\sqrt{\frac{1+\frac{b^2+c^2-a^2}{2bc}}{2}}$$

$$\cos\left(\frac{A}{2}\right)=\sqrt{\frac{2bc+b^2+c^2-a^2}{4bc}}$$

$$\cos\left(\frac{A}{2}\right)=\sqrt{\frac{(b+c)^2-a^2}{4bc}}$$

$$\cos\left(\frac{A}{2}\right)=\sqrt{\frac{(b+c+a+)(b+c-a)}{4bc}}$$

$$\cos\left(\frac{A}{2}\right)=\sqrt{\frac{(2p)(2p-a-a)}{4bc}}$$

$$\cos\left(\frac{A}{2}\right)=\sqrt{\frac{(p)(p-a)}{bc}}$$

$$\tan\left(\frac{A}{2}\right)=\frac{\text{sen}\left(\frac{A}{2}\right)}{\cos\left(\frac{A}{2}\right)}$$

$$\tan\left(\frac{A}{2}\right)=\sqrt{\frac{\frac{(p-b)(p-c)}{bc}}{\frac{p(p-a)}{bc}}}$$

$$\tan\left(\frac{A}{2}\right)=\sqrt{\frac{(p-b)(p-c)}{p(p-a)}}$$

$$\therefore\ \text{sen}\left(\frac{A}{2}\right)=\sqrt{\frac{(p-b)(p-c)}{bc}}$$

$$\cos\left(\frac{A}{2}\right)=\sqrt{\frac{(p)(p-a)}{bc}}$$

$$\tan\left(\frac{A}{2}\right)=\sqrt{\frac{(p-b)(p-c)}{p(p-a)}}$$

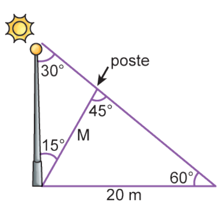
Clave E

Nivel 3 (página 95) Unidad 4

Comunicación matemática

24.

En M:



Por ley de senos:

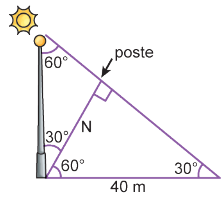
$$\frac{20}{\text{sen}45°}=\frac{M}{\text{sen}60°}$$

$$20\left(\frac{\sqrt{3}}{2}\right)\left(\frac{2}{\sqrt{2}}\right)=M$$

$\therefore M=(1{,}22)(20\text{ m})$

$M=24{,}5\text{ m}$

En N:



Por ley de senos:

$$\frac{N}{\text{sen}30°}=\frac{40\text{ m}}{\text{sen}90°}$$

$$N=\frac{1}{2}(40\text{ m})$$

$\therefore N=20\text{ m}\qquad M>N$

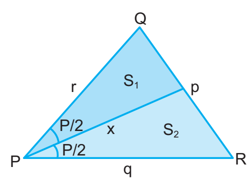
Clave A

25. Por teorema de tangentes:

I. $$\frac{q-r}{q+r}=\frac{\tan\left(\frac{P-Q}{2}\right)}{\tan\left(\frac{P+Q}{2}\right)}\qquad\text{(F)}$$

II. $$\frac{p+q}{p-q}=\frac{\tan\left(\frac{P+Q}{2}\right)}{\tan\left(\frac{P-Q}{2}\right)}\qquad\text{(V)}$$

III.



$$S_{PQR}=S_1+S_2$$

$$\frac{rq}{2}\text{sen}(P)=\frac{rx}{2}\text{sen}\left(\frac{P}{2}\right)+\frac{qx}{2}\text{sen}\left(\frac{P}{2}\right)$$

$$\frac{rq}{2}2\text{sen}\left(\frac{P}{2}\right)\cos\left(\frac{P}{2}\right)=\frac{rx}{2}\text{sen}\left(\frac{P}{2}\right)+\frac{qx}{2}\text{sen}\left(\frac{P}{2}\right)$$

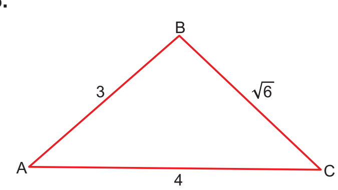
$$2rq\cos\left(\frac{P}{2}\right)=rx+qx$$

$$2rq\cos\left(\frac{P}{2}\right)=x(r+q)$$

$$\therefore x=\frac{2rq}{(r+q)}\cos\left(\frac{P}{2}\right)\qquad\text{(V)}$$

Razonamiento y demostración

26.



Por la ley de cosenos:

$$(\sqrt{6})^2=3^2+4^2-2(3)(4)\cos A$$

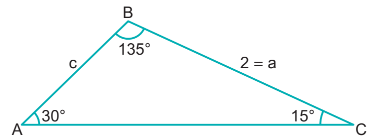
$$6=9+16-24\cos A$$

$$24\cos A=19$$

$$\therefore \cos A=\frac{19}{24}$$

Clave D

27. Por dato:



Se cumple:

$$m\angle A+m\angle B+m\angle C=180°$$

$$30°+135°+m\angle C=180°$$

$$m\angle C=15°$$

Luego, por ley de senos:

$$\frac{c}{\text{sen}15°}=\frac{2}{\text{sen}30°}\Rightarrow\frac{c}{\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)}=\frac{2}{\left(\frac{1}{2}\right)}$$

$$\Rightarrow c=4\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$$

$$\therefore c=\sqrt{6}-\sqrt{2}$$

Clave A

28. Piden:

$$N=\frac{a\cos B+b\cos A}{b\cos C+c\cos B}$$

Por la ley de proyecciones:

$c=a\cos B+b\cos A$

$a=b\cos C+c\cos B$

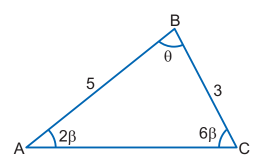
Reemplazando en la expresión N:

$$N=\frac{a\cos B+b\cos A}{b\cos C+c\cos B}=\frac{c}{a}$$

$$\therefore N=\frac{c}{a}$$

Clave C

29.



Por ley de senos:

$$\frac{3}{\text{sen}2\beta}=\frac{5}{\text{sen}6\beta}\Rightarrow\frac{\text{sen}6\beta}{\text{sen}2\beta}=\frac{5}{3}$$

Por identidad del ángulo triple:

$$\frac{\text{sen}2\beta(2\cos4\beta+1)}{\text{sen}2\beta}=\frac{5}{3}$$

$$2\cos4\beta=\frac{5}{3}-1$$

$$\cos4\beta=\frac{1}{3}$$

En el $\triangle ABC$, se cumple:

$2\beta + \theta + 6\beta = 180°$

$\Rightarrow \theta = 180° - 8\beta$

Piden:

$1 - \cos\theta = 1 - \cos(180° - 8\beta)$

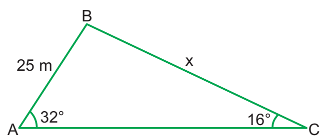
$1 - \cos\theta = 1 - (-\cos 8\beta) = 1 + \cos 8\beta$

$\Rightarrow 1 - \cos\theta = 2\cos^2 4\beta = 2\left(\frac{1}{3}\right)^2$

$\therefore 1 - \cos\theta = \frac{2}{9}$

Clave E

30. Del enunciado:



Por ley de senos:

$$\frac{x}{\text{sen}32°} = \frac{25}{\text{sen}16°} \Rightarrow x = \frac{25\text{sen}32°}{\text{sen}16°}$$

$$\Rightarrow x = \frac{25(2\text{sen}16°\cos 16°)}{\text{sen}16°} = 50\cos 16°$$

Sabemos: $\cos 16° = \frac{24}{25}$

$$\Rightarrow x = 50\left(\frac{24}{25}\right) = 48$$

$\therefore x = 48$ m

Clave C

31. Por dato: $A + B + C = 180°$

Además:

$$\frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C}$$

Empleando ley de senos:

$$\frac{(2R\text{sen}A)}{\cos A} = \frac{(2R\text{sen}B)}{\cos B} = \frac{(2R\text{sen}C)}{\cos C}$$

$\Rightarrow \tan A = \tan B = \tan C$

Entre A y B:

$A = B \;\vee\; A = 180° + B$

Pero: $0° < A < 180° \Rightarrow A = B$

Entre B y C:

$B = C \;\vee\; B = 180° + C$

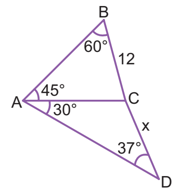
Pero: $0° < B < 180° \Rightarrow B = C$

Entonces se deduce: $A = B = C$

Por lo tanto, como los tres ángulos internos son iguales, entonces el triángulo es equilátero.

Clave D

32.



En el triángulo ABC por ley de senos:

$$\frac{AC}{\text{sen}60°} = \frac{12}{\text{sen}45°} \Rightarrow \frac{AC}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{12}{\left(\frac{\sqrt{2}}{2}\right)}$$

$\Rightarrow AC = 6\sqrt{6}$

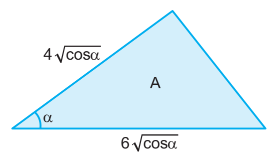
En el triángulo ACD por ley de senos:

$$\frac{AC}{\text{sen}37°} = \frac{x}{\text{sen}30°} \Rightarrow \frac{(6\sqrt{6})}{\left(\frac{3}{5}\right)} = \frac{x}{\left(\frac{1}{2}\right)}$$

$\therefore x = 5\sqrt{6}$

Clave A

33.



Piden: el área del triángulo (A).

$$A = \frac{(4\sqrt{\cos\alpha})(6\sqrt{\cos\alpha})}{2}\text{sen}\alpha$$

$$A = \frac{(24\cos\alpha)}{2}\text{sen}\alpha = (12\cos\alpha)\text{sen}\alpha$$

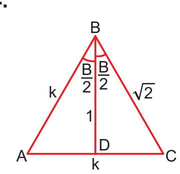
$\Rightarrow A = 6(2\text{sen}\alpha\cos\alpha) = 6(\text{sen}2\alpha)$

$\therefore A = 6\text{sen}2\alpha$

Clave B

Resolución de problemas

34.



En $\triangle ABC$:

$k^2 = k^2 + (\sqrt{2})^2 - 2(k)(\sqrt{2})\cos B$

$2k\sqrt{2}\cos B = 2$

$$\cos B = \frac{1}{k\sqrt{2}} \qquad \text{...(1)}$$

$$BD = \frac{2(AC)(BC)}{(AB + BC)}\cos\frac{B}{2}$$

$$1 = \frac{2(k)\sqrt{2}}{(k + \sqrt{2})}\cos\frac{B}{2}$$

$$\frac{k + \sqrt{2}}{2(k)(\sqrt{2})} = \cos\frac{B}{2} \qquad \text{... (2)}$$

Sabemos:

$$\cos\frac{B}{2} = \sqrt{\frac{1 + \cos B}{2}} \qquad \text{... (3)}$$

Reemplazamos (1) y (2) en (3):

$$\frac{k + \sqrt{2}}{2(k)(\sqrt{2})} = \sqrt{\frac{1 + \frac{1}{k\sqrt{2}}}{2}}$$

$$\frac{(k + \sqrt{2})^2}{(2k(\sqrt{2}))^2} = \frac{k\sqrt{2} + 1}{2k(\sqrt{2})}$$

$k^2 + 2 + 2k\sqrt{2} = 2k\sqrt{2}(k\sqrt{2} + 1)$

$k^2 + 2 + 2k\sqrt{2} = 4k^2 + 2k\sqrt{2}$

$2 = 3k^2 \Rightarrow k = \sqrt{\frac{2}{3}}$

Reemplazando en (1):

$$\cos B = \frac{1}{k\sqrt{2}} = \frac{1}{\sqrt{\frac{2}{3}} \times \sqrt{2}}$$

$$\cos B = \frac{\sqrt{3}}{2}$$

$\cos 3B = 4\cos^3 B - 3\cos B$

$$\cos 3B = 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3\left(\frac{\sqrt{3}}{2}\right)$$

$$\cos 3B = 4\frac{3\sqrt{3}}{8} - 3\frac{\sqrt{3}}{2}$$

$\cos 3B = 0$

Clave C

35. Sabemos:

$A + B + C = 180°$

$$\frac{A}{2} = 90° - \left(\frac{B + C}{2}\right)$$

$$\tan\left(\frac{A}{2}\right) = \tan\left[90° - \left(\frac{B + C}{2}\right)\right]$$

$$\tan\left(\frac{A}{2}\right) = \cot\left(\frac{B + C}{2}\right)$$

Reemplazamos en la condición:

$$\tan\frac{A}{2}\tan\left(\frac{B - C}{2}\right) = \frac{1}{4}$$

$$\cot\left(\frac{B + C}{2}\right)\tan\left(\frac{B - C}{2}\right) = \frac{1}{4}$$

$$\frac{\tan\left(\frac{B - C}{2}\right)}{\tan\left(\frac{B + C}{2}\right)} = \frac{1}{4}$$

Por teorema de tangentes:

$$\frac{\tan\left(\frac{B+C}{2}\right)}{\tan\left(\frac{B-C}{2}\right)} = \frac{b+c}{b-c} = 4$$

$b + c = 4b - 4c$

$5c = 3b \Rightarrow \frac{b}{c} = \frac{5}{3}$

Por la ley de senos:

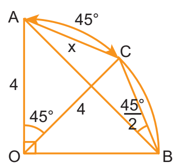
$$\frac{b}{\text{sen}B} = \frac{c}{\text{sen}C}$$

$$\frac{b}{c} = \frac{\text{sen}B}{\text{sen}C} = \frac{5}{3}$$

Clave A

MARATÓN MATEMÁTICA (página 96)

1. Por teorema de cosenos:



$x^2 = 4^2 + 4^2 - 2(4)(4)\cos 45°$

$x^2 = 32 - 32\,\frac{\sqrt{2}}{2}$

$x^2 = 16(2 - \sqrt{2})$

$\therefore\ x = 4\sqrt{2 - \sqrt{2}}$

Clave D

2. Se cumple: $M + N + P = 180°$

$$\frac{N}{2} = 90° - \left(\frac{M+P}{2}\right)$$

$$5\cot\left(\frac{N}{2}\right) = 17\tan\left(\frac{M-P}{2}\right)$$

$$5\cot\left(90° - \left(\frac{M+P}{2}\right)\right) = 17\tan\left(\frac{M-P}{2}\right)$$

$$\frac{\tan\left(\frac{M+P}{2}\right)}{\tan\left(\frac{M-P}{2}\right)} = \frac{17}{5}$$

$$\frac{m+p}{m-p} = \frac{17}{5} \Rightarrow 5m + 5p = 17m - 17p$$

$22p = 12m$

$33 = 12m$

$\therefore\ m = 11/4$

Clave A

3. $A = \arctan 1/8 + \arctan 1/5$

$$A = \arctan\left(\frac{\frac{1}{8}+\frac{1}{5}}{1-\frac{1}{8}\times\frac{1}{5}}\right) = \arctan\left(\frac{\frac{13}{40}}{\frac{39}{40}}\right)$$

$\therefore A = \arctan(1/3)$

Clave B

4. $R = \arccos[\text{sen}(-\pi/5)]$

$R = \pi/2 - \text{arcsen}[-\text{sen}\ (\pi/5)]$

$R = \pi/2 - [-\text{arcsen}[\text{sen}(\pi/5)]]$

$R = \pi/2 + \pi/5 = 7\pi/10$

Clave E

5. Para $x = 0 \Rightarrow y = 2$

$f(0) = 2 = A\cos(B(0))$

$2 = A\cos 0° \Rightarrow A = 2$

Para $x = \pi/4 \Rightarrow y = 0$

$f(\pi/4) = 2\cos(B(\pi/4)) = 0$

$\cos(B\pi/4) = 0 \Rightarrow B = 2$

Luego $A - B = 2 - 2 = 0$

Clave C

6. La función tendrá la forma:

$f(x) = \tan(Ax + B) + C$

$-\pi/6 < Ax + 5\pi/6$; $C = -1/2 \wedge B = \pi/3$

$-\pi/6 < Ax + \pi/3 < 5\pi/6$

$-\pi/2 < Ax < \pi/2 \Rightarrow A = 1$

Clave D

7. $\cos(x/4) = 0$

$x/4 = (2n + 1)\pi/2;\ n \in \mathbb{Z}$

$x/4 = n\pi + \pi/2$

$x = 4n\pi + 2\pi$

Clave B

8. Ley de senos:

$$\frac{AB}{\text{sen}30°} = \frac{BC}{\text{sen}53°} \Rightarrow BC = 5$$

$$A_S = \frac{\overline{BC}(\overline{DE})}{2} = \frac{5(6)}{2} = 15$$

$\therefore\ A_S = 15$

Clave A

9. Factorizamos:

$$\cos x = \text{sen}3x + \text{sen}x = 2\text{sen}\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)$$

$\cos x = 2\text{sen}2x\ .\ \cos x$

$(2\text{sen}2x - 1)\cos x = 0$

$\Rightarrow\ 2\text{sen}2x - 1 = 0$

$\text{sen}2x = 1/2$

Entonces:

$2x = n\pi + (-1)^n\ \pi/6;\ n \in \mathbb{Z}$

$x = n\pi/2 + (-1)^n\ \pi/12\ ,\ n \in \mathbb{Z}$

Para $n = 0$:

$x = 0 + (1)\ \pi/12 = \pi/12$

$\therefore\ x = \pi/12$

Clave B